EXPONENT AND RADICALS

EXPONENTS:

- Is the repeated product of real number by itself

e.g. i) $2 \times 2 \times 2 \times 2 = 2^4$

ii) $6 \times 6 \times 6 \times 6 \times 6 = 6^5$

iii) $a \times a \times a \times a \times a = a^5$

LAWS OF EXPONENTS

MULTIPLICATION RULE

Suppose;

$4 \times 4 \times 4 = 4^3$

Then, 4^3 = power

4 = base

3 = exponent

Suppose, $3^2 \times 3^4 = 3^{(2+4)} = 3^6$

$3^2 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Generally when powers with the same base are multiplied,the exponents are added $a^m \times a^n = a^{m+n}$

Example 1

Simplify the following

i) $6^4 \times 6^8 \times 6^6 \times 6^1$

ii) $y^4 \times y^0 \times y^3$

Solution:

i) $6^4 \times 6^8 \times 6^6 \times 6^1 = 6^{4+8+6+1}$

$= 6^{19}$

ii) $y^4 \times y^0 \times y^3$

Solution:

$Y^4 \times y^0 \times y^3 = y^{4+0+3}$

$= y^7$

Example 2

Simplify the following

i) $3^2 \times 5^4 \times 3^3 \times 5^2$

ii) $a^3 \times b^3 \times b^4 \times a^5 \times b^2$

Solution:

i) $3^2 \times 5^4 \times 3^3 \times 5^2 = 3^{2+3} \times 5^{4+2}$

$= 3^5 \times 5^6$

ii) $a^3 \times b^3 \times b^4 \times a^5 \times b^2 = a^{3+5} \times b^3$

$= a^8 \times b^9$

Example 3

If 2^Y x 16 x 8^Y = 256, find y

Solution:

2^y x 2^4 x 8^y = 256

2^y x 2^4 x 8^y = 2^8

2^y x 2^4 x $(2^3)^y$ = 2^8

y + 4 + 3y = 8

y + 3y = 8 – 4

4y = 4

Y = 1

Exercise 1:

1. Simplify

i) 3^4 x 4^3 x 3^8 x 3^4 x 4^2 = 3^{4+8+4} x 4^{3+2} = 3^{16} x 4^5

ii) a^2 x a^3 x a^4 x b^2 x b^3 = a^{2+3+4} x b^{2+3} = a^9 x b^5

2. If 125^m x 25^2 = 5^{10} find m

Solution:

125^m x 25^2 = 5^{10}

5^{3m} x 5^4 = 5^{10}

3m + 4 = 10

3m = 10 – 4

3m=6

m = 2

3. If $x^7 = 2187$. Find x

Solution:

$X^7 = 2187$

$X^7 = 3^7$

X = 3

QUOTIENT LAW

$\frac{3^4}{3^2} = \frac{3 \times 3 \times 3 \times 3}{3 \times 3} = 3 \times 3$

$= 3^2$

Also $\frac{3^4}{3^2} = 3^{4-2} = 3^2$

Generally:

when power is divided by another power of the same base subtract the ex

$\frac{a^m}{a^n} = a^{m-n}$ where: $a \neq 0$

Example 1.

Find i) $\frac{8^7}{8^5} = 8^{7-5}$

$= 8^2$

ii) $\frac{5^{2n}}{5^n} = 5^{2n-n}$

$= 5^n$

Example 2.

If $\frac{27^n}{3^4} = 81$ find n

Solution:

$\frac{27^n}{3^4} = 81$

$(\frac{3^{3n}}{3^4}) =$
3^4

$3^{3n - 4} = 3^4$

Equate the exponents

$3n - 4 = 4$

$n = {}^{8}/_{3}$

NEGATIVE EXPONENTS

Suppose $\frac{3^2}{3^4} = 3^{2-4} = 3^{-2}$

Also $\frac{3^2}{3^4} = \frac{3 \times 3}{3 \times 3 \times 3 \times 3}$

$= \frac{1}{3^2}$

Generally For any none-zero number, X: $\frac{1}{x^n} = X^{-1}$

and Inversely $x^n = \frac{1}{x^{-n}}$

Example

Find

(i) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(ii) $9^{-1/2} = \frac{1}{3}$

(iii) $\frac{1}{3^{-3}} = 3^3 = 27$

EXERCISE 2

1. Given $2^{3n} \times 16 \times 8^n = 4096$ find n

2. Given $\frac{625 \times 5^y}{125^2} = 5^6$ find y

3. If $3^{2n+1} - 5 = 76$ find n

4. Given $2^y = 0.0625$.Find y

5. Simplify $\left(\frac{a^5 \times b^5}{a^4 \times b^2}\right)^3$

6. Find the value of x

(i). $81^{-1/2} = x$

ii) $2^{-x} = 8$

ZERO EXPONENTS

Suppose,

$\frac{3^4}{3^4} = \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} =$

1

Also $\frac{3^4}{3^4} = 3^{4-4} = 3$

$3^0 = 1$

Generally For any non-zero number x, $x^0 = 1$

Example

Show that $9^0 = 1$

Consider $\frac{9^2}{9^2} = \frac{9 \times 9}{9 \times 9} = \frac{81}{81} =$

1

Also $\frac{9^2}{9^2} = 9^{2-2} = 9^0$

$9^0 = 1$ hence shown

Also

(i) $\left(\frac{x}{y}\right)^m = \frac{x^m}{x^m}$

(ii) $(x \times y)^m = x^m \times y^m$

Example

(1)Find

i) $(5 \text{ x } 4)^2$

Solution:

$(5 \text{ x } 4)^2 = 5^2 \text{ x } 4^2$

5 x 5 x 4 x 4 = 400

ii) $(^2/_3)^3$

$^{2^3}/_{3^3} = \frac{2 \text{ x } 2 \text{ x } 2}{3 \text{ x } 3 \text{ x } 3} = \frac{8}{27}$

2. Show that $2^{-1} = {}^1/_2$

Solution:

$2^{-1} = {}^1/_2$

${}^1/_2 = {}^1/_2$

consider LHS

$2^{-1} = {}^{1}/_{2}$

L H S = R H S

Therefore

$2^{-1} = {}^{1}/_{2}$ hence shown

FRACTIONAL EXPONENTS AND EXPONENTS OF POWERS

EXPONENTS OF POWERS

Consider $(5^4)^3 = (5x5x5x5)^3$

$= (5x5x5x5)x(5x5x5x5)x(5x5x5x5)$

$= 5x5x5x5x5x5x5x5x5x5x5x5$

$= 5^{12}$

Similarly $(5^4)^3 = 5^{4x3}$

Generally:- $(X^m)^n = \underbrace{(x \times x \times x \times x \times \dots\dots\dots\dots . \times x)^m}_{\text{n times}}$

$= X^{nm}$

When you take an Exponent of power, multiply the exponents

$(X^n)^m = x^{nxm}$

Examples:

1.Simplify (a $(x^4)^5$

(b) $(8^6)^3$

Solution

(a) $(x^4)^5 = x^{4x5}$

$= x^{20}$

(b) $(8^6)^3 = 8^{6x3}$

$= 8^{18}$

2.Write 23x 42 as a power of single number

Solution

2^3x 4^2 ,but $4=2^2$

therefore 42=(22)2

42=22x2

=24

23x 24=23+4

∴23x 24=27

FRACTIONAL EXPONENT

Since $2^0=1$ and $2^{-1} = \frac{1}{2}$

Find $2^{1/2}$

Solution

Consider the exponents of powers when $2^{1/2}$ is squared, we get

$\left(2^{1/2}\right)^2 = 2^{1/2 \times 2}$

$= 2^1$

$= 2$

Then $2^{1/2}$ is the square root of 2.

$$2^{1/2} = \sqrt{2}$$

Similarly

$\left(2^{1/3}\right)^3 = 2^{1/3 \times 3}$

$= 2^1$

$= 2$

Hence $2^{1/3} = \sqrt[3]{2}$

Let x be positive number and let n be a natural number. Then

$\left(x^{1/n}\right)^n = X^{1/n \times n}$

$= x^1$

$= x$

Hence $x^{1/n} = \sqrt[n]{X}$, the n^th^ root of X

Generally

If x is a positive number, then $X^{1/n} = \sqrt[n]{X}$

Examples:

(1) Find $36^{1/2}$

Solution

$36^{1/2} = \sqrt{36}$

$= \sqrt{6x6}$

$36^{1/2} = 6$

(2) $8^{1/3} = \sqrt[3]{\frac{1}{8}}$

$$= \frac{\sqrt[3]{1}}{\sqrt[3]{8}}$$
$$= \frac{1}{\sqrt[3]{2\times 2\times 2}}$$
$$= \frac{1}{2}$$

(3) $(-8)^{1/3}$

<u>Solution</u>

$(-8)^{1/3} = \sqrt[3]{8}$

$(-8)^{1/3} = \sqrt[3]{-2 \times -2 \times -2}$

$\therefore (-8)^{1/3}$ =-2

Thus if x is a negative number, and n is an odd number

Then $X^{1/n} = \sqrt[n]{X}$

Exercise 2.

1. Show that 2^{-2} = $^1/_4$

Solution:

Consider LHS

$2^{-2} = {}^1/_{2^2} = {}^1/_4$

$2^{-2} = {}^1/_4$

LHS = RHS hence shown

2. Evaluate

$27^{2/3}$ x $729^{1/3}$ ÷ 243

Solution:

$27^{2/3}$ x $729^{1/3}$ ÷ 243

$(3^3)^{2/3}$ x $(3^6)^{1/3}$ ÷ 3^5

3^2 x 3^2 ÷ 3^5

3^{2+2-5}

= 3^{-1} or $^1/_3$

3. Find the value of m

$(1/9)^{2m}$ x $(1/3)^{-m}$ ÷ $(1/27)^2$ = $(1/3)^{-3m}$

Solution:

$(1/3^2)^{2m}$ x $1/3^{-m}$ ÷ $(1/3^3)^2$ = $1/3^{-3m}$

$(1/3)^{4m}$ x $(1/3)^{-m}$ ÷ $(1/3)^6$ = $(1/3)^{3m}$

3^{-4m} x 3^{-m} ÷ 3^{-6} = 3^{-3m}

-4m + -m – 6 = -3m

-5m – 6 = -3m

$6 = -2m$

$m = -3$

4. Given $2^x \times 3^y = 5184$ find x and y

Solution:

$2^x = 5184$ $2^x \times 3^y = 2^6 \times 3^y$

$2^x = 2^6$ By comparison

$2^x = 2^6$ $2^x = 2^6$

$X = 6$

$3^y = 5184$ $3^x = 3^4$

$3^y = 3^4$

$y = 4$

The value of x and y is 6 and 4 respectively

RADICALS

-A number involving roots is called a surd or radical.

-Radical is a symbol used to indicate the square root, cube root or n^{th} root of a number.

-The symbol of a radical is $\sqrt{\ }$

.Example of Radicals

(i) $\sqrt{9}$

(ii) $\sqrt[3]{8}$

(iii) $\sqrt[4]{81}$

PRIME FACTORS

Example 1

Find (i) $\sqrt{196}$ by prime factorization

Solution:

$\sqrt{196} = \sqrt{2 \, x \, 2 \, x \, 7 \, x \, 7}$

= 2x7

= 14

ii) $\sqrt[3]{216}$ by prime factorization

solution:

$\sqrt[3]{216} = \sqrt[3]{2 \text{ x } 2 \text{ x } 2 \text{ x } 3 \text{ x } 3 \text{ x } 3}$

= 2 x 3

= 6

iii) $\sqrt{20}$ by prime factorization

solution:

$\sqrt{20} = \sqrt{2 \, x \, 2 \, x \, 5}$

= 2 $\sqrt{5}$

Example 2

If $\sqrt[3]{8} = 8^x$ find x

Solution:

$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 8^x$

$= (2^3)^{1/3} = 2^{3x}$

$= 2^1 = 2^{3x}$

$x = {}^{1}/_{3}$

Generally ; $\sqrt[n]{x} = x^{1/n}$

Exercise 3

1. Find the following

i) $\sqrt{1024}$

Solution

$\sqrt{1024} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= 2 \times 2 \times 2 \times 2 \times 2$

$= 32$

$\sqrt{1024} = 32$

ii) $\sqrt[3]{125}$

Solution

$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5}$

$= 5$

2. Simplify

a) $\sqrt[3]{250}$ Solution

$\sqrt[3]{250} = \sqrt[3]{2 \times 5 \times 5 \times 5}$

$= 5\sqrt[3]{2}$

b) $\sqrt{675} = \sqrt{3 \times 3 \times 3 \times 5 \times 5}$

$= 3 \times 5\sqrt{3}$

$= 15\sqrt{3}$

3. Find $\sqrt[3]{64} = 16^{y}$ find y

$\sqrt[3]{64} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2^{4y}$

$2^{2} = 2^{4y}$

$2 = 4y$

$y = {}^{1}/_{2}$

4. Find x if

$\sqrt[x]{343} = 49^{1/3}$

Solution

$\sqrt[x]{343} = \sqrt[x]{7 \times 7 \times 7} = 49^{1/3}$

$343^{1/x} = 7^{3/x} = (7^2)^{1/3}$

$7^{3/x} = 7^{2/3}$

$3/x = 2/3$

$2x = 9$

$x = 2/9$

ii) $\sqrt[4]{6561} = 81^x$

solution

$\sqrt[4]{6561} = \sqrt[4]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} = 81^x$

$= 3^2 = 3^{4x}$

$= 2 = 4x$

$x = 1/2$

OPERATION ON RADICAL

ADDITION

Example1.

Evaluate

i) $\sqrt{3}+3\sqrt{3}$

Solution: $\sqrt{3}+ 3\sqrt{3} =(1 + 3)\sqrt{3}$

$=4\sqrt{3}$

ii) $\sqrt{108}+\sqrt{48}$

Solution

$=\sqrt{2 \text{ x } 2 \text{ x } 3 \text{ x } 3 \text{ x } 3}+\sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3}$

$(2^2)^{1/2}(3^2)^{1/2}\sqrt{3}+ (2^2)^{1/2}(2^2)^{1/2}\sqrt{3}$

$= (2 \text{ x } 3)\sqrt{3}+ (2 \text{ x } 2)\sqrt{3}$

$= 6\sqrt{3}+ 4\sqrt{3}$

$= 10\sqrt{3}$

Generally $\sqrt{a}+\sqrt{b} = \sqrt{a}+\sqrt{b}$

$\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$

SUBTRACTION

Example

Evaluate

i) 3 $\sqrt{72}$- 2 $\sqrt{32}$

Solution

$= 3\sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 3 \text{ x } 3}$ n-2$\sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 2}$

= (3 x 2 x 3 $\sqrt{2} -$ 2 x 2 x 2 $\sqrt{2}$)

= 18 $\sqrt{2} -$ 8 $\sqrt{2}$

= 10 $\sqrt{2}$

ii) $\sqrt{108} - \sqrt{48}$

Solution

$\sqrt{108} - \sqrt{48} = \sqrt{2 \text{ x } 2 \text{ x } 3 \text{ x } 3 \text{ x } 3} - \sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3}$

=(2 x 3) $\sqrt{3} -$ (2 x 2) $\sqrt{3}$

= 6 $\sqrt{3} -$ 4 $\sqrt{3}$

= 2 $\sqrt{3}$

Generally $\sqrt{a}$ -$\sqrt{b}$ =$\sqrt{a}$ -$\sqrt{b}$

$\sqrt{a}$ -$\sqrt{b} \neq \sqrt{a-b}$

MULTIPLICATION

Example

Find i) $\sqrt{12}$ x $\sqrt{48}$

solution

$\sqrt{12}$ x $\sqrt{48}$ = $\sqrt{12 \text{ x } 48}$

= $\sqrt{576}$

= $\sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3 \text{ x } 3}$

= 2 x 2 x 2 x 3

= 24

ii) 3 $\sqrt{50}$ x 3 $\sqrt{18}$

Solution

3 $\sqrt{2 \, x \, 5 \, x \, 5}$ x 3 $\sqrt{2 \, x \, 3 \, x \, 3}$

(5 x 3) $\sqrt{2}$ x (3 x 3) $\sqrt{2}$

= 15 $\sqrt{2}$ x 9 $\sqrt{2}$

= 135 $\sqrt{2}$

Generally:

$\sqrt{a} \times \sqrt{b} = \sqrt{a \, x \, b} = \sqrt{ab}$

DIVISION

Example 1

Find i) $\frac{\sqrt{72}}{\sqrt{50}}$

Solution: $\frac{\sqrt{72}}{\sqrt{50}} = \sqrt{72/50}$

$= \sqrt{(2 \; x \; 2 \; x \; 2 \; x \; 3 \; x \; 3)/(2 \; x \; 5 \; x \; 5)}$

$= \sqrt{36/25}$

$= {}^{6}/_{5}$

Generally: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

EXERCISE 4.

1. Find $2\sqrt{108} + 3\sqrt{48}$

Solution: $2\sqrt{2 \text{ x } 2 \text{ x } 3 \text{ x } 3 \text{ x } 3} + 3\sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3}$

$= (2 \text{ x } 2 \text{ x } 3)\sqrt{3} + (3 \text{ x } 2 \text{ x } 2)\sqrt{3}$

$= 12\sqrt{3} + 12\sqrt{3}$

$= 24\sqrt{3}$

(ii)$3(\sqrt{12} + \sqrt{48})$

Solution:

$3(\sqrt{12} + \sqrt{48}) = 3\sqrt{12} + 3\sqrt{48}$

$= 3\sqrt{2 \text{ x } 2 \text{ x } 3} + 3\sqrt{2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3}$

$=(3 \text{ x } 2)\sqrt{3} + (3 \text{ x } 2 \text{ x } 3)\sqrt{3}$

$= 6\sqrt{3} + 12\sqrt{3}$

$= 18\sqrt{3}$

(iii) $6\sqrt{28} - 2\sqrt{63}$

Solution:

$6\sqrt{28} - 2\sqrt{63}\, 6 = \sqrt{2 \text{ x } 2 \text{ x } 7} - 2\sqrt{3 \text{ x } 3 \text{ x } 7}$

$= (6 \text{ x } 2)\sqrt{7} - (2 \text{ x } 3)\sqrt{7}$

$= 12\sqrt{7} - 6\sqrt{7}$

$= 6\sqrt{7}$

iv) $\sqrt{x + y} + \sqrt{9x + 9y}$

Solution:

$\sqrt{x + y} + \sqrt{9(x + y)}$

$\sqrt{x + y} + 3\sqrt{x + y}$

$4\sqrt{x + y}$

(v) $\sqrt{40}$ + 2250

Solution:

$\sqrt{40} + 2250 = \sqrt{2 \times 2 \times 2 \times 5}$ +2250

= $2\sqrt{2 \times 5}$ + 2250

=$2\sqrt{10}$ + 2250

=$2\sqrt{10}$+ 2250

2. Simplify

(i) $\sqrt{32} \times \sqrt{18}$

= $\sqrt{576}$

= $\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}$

= $\sqrt{2 \times 2 \times 2 \times 2}$

= 24

ii) $\sqrt{2}\,(\sqrt{72} - \sqrt{32})$

$\sqrt{2}\,(\sqrt{2 \times 2 \times 2 \times 3 \times 3} - \sqrt{2 \times 2 \times 2 \times 2 \times 2})$

= $\sqrt{2}(2 \times 3\sqrt{2} - 4\sqrt{2})$

$= \sqrt{2}(6\sqrt{2} - 4\sqrt{2})$

$= \sqrt{2}(2\sqrt{2})$

= 4

(iii) $3\sqrt{36} \times 2\sqrt{24}$

Solution:

$= 3\sqrt{2 \times 2 \times 3 \times 3} \times 2\sqrt{2 \times 2 \times 2 \times 3}$

$= 3 \times 2 \times 3 \times (2 \times 2)\sqrt{2 \times 3}$

$= 18 \times 4\sqrt{6}$

$= 72\sqrt{6}$

(iv) $\sqrt{3}\ (15\sqrt{3})$

Solution:

$\sqrt{3}\ (15\sqrt{3}) = 15\sqrt{3 \times 3}$

= 15 X 3

= 45

RATIONALIZATION OF THE DENOMINATOR

- Rationalizing the denominator involves the multiplication of the denominator by a suitable radical resulting in a rational denominator.

The best choice can follow the following rules:-

(i) If a radical is a single term(that is does not involve + or -),the proper choice is the radical itself,that is

For $a\sqrt{x}$, Choose $a\sqrt{x}$

So $(a\sqrt{x})(a\sqrt{x}) = a^2\sqrt{x \times x}$

$a^2 x$

(ii)If the radical involves operations(+ or -),choose a radical with the same format but with one term with the opposite operation.

Examples

RADICALS	CHOICE	SINCE
(i) $\sqrt{2}$	$\sqrt{2}$	$(\sqrt{2})(\sqrt{2})=2$
(ii) $\sqrt{3}-1$	$\sqrt{3}+1$	$(\sqrt{3}-1)(\sqrt{3}+1)=(\sqrt{3})(\sqrt{3})-(1)1$ $=3-1$ $=2$
(iii) $\sqrt{5}+\sqrt{2}$	$\sqrt{5}-\sqrt{2}$	$(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})$ $=(\sqrt{5})(\sqrt{5})-(\sqrt{2})(\sqrt{5})+(\sqrt{2})(\sqrt{5})-(\sqrt{2})(\sqrt{2})$ $=5-2$ $=3$
(iv) $4\sqrt{3}-2\sqrt{5}$	$4\sqrt{3}-2\sqrt{5}$	$(4\sqrt{3}-2\sqrt{5})(4\sqrt{3}+2\sqrt{5})$ $=(4\sqrt{3})(4\sqrt{3})+(4\sqrt{3})(2\sqrt{5})-(4\sqrt{3})(2\sqrt{5})-(2\sqrt{5})(2\sqrt{5})$ $=16(3)+8\sqrt{15}-8\sqrt{15}-(4)(5)$ $=48-20$ $=28$

The same technique can be used to rationalize the denominator.

Example 1

Rationalize i) $\frac{7}{\sqrt{6}}$

Solution $\frac{7}{\sqrt{6}} = \frac{7}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{7\sqrt{6}}{\sqrt{6}}$

(ii) $\frac{\sqrt{2}}{\sqrt{3}}$

Solution:

$\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$ x $\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{6}}{\sqrt{9}}$

$= \frac{\sqrt{6}}{3}$

(iii) $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}}$

Solution:

$\frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}}$ x $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$= \frac{\sqrt{15}+\sqrt{6}}{\sqrt{5}(\sqrt{5}+\sqrt{2}) - \sqrt{2}(\sqrt{5}+\sqrt{2})}$

$= \frac{\sqrt{15}+\sqrt{6}}{(\sqrt{25}+\sqrt{10}) - (\sqrt{10}+\sqrt{4})}$

$= \frac{\sqrt{15}+\sqrt{6}}{5-2}$

$= \frac{\sqrt{15}+\sqrt{6}}{3}$

Example 2:

Rationalize (i) $\frac{3\sqrt{3}-2}{4\sqrt{3}-2\sqrt{2}}$

Solution:

$$\frac{3\sqrt{3}-2}{4\sqrt{3}-2\sqrt{2}} = \frac{3\sqrt{3}-2}{4\sqrt{3}-2\sqrt{2}} \times \frac{4\sqrt{3}+2\sqrt{2}}{4\sqrt{3}+2\sqrt{2}}$$

$$= \frac{3\sqrt{3}(4\sqrt{3}+2\sqrt{2})-2(4\sqrt{3}+2\sqrt{2})}{4\sqrt{3}(4\sqrt{3}+2\sqrt{2})-2\sqrt{2}(4\sqrt{3}+2\sqrt{2})}$$

$$= \frac{3 \times 4 \times 3 + 3\times2\sqrt{6} - 8\sqrt{3} - 4\sqrt{2})}{4 \times 4 \times 3 + 8\sqrt{6} - 8\sqrt{6} - 2 \times 2 \times 2}$$

$$= \frac{36 + 6\sqrt{6} - 8\sqrt{3} - 4\sqrt{2}}{48 - 8}$$

$$= \frac{36 + 6\sqrt{6} - 8\sqrt{3} - 4\sqrt{2}}{40}$$

$$= \frac{2(18 + 3\sqrt{6} - 4\sqrt{3} - 2\sqrt{2})}{40}$$

$$= \frac{18 + 3\sqrt{6} - 4\sqrt{3} - 2\sqrt{2}}{20}$$

(ii) Rationalize $\frac{3\sqrt{5}-\sqrt{3}}{3\sqrt{5}-3\sqrt{2}}$

Solution:

$$\frac{3\sqrt{5}-\sqrt{3}}{3\sqrt{5}-3\sqrt{2}} = \frac{3\sqrt{5}-\sqrt{3}}{3\sqrt{5}-3\sqrt{2}} \times \frac{3\sqrt{5}+3\sqrt{2}}{3\sqrt{5}+3\sqrt{2}}$$

$$= \frac{3\sqrt{5}(3\sqrt{5}+3\sqrt{2}) - \sqrt{3}(3\sqrt{5}+3\sqrt{2})}{3\sqrt{5}(3\sqrt{5}+3\sqrt{2}) - 3\sqrt{2}(3\sqrt{5}+3\sqrt{2})}$$

$$= \frac{9\sqrt{25} + 9\sqrt{10} - 3\sqrt{15} - 3\sqrt{6}}{9\sqrt{25} + 9\sqrt{10} - 9\sqrt{10} - 9\sqrt{4}}$$

$$= \frac{9 \times 5 + 9\sqrt{10} - 3\sqrt{15} - 3\sqrt{6}}{9 \times 5 + 9 \times 2}$$

$$= \frac{45 + 9\sqrt{10} - 3\sqrt{15} - 3\sqrt{6}}{45 - 18}$$

$$= \frac{45 + 9\sqrt{10} - 3\sqrt{15} - 3\sqrt{6}}{27}$$

$$= \frac{3(15 + 3\sqrt{10} - \sqrt{15} - \sqrt{6})}{27}$$

$$= \frac{15 + 3\sqrt{10} - \sqrt{15} - \sqrt{6}}{9}$$

EXERCISE 5

1. Evaluate

(i) $(2\sqrt{3} - 4)(3\sqrt{5} - 3\sqrt{2})$

Solution:

(1) $(2\sqrt{3} - 4)(3\sqrt{5} - 3\sqrt{2}) = (2\sqrt{3}(3\sqrt{5} - 3\sqrt{2}) - 4(3\sqrt{5} - 3\sqrt{2})$

$= 6\sqrt{15} - 6\sqrt{6} - 12\sqrt{5} + 12\sqrt{2}$

(ii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b})$

Solution:

(iii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b}) = \sqrt{a}(\sqrt{a} + \sqrt{b}) + \sqrt{b}(\sqrt{a} + \sqrt{b})$

$= a + \sqrt{ab} + \sqrt{ab} + b$

$= a + b + 2\sqrt{ab}$

(iv) $(\sqrt{m} + \sqrt{n})(\sqrt{m} - \sqrt{n})$

Solution:

$$(\sqrt{m} + \sqrt{n})(\sqrt{m} - \sqrt{n}) = \sqrt{m}(\sqrt{m} + \sqrt{n}) + \sqrt{n}(\sqrt{m} - \sqrt{n})$$

$$= m + \sqrt{mn} - \sqrt{mn} - n$$

$$= m - n$$

(v) $(\sqrt{m} + \sqrt{n})(\sqrt{m} - \sqrt{n})$

Solution:

$$(\sqrt{p} - \sqrt{q})(\sqrt{p} + \sqrt{q}) = \sqrt{p}(\sqrt{p} + \sqrt{q} - \sqrt{p}(\sqrt{p} + \sqrt{q})$$

$$= p - \sqrt{pq} + \sqrt{pq} - q$$

$$= p - q$$

2. Rationalize

(i) $\frac{4\sqrt{7} - 3\sqrt{5}}{3\sqrt{3} + 2\sqrt{2}}$

Solution:

$$\frac{4\sqrt{7}-3\sqrt{5}}{3\sqrt{3}+2\sqrt{2}} = \frac{4\sqrt{7}-3\sqrt{5}}{3\sqrt{3}+2\sqrt{2}} \text{ x } \frac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$$

$$= \frac{4\sqrt{7}(3\sqrt{3}-2\sqrt{2})-3\sqrt{5}(3\sqrt{3}-2\sqrt{2})}{3\sqrt{3}\left(3\sqrt{3}-2\sqrt{2}\right)+2\sqrt{2}(3\sqrt{3}-2\sqrt{2})}$$

$$= \frac{12\sqrt{21}-8\sqrt{14}-9\sqrt{15}+6\sqrt{10})}{9\sqrt{9}-6\sqrt{6}+6\sqrt{6}-4\sqrt{4}}$$

$$= \frac{12\sqrt{21}-8\sqrt{14}-9\sqrt{15}+6\sqrt{10}}{9 \text{ x } 3-4 \text{ x } 2}$$

$$= \frac{12\sqrt{21}-8\sqrt{14}-9\sqrt{15}+6\sqrt{10}}{19}$$

$$= \frac{2(18+3\sqrt{6}-4\sqrt{3}-2\sqrt{2})}{40}$$

(ii) $\frac{2+2\sqrt{5}}{\sqrt{7}-3}$

Solution:

$$\frac{2+2\sqrt{5}}{\sqrt{7}-3} = \frac{(2+2\sqrt{5})(\sqrt{7}+3)}{(\sqrt{7}-3)(\sqrt{7}+3)}$$

$$= \frac{2\sqrt{7}+6+2\sqrt{35}+6\sqrt{5}}{\sqrt{49}-3\sqrt{7}-3\sqrt{7}-9}$$

$$= \frac{2\sqrt{7}+6+2\sqrt{35}+6\sqrt{5}}{-2}$$

$$= -\left(\frac{2\sqrt{7}+6+2\sqrt{35}+6\sqrt{5})}{2}\right)$$

EXERCISE 6

Rationalize the following denominator

(i) $\frac{\sqrt{7}+2\sqrt{2}}{8-2\sqrt{3}}$

Solution:

$$\frac{\sqrt{7}+2\sqrt{2}}{8-2\sqrt{3}} = \frac{(\sqrt{7}+2\sqrt{2})(8+2\sqrt{3})}{(8-2\sqrt{3})(8+2\sqrt{3})}$$

$$= \frac{\sqrt{7}(8+2\sqrt{3})+ 2\sqrt{2}(8+2\sqrt{3})}{8(8+2\sqrt{3}) - 2\sqrt{3}(8+2\sqrt{3})}$$

$$= \frac{8\sqrt{7}+2\sqrt{21}+16\sqrt{2}+4\sqrt{6}}{64+16\sqrt{3}-16\sqrt{3}-4\sqrt{9}}$$

$$= \frac{8\sqrt{7}+2\sqrt{21}+16\sqrt{2}+4\sqrt{6}}{64-12}$$

$$= \frac{8\sqrt{7}+2\sqrt{21}+16\sqrt{2}+4\sqrt{6}}{52}$$

(ii) $\frac{8\sqrt{7}-9}{3\sqrt{5}-2\sqrt{7}}$

Solution:

$$\frac{8\sqrt{7}-9}{3\sqrt{5}-2\sqrt{7}} = \frac{(8\sqrt{7}-9)(3\sqrt{5}+2\sqrt{7})}{(3\sqrt{5}-2\sqrt{7})(3\sqrt{5}+2\sqrt{7})}$$

$$= \frac{8\sqrt{7}\left(3\sqrt{5}+2\sqrt{7}\right) - 9(3\sqrt{5}+2\sqrt{7})}{3\sqrt{5}(3\sqrt{5}+2\sqrt{7}) - 2\sqrt{7}(3\sqrt{5}+2\sqrt{7})}$$

$$= \frac{24\sqrt{35} + 16\sqrt{49} - 27\sqrt{5} - 18\sqrt{7}}{9\sqrt{25} + 6\sqrt{35} - 6\sqrt{35} - 4\sqrt{49}}$$

$$= \frac{24\sqrt{35} + 112 - 27\sqrt{5} - 18\sqrt{7}}{45 - 28}$$

$$= \frac{24\sqrt{35} + 112 - 27\sqrt{5} - 18\sqrt{7}}{17}$$

(iii) $\frac{6\sqrt{6}+5\sqrt{3}}{3\sqrt{5}+2\sqrt{3}}$

Solution:

$$\frac{6\sqrt{6}-5\sqrt{3}}{3\sqrt{5}+2\sqrt{3}} = \frac{(6\sqrt{6}+5\sqrt{3})(3\sqrt{5}-2\sqrt{3})}{(3\sqrt{5}+2\sqrt{3})(3\sqrt{5}-2\sqrt{3})}$$

$$= \frac{6\sqrt{6}\left(3\sqrt{5}-2\sqrt{3}\right) - 5\sqrt{3}(3\sqrt{5}-2\sqrt{3})}{3\sqrt{5}\left(3\sqrt{5}-2\sqrt{3}\right) + 2\sqrt{3}(3\sqrt{5}-2\sqrt{3})}$$

$$= \frac{18\sqrt{30} - 12\sqrt{18} - 15\sqrt{15} + 10\sqrt{9}}{9\sqrt{25} - 6\sqrt{15} + 6\sqrt{15} - 4\sqrt{9}}$$

$$= \frac{18\sqrt{30} - 12\sqrt{18} - 15\sqrt{15} + 10\sqrt{9}}{45 - 12}$$

$$= \frac{18\sqrt{30} - 12\sqrt{18} - 15\sqrt{15} + 10\sqrt{9}}{33}$$

(iv) $\frac{a\sqrt{m} - b}{b\sqrt{n} + a}$

Solution:

$$\frac{a\sqrt{m} - b}{b\sqrt{n} + a} = \frac{(a\sqrt{m} - b)(b\sqrt{n} - a)}{(b\sqrt{n} + a)(b\sqrt{n} - a)}$$

$$= \frac{a\sqrt{m}\left(b\sqrt{n} - a\right) - b(b\sqrt{n} - a)}{b\sqrt{n}\left(b\sqrt{n} - a\right) + a(b\sqrt{n} - a)}$$

$$= \frac{ab\sqrt{mn} - a^2\sqrt{n} - b^2\sqrt{n} + ba}{b^2\sqrt{n^2} - ba\sqrt{n} + ba\sqrt{n} - a^2}$$

$$= \frac{ab\sqrt{mn} - a^2\sqrt{n} - b^2\sqrt{n} + ab}{b^2 n - a^2}$$

SQUARE ROOT OF A NUMBER

Example

Find(i) $\sqrt{6561}$

Solution

```
        8   1
     ____________
8   | 65, 61     |____
    | -64
161 |   161
    |  -161
```

$\sqrt{6561} = 81$

ii) $\sqrt{724201}$

Solution:

	8 5 1
8	72,42,01
	-64
165	842
	-825
1701	1701
	-1701

$\sqrt{724201} = 851$

(iii) $\sqrt{54007801}$

Solution:

	7 3 4 9
7	54, 00, 78, 01
	-49
143	500
	-429
1464	7178
	-5856
14689	132201
	-132201

$\sqrt{724201} = 851$

TRANSPOSITION OF FORMULA

A formula expresses a rule which can be used to calculate one quantity where others are given,when one of the given quantity is expressed in terms of the other quantity the process is called transposition of formula.

Example 1

The following are examples of a formula

a. A = $^{1}/_{b}$

b. v = $\pi r^2 h$

c. I = $\frac{PRT}{100}$

d. A = $^{1}/_{2}$ (a +b)h

e. T = $2^{\pi}r\sqrt{l/g}$

Example 2

The simple interest (I) on the principal (p) for time (T) years. Calculated at the rate of R% per annual is given by formula

I = $\frac{PRT}{100}$

Make T the subject of a formula

Solution:

$100 \times I = \frac{PRT}{100} \times$
100

$\frac{100}{PR} = \frac{PRT}{PR}$

$\frac{100}{PR} = T$

$T = \frac{100}{PR}$

Example 3.

Given that

$Y = mx + c$, make m the subject

Solution:

$Y = mx + c$

$\frac{Y-c}{x} = \frac{mx}{x}$

$m = \frac{Y-c}{x}$

Example 4

Given that $p = w\frac{(1+a)}{1-a}$

Make a the subject.

Solution:

$P = w\frac{(1+a)}{1-a}$

Divide by w both sides

$$\frac{p}{w} = \frac{w}{w}\frac{(1+a)}{1-a}$$

$$\frac{p}{w} = \frac{(1+a)}{1-a}$$

Multiply by (1 – a) both sides

$$\frac{p}{w}(1-a) = (1-a)\frac{(1+a)}{1-a}$$

$$\frac{p}{w}(1-a) = 1 + a$$

$$\frac{p}{w} - \frac{pa}{w} = 1 + a$$

$$\frac{p}{w} - 1 = a + \frac{pa}{w}$$

$$\frac{p}{w} - 1 = a(1 + \frac{p}{w})$$

Divide by $1 + \frac{p}{w}$ both sides

$$\frac{\frac{p}{w} - 1}{1 + \frac{p}{w}} = \frac{a(1 + \frac{p}{w})}{1 + \frac{p}{w}}$$

$$a = \frac{\frac{p}{w} - 1}{1 + \frac{p}{w}}$$

Alternatively

$P=w\left(\frac{1+a}{1-a}\right)$

Multiply by (1-a) both side

$P(1-a)=w\left(\frac{1+a}{1-a}\right) \times (1-a)$

p-pa=w(1+a)

P − pa=w+wa

P − w = pa+wa

P − w = a(p + w)

$a = \frac{p-w}{p+w}$

Example 5

Given that $T = 2^{\pi}\sqrt{l/g}$ write g in terms of other letters

Solution:

$$T = 2^{\pi}\sqrt{l/g}$$

Divide by 2^{π} both side

$$\frac{T}{2\pi} = \frac{2\pi}{2\pi}\sqrt{l/g}$$

Remove the radical by squares both sides

$\left(\frac{T}{2\pi}\right)_2 = \left(\sqrt{l/g}\right)_2$

$\frac{T^2}{4\pi} = l/g$

Multiply by g both sides

$\frac{T^2 g}{4\pi^2} = (l/g)_g$

$l = \frac{T^2 g}{4\pi^2}$

Multiply by 4^{π_2} both sides

$4^{\pi_2} \times \frac{T^2 g}{4\pi^2} = l \times 4^{\pi_2}$

$T^2g = 4^{\pi_2}\, l$

Divide by T^2 both sides

$\therefore g = \frac{4\pi 2\ l}{T^2}$

Example 6

If $A = p + \frac{PRT}{100}$

(i) Make R as the subject formula

(ii) Make P as the subject formula

Solution:

(i) $A = p + \frac{PRT}{100}$

$= A - P = \frac{PRT}{100}$

Multiply by 100 both sides

$= \frac{100(A - P)}{PT} = R$

$R = \frac{100(A - P)}{PT}$

(ii) $A = P + \frac{PRT}{100}$

Solution:

Multiply by 100 both sides

100A = 100P + PRT

100A = P(100 + RT)

Divide by 100 + RT both sides

$\frac{100A}{100 + RT} = P$

$$P = \frac{100A}{100 + RT}$$

Exercise 7

1. If S = $^1/_2$ at^2. Make t the subject of the formula

2. If c = $^5/_9$ (F – 32) make F the subject of the formula

Solution:

$S = {^1/_2}at^2$

Multiply by 2 both sides

$s \times 2 = {^1/_2}at^2 \times 2$

$2s = at^2$

Divide by a both sides

$$\frac{2s}{a} = \frac{at^2}{a}$$

$$t^2 = \frac{2s}{a}$$

Square root both sides

$$\sqrt{t^2} = \sqrt{\frac{2s}{a}}$$

$t = \sqrt{\frac{2s}{a}}$

2. $C = {}^{5}/_{9}(F - 32)$

$C = {}^{5}/_{9}F - {}^{160}/_{9}$

$C + {}^{160}/_{9} = {}^{5F}/_{9}$

Multiply by 9 both sides

$9C + 160 = 5F$

Divide by 5 both sides

$F = \frac{9c + 160}{5}$

More Examples

1. If $A = \frac{1}{2}h(a + b)$

(i) Make h the subject formula

(ii) Make b the subject formula

2. If ${}^{1}/_{f} = {}^{1}/_{v} - {}^{1}/_{u}$

(i) Make f the subject formula

(ii) Make u the subject formula

Solution:

1. A = $\frac{1}{2}h(a + b)$

2A = $\frac{1}{2}h$(a + b)x 2

2A = h(a + b)

Divide by a + b both sides

$\frac{2A}{a+b} = \frac{h(a+b)}{a+b}$

h = $\frac{2A}{a+b}$

(ii) Make b the subject formula.

Solution:

A = $\frac{1}{2}h(a + b)$

2A = $\frac{1}{2}h$(a + b)x 2

2A = h(a + b)

2A = ah + bh

2A − ah = bh

Divide by h both sides

$\frac{2A - ah}{h} = b$

$b = \frac{2A - ah}{h}$

2. $^1/_f = {}^1/_v - {}^1/_u$

Solution:

$^1/_f = {}^1/_v - {}^1/_u$

$^1/_f = \frac{u-v}{vu}$

$vu = f(u - v)$

Divide by u – v both sides

$f = \frac{vu}{u-v}$

ii) Make u the subject formula

$^1/_f = {}^1/_v - {}^1/_u$

Solution:

$^1/_f = \frac{u-v}{vu}$

Multiply by uv both sides

$uv = f(u - v)$

$uv = fu - fv$

$fv = fu - uv$

$fv = u (f - v)$

Divide by f – v both sides

$u = \frac{fv}{f - v}$

Exercise 8

1. If T = $^3/_2\sqrt{t/g}$

(i) Make t the subject formula

(ii) Make g the subject

2. If P = w $\left(\frac{1+a}{1-a}\right)$

(i) Make w as the subject formula

(ii) Make a the subject formula

Solution:

1. (i)T = $^3/_2\sqrt{t/g}$

Square both sides

$T^2 = {}^9/_4\,(^t/_g)$

Multiply by 4 both sides

$4T^2 = {}^{9t}/_g$

$4T^2g = 9t$

Divide by 9 both sides

t = $^{4T^2g}/_9$

(ii) Make g the subject formula

T = $^3/_2\sqrt{t/g}$

Solution:

Square both sides

$T^2 = {}^9/_4\,({}^t/_g)$

Multiply by 4 both sides

$4T^2 = {}^{9t}/_g$

$4T^2g = 9t$

Divide by $4T^2$ both sides

$g = {}^{9t}/_{4T^2}$

2)(i) Make w was the subject

Make a the subject

Solution:

$P = w\left(\frac{1+a}{1-a}\right)$

$P(1-a) = w(1+a)$

Divide by $(1+a)$ both sides

$w = P\,\frac{(1-a)}{(1+a)}$

ii) Make a the subject formula

Solution:

$P = w\,\frac{(1+a)}{1-a}$

Divide by w both sides

$$\frac{p}{w} = \frac{w}{w}\frac{(1+a)}{1-a}$$

$$\frac{p}{w} = \frac{(1+a)}{1-a}$$

Multiply by (1 – a) both sides

$$\frac{p}{w}(1-a) = (1-a)\frac{(1+a)}{1-a}$$

$$\frac{p}{w}(1-a) = 1 + a$$

$$\frac{p}{w} - \frac{pa}{w} = 1 + a$$

$$\frac{p}{w} - 1 = a + \frac{pa}{w}$$

$$\frac{p}{w} - 1 = a(1 + \frac{p}{w})$$

Divide by $1 + \frac{p}{w}$ both sides

$$\frac{\frac{p}{w} - 1}{1 + \frac{p}{w}} = \frac{a(1 + \frac{p}{w})}{1 + \frac{p}{w}}$$

$$a = \frac{\frac{p}{w} - 1}{1 + \frac{p}{w}}$$

Exercise 9

I. If $v = \frac{24R}{r + R}$ Make R the subject formula

Solution:

$v = \frac{24R}{r + R}$

Multiply by r + R both sides

$v(r + R) = 24R$

$vr + Rv = 24R$

$vr = 24R – Rv$

$vr = R(24 – v)$

Divide by 24 – v both sides

2. If $m = n\frac{(x - y)}{(x + y)}$

(i) Make x the subject formula

Solution:

$m = n\frac{(x - y)}{(x + y)}$

Multiply by x + y both sides

$mx + my = nx – ny$

$my + ny = nx – mx$

$my + ny = x(n - m)$

divide by n – m both sides

$x = \frac{(my + ny)}{(n - m)}$

(ii)If $T = 2^{\pi\sqrt{kt/a}}$

Make t the subject formula

Solution:

$T = 2^{\pi\sqrt{kt/a}}$

Square both sides

$T^2 = 4\pi_2 kt/a$

Multiply by a both sides

$T^2a = 4\pi_2 kt$

Divide by $4\pi_2 k$ both sides

$t = T_2 a / (4\pi^2 k)$

ALGEBRA

- **BINARY OPERATIONS**

This is the operation in which the two numbers are combined according to the instruction

The instruction may be explained in words or by symbols e.g. x, *, △

- Bi means two

Example1.

Evaluate

(i) 5 x 123

Solution:

5 x 123 = 5(100 + 20 + 3)

= 500 + 100 + 15

= 615

(ii) (8 x 89) – (8 x 79)

= 8(89 – 79)

= 8(10)

= 80

Example2

If a * b = 4a – 2b

Find 3 * 4

Solution:

a * b = 4a – 2b

3 * 4 = 4(3) – 2(4)

= 12 – 8

3 * 4 = 4

Example 3

If p * q = 5q – p

Find 6 * (3 * 2)

Solution:

- consider 3 * 2

From p * q = 5q – p

3 * 2 = 5q – p

= 10 – 3

= 7

Then, 6 * 7 = 5q – p

6 * 7 = 5(7) – p

35 - 6 = 29

6 *(3 * 2) = 29

35 – 6 = 29

6 * (3 * 2) = 29

BRACKETS IN COMPUTATION

- In expression where there are a mixture of operations, the order of performing the operation is

BODMAS

(ii) B = BRACKET

O = OPEN

D = DIVISION

M = MULTIPLICATION

A = ADDITION

S = SUBTRACTION

Example

Simplify the following expression

(i) 10x – 4(2y + 3y)

Solution

10x – 4(2y + 3y)

= 10x – 4(5y)

= 10x – 20y

IDENTITY

- Is the equation which are true for all values of the variable

Example

Determine which of the following are identity.,

(i) $3y + 1 = 2(y + 1)$

Solution:

$3y + 1 = 2(y + 1)$

Test $y = 3$

$3(3) + 1 = 3(2 + 1)$

$9 + 1 = 3(3)$

$10 = 9$

Now, LHS ≠ RHS (The equation is not an identity)

(ii) $2(p – 1) + 3 = 2p + 1$

Test $p = 4$

$2(4 – 1) + 3 = 2(4) + 1$

$2(3) + 3 = 8 + 1$

$6 + 3 = 9$

$9 = 9$

Now, LHS = RHS (The equation is an identity)

EXERCISE

1. If $a * b = 3a^3 + 2b$

Find $(2* 3) * (3 * 2)$

Solution:

$a* b = 3a^3 + 2b$

$(2 * 3) = 3(2)^3 + 2 \times 3$

= 3(8) + 6

= 24 + 6 = 30

Then

$(3 * 2) = 3(3)^3 + 2(2)$

a * b = 30 * 85

$30 * 85 = 3(30)^3 + 2(85)$

= 3(27000) + 170

= 81000 + 170

(2 * 3) * (3 * 2) = 81170

2. If x * y = 3x + 6y, find 2*(3 * 4)

Solution:

Consider (3 * 4)

From x * y = 3x + 6y

3 * 4 = 3(3) + 6(4)

= 9 + 24

= 33

Then 2 * 33 = 3x + 6y

2 *33 = 3(2) + 6(33)

= 6 + 198 = 204

2 * (3 * 4) = 204

3. If $m*n = 4m^2 – n$

Find y if 3 * y = 34

Solution:

$= m * n = 4m^2 – n$

$= 3 * y = 34$

$= 3 * y = 4(3)^2 – y = 34$

$= 4(3^2) – y = 34$

$= 4(9) – y = 34$

$36 – y = 34$

$y = 2$

4. Determine which of the following is identities

$2y + 1 = 2(y + 1)$

Solution:

$2y + 1 = 2(y + 1)$

Test y = 7

$2(7) + 1 = 2(7 + 1)$

$14 + 1 = 2(8)$

$15 = 16$

Now, LHS $\neq$ RHS (The equation is not an identity).

QUADRATIC EXPRESSION

Is an expression of the form of **$ax^2 + bx + c$.**

- Is an expression whose highest power is 2.

- General form of quadratic expression is $ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$.

Note

(i) $a \neq o$

bx – middle term

$y = mx^2 + cx$ – linear equation

$y = ax + b$

$y = mx^2 + 2$ – quadratic equation

$y = mx^2 + c$

example

(i) $2x^2 + 3x + 6$ (a =2, b =3, c =6)

ii) $3x^2 – x$ (a =3, b = -1, c = 0)

iii) $1/2x^2 – 1/yx – 5$ (a = ½, b = -1/4, c = -5)

iv) $–x^2 – x – 1$ (a = -1, b = -1, c = -1)

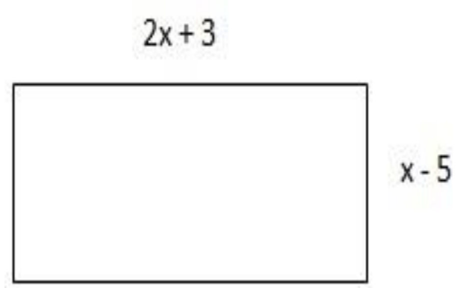
v) $x^2 – 4$ (a = 1, b = 0, c = -4)

vi) x^2 (a = 1, b = 0, c = 0)

Example

If a rectangle has length 2x + x and width x – 5 find its area

Solution:



From, A = l x w   where A is area, l is length and w is width

$= (2x + 3)(x - 5)$

$= 2x(x - 5) + 3(x - 5)$

$= 2x^2 - 10x + 3x - 15$

$2x^2 - 7x - 15$unit area

Alternative way:

$(2x + 3)$ X $(x-5)$

$2x^2$ -10N + 3x-15

<u>$2x^2 - 7x-15$ Unit area</u>

EXPANSION

Example 1

Expand i) $(x + 2)(x + 1)$

Solution:

$(x + 2)(x + 1)$

$x(x + 1) + 2(x + 1)$

$= x^2 + x + 2x + 2$

$= x^2 + 3x + 2$

Alternative way:

$(x+2)(x+1)$

$x^2 +x+2x+2$

<u>x^2+3x+2</u>

ii) (x – 3) (x + 4)

x (x + 4) – 3(x + 4)

$x^2 + 4x – 3x – 12$

$= x^2 + x – 12$

Alternative way:

(x-3) (x+4)

$x^2+4x-3x-12$

$\underline{x^2+x-12}$

iii) (3x + 5) (x – 4)

3x(x -4) + 5 (x – 4)

$= 3x^2 – 12x + 5x – 20$

$= 3x^2 – 7 – 20$

Alternative way:

(3x+5) (x-4)

$3x^2-12x+5x-20$

$\underline{3x^2-7x-20}$

iv) (2x + 5) (2x – 5)

2x (2x – 5) + 5(2x – 5)

$4x^2 – 10x + 10x – 25$

$= 4x^2 – 25$

Alternative way:

(2x+5) (2x-5)

$4x^2-10x+10x-25$

$\underline{4x^2-25}$

EXERCISE

I. Expand the following

(x + 3) (x + 3)

Alternative way:

x(x + 3) + 3x + 9 (x+3) (x+3)

= $x^2 + 3x + 3x + 9$ $x^2+3x+3x+9$

= $x^2 + 6x + 9$ $\underline{x^2+6x+9}$

iii) (2x – 1) (2x – 1)

Solution:

2x(2x – 1) – 1 (2x – 1)

=(2x-1) (2x-1)

= $4x^2 - 2x - 2x + 1$

= $\underline{4x^2 - 4x + 1}$

iii) (3x – 2) (x +2)

Solution:

3x(x + 2) – 2(x + 2)

= 3x2 + 6x – 2x – 4

= $3x^2 + 4x - 4$

Alternative way:

(3x-2) (x+2)

$3x^2+6x-2x-4$

$\underline{3x^2+4x-4}$

2) Expand the following

i) (a + b) (a + b)

Solution:

a(a + b) + b(a + b)

=(a+b) (a+b)

$= a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2$

ii) (a + b) (a –b)

Solution:

a(a + b) - b(a + b)

= (a+b) (a-b)

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

iii) (p + q) (p – q)

Solution:

p(p - q) + q(p – q)

$= p^2 – pq + qp – q^2$

$= p^2 – q^2$

Alternative way:

(p+q) (p-q)

p2-pq+pq- q2

p2 – q2

iv) (m – n) (m + n)

Solution:

m(m + n) – n(m + n)

Alternative way:

$= m^2 + mn – nm + n^2$

(m-n) (m+n)

$= m^2 - n^2$

m2 + mn -nm - n2

m2 - n2

v) (x – y) (x – y)

Solution:

x(x – y) – y(x – y)

= (x-y) (x-y)

$= x^2 – xy – yx + y^2$

$= x^2 – 2xy + y^2$

FACTORIZATION

- Is the process of writing an expression as a product of its factors

(i) BY SPLITTING THE MIDDLE TERM

- In quadratic form

$ax^2 + bx + c$

Sum = b

Product =ac

Example i) $x^2 + 6x + 8$

Solution:

Find the number such that

i) Sum = 6; coefficient of x

ii) Product = 1 x 8; Product of coefficient of x^2 and constant term

= 8 = 1 x 8

= 2 x 4

Now

$x^2 + 2x + 4x + 8$

$(x^2 + 2x) + (4x + 8)$

x (x + 2) + 4(x + 2)

= (x + 4) + (x + 2)

ii) $2x^2 + 7x + 6$

Solution:

Sum = 7

Product, = 2 x 6 = 12

- 12 = 1 x 12

= 2 x 6

= 3 x 4

Now,

$2x^2 + 3x + 4x + 6$

$(2x^2 + 3x) + (4x + 6)$

= x (2x + 3) + 2(2x + 3)

= (x + 2) (2x + 3x)

iii) $3x^2 – 10x + 3$

Solution:

Sum = -10

Product = 3 x 3 = 9

9 = 1 x 9

= 3 x 3

Now,

$3x^2 – x - 9x + 3$

$(3x^2 – x) - (9x + 3)$

x(3x – 1) - 3(3x + 1)

(x - 3) (3x - 1)

iv) $x^2 + 3x - 10$

Solution:

Sum = 3

Product = 1 x -10 = -10

= -2 x 5

Now,

$X^2 - 2x + 5x - 10$

$(x^2 - 2x) + (5x - 10)$

$x (x - 2) + 5(x - 2)$

$= (x + 5) (x - 2)$

EXERCISE

i) Factorize the following

$4x^2 + 20x + 25$

Solution:

Sum = 20

Product = 4 x 25 = 100

100 = 1 x 100

$= 2 \times 50$

$= 4 \times 25$

$= 5 \times 20$

$= 10 \times 10$

$= 4x^2 + 10x + 10x + 25$

$(4x^2 + 10x) + (10x + 25)$

$2x(2x + 5) + 5(2x + 5)$

$= (2x + 5)(2x + 5)$

ii) $2x^2 + 5x - 3$

Solution:

Sum = 5

Product = -6

number = (- 1,6)

$= 2x^2 - x + 6x - 3$

$= 2x^2 + 5x - 3$

$(2x^2 - x) + (6x - 3)$

$x(2x - 1) + 3(2x - 1)$

$= (x + 3)(2x - 1)$

iii) $x^2 - 11x + 24$

Solution:

Sum = -11

Product = 1 x 24 = 24

24 = 1 x 24

= 1 x 24

= 2 x 12

= 3 x 8 = -3 x -8

= 4 x 6

$x^2 - 3x - 8x + 24$

$(x^2 - 3x) - (8x - 24)$

$x(x - 3) - 8(x - 3)$

$= (x - 8)(x - 3)$

iv) $x^2 - 3x - 28$

Solution:

Sum = -3

Product = 1 x -28 = -28

28 = 1 x 28

= 2 x 14

= 4 x ¯ 7

= x^2 + 4x - 7x - 28

(x^2 + 4x) - (7 + 28)

x(x +4) - 7(x +4)

(x - 7) (x + 4)

BY INSPECTION

Example

Factorize

i) $x^2 + 7x + 10$

Solution:

(x + 2) (x + 5)

ii) $x^2 + 3x – 40$

Solution:

(x – 5) (x + 8)

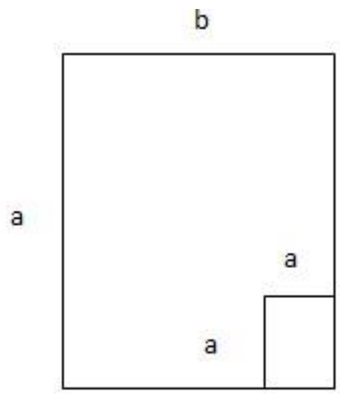
iii) $x^2 + 6x + 7$

Solution:

Has no factor.

DIFFERENT OF TWO SQUARE

Consider a square with length ''a'' unit



1st case, At = (a x a) – (b x b)

$= a^2 - b^2$

2nd case

$A_1 = a(a - b)$ …….(i)

$A_2 = b(a – b)$…….(ii)

Now, 1st case = 2nd case

$A_T = A_1 + A_2$

$a^2 – b^2 = a(a – b) + b(a – b)$

$= (a + b)(a – b)$

Generally $a^2 – b^2 = (a + b)(a – b)$

Example 1

Factorize i) $x^2 – 9$

ii) $4x^2 – 25$

iii) $2x^2 – 3$

Solution:

i) $x^2 – 9 = x^2 – 3^2$

$= (x + 3)(x – 3)$

ii) $4x^2 – 25 = 2^2x^2 – 5^2$

$= (2x)^2 - 5^2$

iii) $2x^2 – 3 = (\sqrt{2})^2 x^2 – (\sqrt{3})^2$

$= (\sqrt{2}x)^2 - (\sqrt{3})^2$

$= (\sqrt{2}x + \sqrt{3})(\sqrt{2}x - \sqrt{3})$

EXERCISE

I. Factorize by inspection

i) $x^2 + 11x - 26$

Solution:

$(x + 13)(x - 2)$

ii) $x^2 - 3x - 28$

Solution:

$(x - 7)(x + 4)$

2. Factorization by difference of two square

i) $x^2 - 1$

Solution:

$X^2 - 1 = (\sqrt{x})^2 - (\sqrt{1})^2$

$= (x)^2 - 1$

$= (x + 1)(x - 1)$

ii) $64 - x^2$

Solution:

$64 - x^2 = 8^2 - x^2$

$= (8 + x)(8 - x)$

iii) $(x + 1)^2 - 169$

solution:

$(x + 1)^2 - 169$

$(x + 1)^2 - 13^2$

$= (x + 1 - 13)(x + 1 + 13)$

$= (x - 12)(x + 14)$

iv) $3x^2 - 5$

Solution:

$3x^2 - 5 = (\sqrt{3}x)^2 - (\sqrt{5})^2$

$= (\sqrt{3}x - \sqrt{5})(\sqrt{3}x + \sqrt{5})$

APPLICATION OF DIFFERENCES OF TWO SQUARE

Example 1

Find the value of i) $755^2 - 245^2$

ii) $5001^2 - 4999^2$

Solution:

i) $755^2 – 745^2$

From $a^2 – b^2 = (a + b) (a – b)$

$755^2 - 245^2 = (755 – 245)(755 + 245)$

$= (510) (1000)$

$= 510, 000$

ii) $5001^2 – 4999^2$

$5001^2 – 4999^2 = (5001 – 4999) (5001 + 4999)$

$5001^2 – 4999^2 = (5001 + 4999)$

$= (2) (10000)$

$= 20,000$

PERFECT SQUARE

Note

$(a + b)^2 = (a + b) (a + b)$

$(a - b)^2 = (a – b) (a – b)$

Example

Factorize i) $x^2 + 6x + 9$

Sum = 6

Product = 9 x 1 = 9

= 9 = 1 x9

= 3 x 3

$x^2 + 3x + 3x + 9$

$(x^2 + 3x) + (3x + 9)$

= x (x + 3)+3 (x + 3)

$= (x + 3)^2$

ii) $2x^2 + 8x + 8$

Sum = 8

Product = 2 x 8 = 16

16 = 1 x 16

= 2 x 8

= 4 x4

$2x^2 + 4x + 4x + 8$

$(2x^2 + 4x) + (4x + 8)$

2x(x + 2) +4(x + 2)

(x +2) (2x + 4)

For a perfect square $ax^2 + bx + c$

Then $4ac = b^2$

Example 1

If $ax^2 + 8x + 4$ is a perfect square find the value of a

Solution:

$ax^2 + 8x + 4$

a = a, b = 8, c = 4

From,

$4ac = b^2$

$4(a)(4) = 8^2$

$16a/16 = 64/16$

$a = 4$

Example2

If $2x^2 + kx + 18$ is a perfect square find k.

Solution:

$2x^2 + kx + 18$

a = 2, b = kx, c = 18

from

$4ac = b^2$

$4(2)(18) = k^2$

From

$4ac = b^2$

$4(2)(18) = k^2$

$\sqrt{144} = \sqrt{k^2}$

$K = \sqrt{144}$

$K = 12$

- Other example

Factorize i) $2x^2 - 12x$

Solution:

$2x(x - 6)$

ii) $x^2 + 10x$

$= x(x + 10)$

QUADRATIC EQUATION

QUADRATIC EQUATION

Is any equation which can be written in the form of $ax^2 + bx + c=0$ where $a \neq 0$ and a, b and c are real numbers.

SOLVING QUADRATIC EQUATION

i) **BY FACTORIZATION**

Example 1

solve $x^2 + 3x - 10 = 0$

Solution:

$x^2 + 3x - 10 = 0$

$(x^2 - 2x) + 5(x - 2) = 0$

$x(x - 2) + 5(x - 2) = 0$

$(x + 5)(x - 2) = 0$

Now $x + 5 = 0$ or $x - 2 = 0$

$x = -5$ or $x = 2$

$x = -5$ or 2

Example 2

Solve for x

i) $2x^2 + 9x + 10 = 0$

Solution:

Sum = 9

Product = 2 x 10 = 20

$20 = 1 \times 20$

$= 2 \times 10$

$= 4 \times 5$

$(2x^2 + 4x) + (5x + 10) = 0$

$2x(x + 2) + 5(x + 2) = 0$

$(2x + 5)(x + 2) = 0$

Now,

$2x + 5 =$ or $x + 2 = 0$

$x = -2.5$ or -2

ii) $2x^2 – 12x = 0$

Solution:

$2x(x – 6) = 0$

$2x = 0$ or $x – 6 = 0$

$X = 0$, or $x = 6$

$X = 0$ or 6

iii) $x^2 – 16 = 0$

Solution:

$x^2 - 16 = 0$

$(x^2) - (4)^2 = 0$

$(x + 4)(x - 4) = 0$

Now, $x + 4 = 0$ or $x - 4 = 0$

$x = -4$ or $x = 4$

EXERCISE

1. Solve for x from

$X^2 - 7x + 12=0$

Solution:

$x^2 - 3x - 4x + 12 = 0$

$(x^2 - 3x) - (4x - 12) = 0$

$x(x - 3) - 4(x - 3) = 0$

$(x - 4)(x - 3) = 0$

Now, $x - 4 = 0$ or $x - 3 = 0$

$x= 4$ or $x = 3$

ii) $4x^2 - 20x + 25 = 0$

Solution:

$4x^2 - 10x - 10x - 25 = 0$

$(4x^2 - 10x) - (10x - 25) = 0$

$2x(2x - 5) - 5(2x - 5) = 0$

$(2x - 5)(2x - 5) = 0$

Now, $2x - 5 = 0$ or $2x - 5 = 0$

$x = \frac{5}{2}$

iii) $4x^2 - 1 = 0$

Solution:

$4x^2 - 1 = 0$

$2^2x^2 - 1 = 0$

$(2x)^2 - (1)^2 = 0$

$(2x + 1)(2x - 1) = 0$

Now, $2x + 1 = 0$, or $2x - 1 = 0$

$X = -\frac{1}{2}$ or $x = \frac{1}{2}$

iv) $(x - 1)^2 - 81 = 0$

Solution:

$(x - 1)^2 - 9^2 = 0$

$(x - 1 - 9)(x - 1 + 9) = 0$

Now, $x - 1 - 9 = 0$, or $x - 1 + 9$

$x - 10 = 0$, $x + 8 = 0$

$x = 10$ or $x = -8$

v) $2x^2 = 10x$

Solution:

$2x^2 - 10x = 0$

$2x(x - 5) = 0$

$2x = 0$ or $x - 5 = 0$

$x = 0$, or $x = 5$

SOLVING BY COMPLETING THE SQUARE

Example 1

Solve i) $2x^2 + 8x - 24 = 0$

Solution:

$$\frac{2x^2}{2} + \frac{8x}{2} - \frac{24}{2} = \frac{0}{2}$$

$x^2 + 4x - 12 = 0$

$x^2 + 4x = 12$

$x^2 + 2x + 2x + 4 = 12 + 4$

$(x^2 + 2x) + (2x + 4) = 16$

$x(x+ 2) + 2(x +2) = 16$

$(x +2) (x +2) = 16$

$(x +2)^2 = 16$

$\sqrt{(x+2)^2} = \sqrt{16}$

$x + 2 = \pm 4$

$X = \pm 4 - 2$

$X = 2$ or $x = -6$

$X = 2$ or -6

ii) $x^2 + 5x – 14 = 0$

solution:

$x^2 + 5x = 14$

$(x^2 + \frac{5x}{2}) + (\frac{5x}{2} + \frac{25}{4}) = 14 + \frac{25}{4}$

$x(x + \frac{5}{2}) + \frac{5}{2} (x + \frac{5}{2}) = \frac{25+56}{4}$

$(x + \frac{5}{2})(x + \frac{5}{2}) = \frac{81}{4}$

$\sqrt{(x+\frac{5}{2})^2} = \sqrt{\frac{81}{4}}$

$x+\frac{5}{2} = \pm\frac{9}{2}$

$x=\frac{9}{2}-\frac{5}{2}$ or $x = -\frac{9}{2}-\frac{5}{2}$

$x = 2$ or $^-7$

iii) $3x^2 - 7x - 6 = 0$

Solution:

$x^2 - \frac{7x}{3} - 2 = 0$

$x^2 - \frac{7x}{3} = 2$

$x^2 - \frac{7x}{3\times 2} - \frac{7x}{3\times 2} + \frac{49}{36} = 2 + \frac{49}{36}$

$(x^2 - \frac{7x}{6}) - (\frac{7x}{6} - \frac{49}{36}) = \frac{72+49}{36}$

$x(x - \frac{7}{6}) - \frac{7}{6}(x - \frac{7}{6}) = \frac{121}{36}$

$(x - \frac{7}{6})(x - \frac{7}{6}) = \frac{121}{36}$

$\sqrt{(x-\frac{7}{6})^2} = \sqrt{\frac{121}{36}}$

$x - \frac{7}{6} = \pm\frac{11}{6}$

Now,

$x - \frac{7}{6} = \frac{11}{6}$, $x - \frac{7}{6} = -\frac{11}{6}$

$x = 3$ or $x = -\frac{2}{3}$

iv) $x^2 - 5x + 2 = 0$

$x^2 - 5x = -2$

$x^2 - \frac{5x}{2} - \frac{5x}{2} + \frac{25}{4} = -2 + \frac{25}{4}$

$x(x - \frac{5}{2}) - \frac{5}{2}(x - \frac{5}{2}) = \frac{25-8}{4}$

$(x - \frac{5}{2})^2 = \frac{17}{4}$

$\sqrt{(x - \frac{5}{2})^2} = \sqrt{\frac{17}{4}}$

$x - \frac{5}{2} = \pm\sqrt{\frac{17}{4}}$

$x = \frac{5}{2} \pm\sqrt{\frac{17}{4}}$

$x = \frac{\sqrt{17}}{2} + \frac{5}{2}$ or $\frac{5}{2} - \frac{\sqrt{17}}{2}$

<u>GENERAL FORMULA</u>

1. Solve $ax^2 + bx + c = 0$

Solution:

$$\frac{ax^2}{a}$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{bx}{a} = -\frac{c}{a}$$

$$x^2 + \frac{bx}{2a} + \frac{bx}{2a} + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$(x^2 + \frac{bx}{2a}) + (\frac{bx}{2a} + \frac{b^2}{4a^2}) = \frac{-4ac + b^2}{4a^2}$$

$$x(x + \frac{b}{2a}) + \frac{b}{2a}(x + \frac{b}{2a}) = \frac{-4ac + b^2}{4a^2}$$

$$(x + \frac{b}{2a})^2 = \frac{-4ac + b^2}{4a^2}$$

$$\sqrt{(x + \frac{b}{2a})^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Generally, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 1.

Solve for x by using generally formula

i) $6x^2 + 11x + 3 = 0$

Solution: a = 6, b = 11, c = 3

From the general equation, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(6)(3)}}{2\,(6)}$$

$$x = \frac{-11 \pm \sqrt{121 - 72}}{12}$$

$$x = \frac{-11 \pm \sqrt{49}}{12}$$

$$x = \frac{-11 \pm 7}{12}$$

$x = \frac{-11 + 7}{12}$ and $x = \frac{-11 \;\; -7}{12}$

$x = \frac{-1}{3}$ and $x = \frac{-3}{2}$

ii) $5x^2 - 6x - 1 = 0$

Solution:

a= 5, b = -6, c =1

From the general equation, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{6 \pm \sqrt{6^2 - 4(5)(1)}}{2\,(5)}$$

$$x = \frac{6 \pm \sqrt{36-20}}{10}$$

$$x = \frac{6 \pm \sqrt{16}}{10}$$

$$x = \frac{6 \pm 4}{10}$$

$x = \frac{6+4}{10}$ and $x = \frac{6-4}{10}$

$x = 1$ and $x = \frac{1}{5}$

iii) $0 = 400 + 20t - t^2$

solution:

$t^2 - 20t - 400 = 0$

a = 1, b = -20, c = -400

From the general equation

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{20 \pm \sqrt{20^2 - 4(1)(-400)}}{2\,(1)}$$

$$t = \frac{20 \pm \sqrt{400+1600}}{2}$$

$$t = \frac{20 \pm \sqrt{2000}}{2}$$

$$t = \frac{20+\sqrt{2000}}{2} \quad \text{or} \quad t = \frac{20-\sqrt{2000}}{2}$$

GRAPHICAL SOLUTION OF QUADRATIC EQUATION

- The general quadratic equation $ax^2 + bx + c = 0$ can be solved graphically
- First draw the graph by setting $ax^2 + bx + c = y$ and then

Drawing graphs

Example 1

Draw the graph of the following equation

i) $y = x^2 - 3$

ii) $y = 2 - x^2$

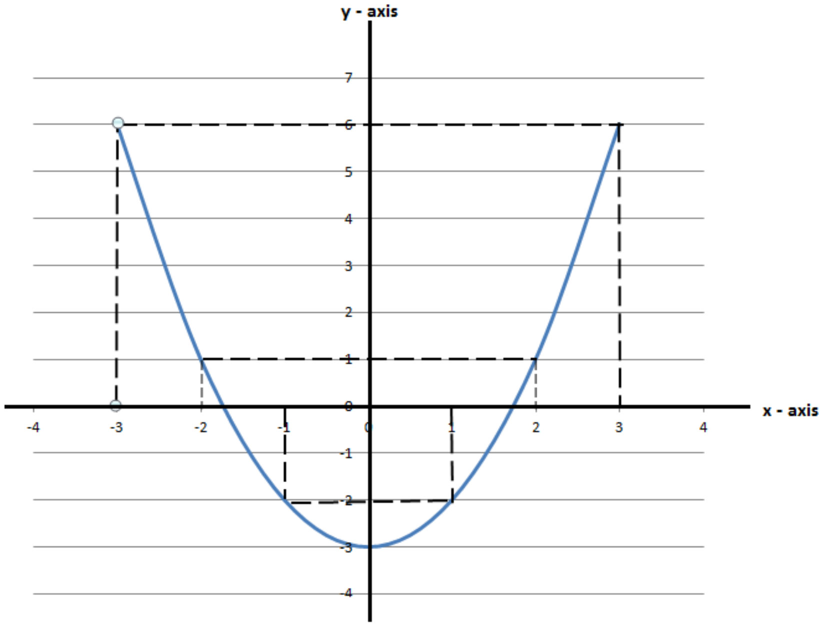
iii) $y = x^2 + x - 1$

Solution:

i) $y = x^2 - 3$

TABLE VALUE

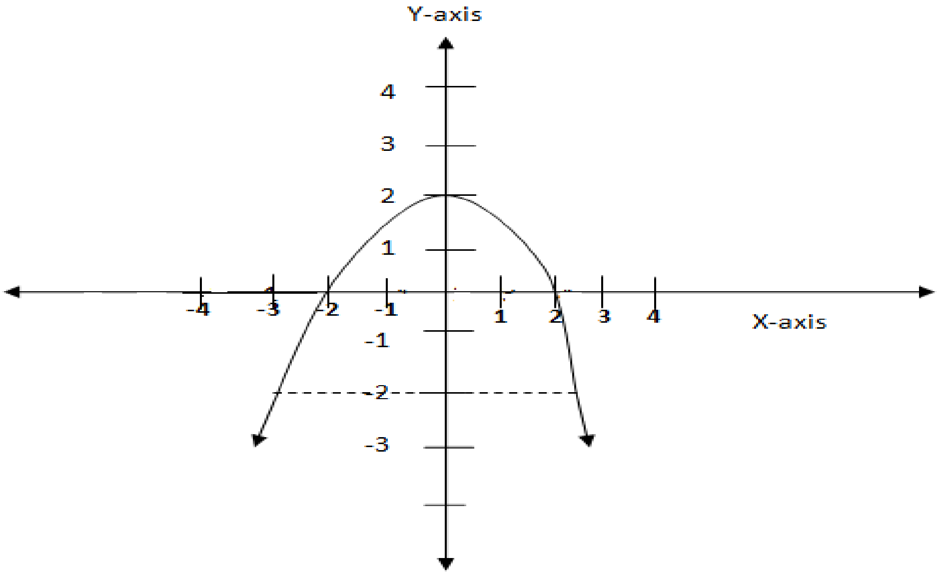
x	-3	-2	-1	0	1	2	3
y	6	1	-2	-3	-2	1	6



ii) $y = 2 - x^2$

x	2	1	0	-1	2
y	-2	1	2	1	-2

A Graph of $y = 2 - x^2$

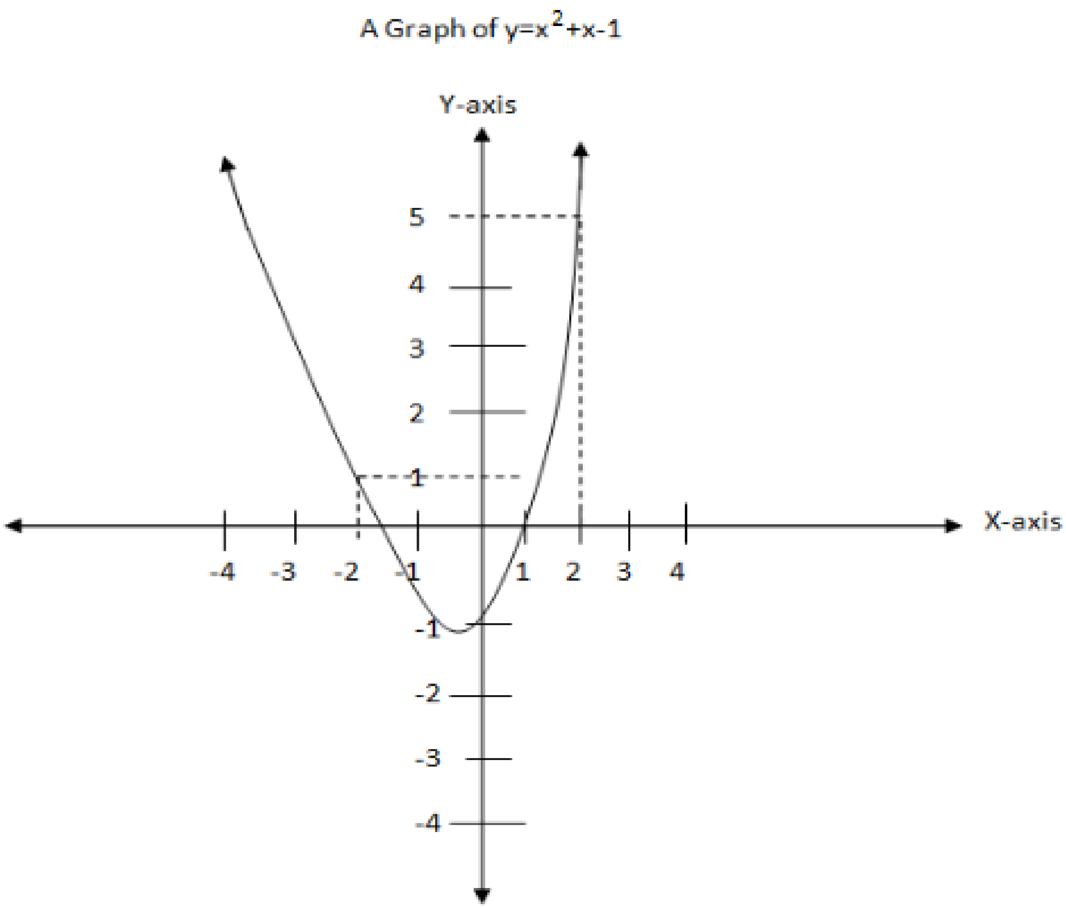


iii) $y = x^2 + x - 1$

Solution:

Table value:

X	-2	-1	0	1	2
Y	1	-1	-1	1	5



APPLICATION OF GRAPHS IN SOLVING QUADRATIC EQUATION

a) Solve graphically the equation $x^2 - x - 6 = 0$

b) Use the graph in a to solve the equation

$x^2 - x - 2 = 0$

Solution:

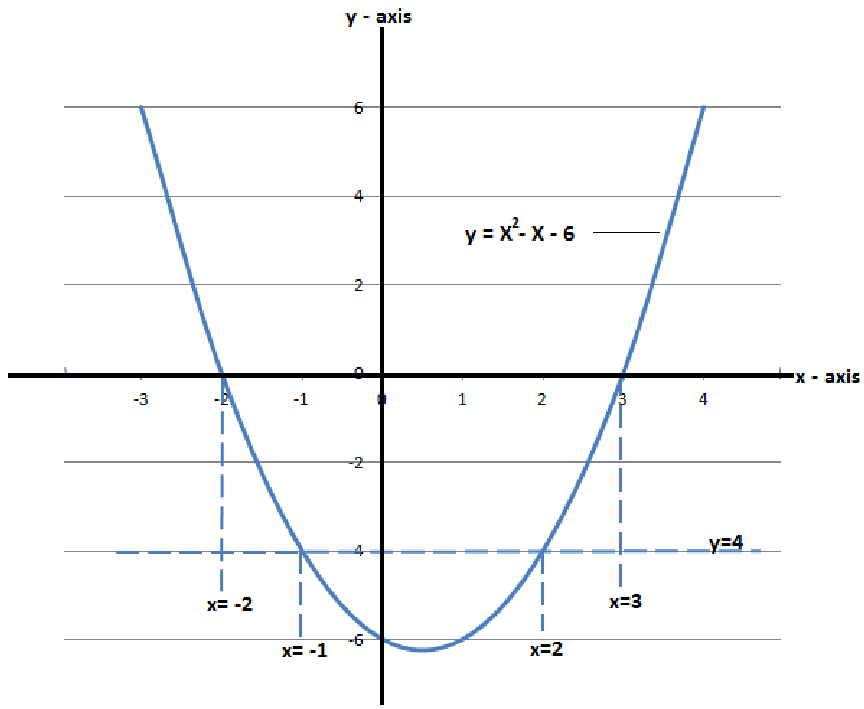
$x^2 - x - 6 = 0$

Let $y=x^2 - x - 6$........................(i) Then

$y=0$.....................(ii)

$y = x^2 - x - 6$

x	-3	-2	-1	0	1	2	3
y	6	0	-4	-6	-6	-4	0



(a) x = 2 or x = 3

(b) From $x^2 - x - 6 = 0$

Then

$x^2 - x - 2 = 0$ can be written as

$x^2 - x - 2 - 4 = 0 - 4$

$x^2 - x - 6 = -4$ But $y=x^2 - x - 6$

$\therefore y=-4$

$\therefore x=-1$ or $x=2$

More examples

1. A man is 4 times as old as his son. In 4 years the product of their ages will be 520.

Find the sons present age

Solution:	son	man
	present x	4x
	after (x + 4)	(4x + 4) = 520

Now

$(x + 4)(4x + 4) = 520$

$4x^2 + 4x + 16x + 16 = 520$

$$\frac{4x^2}{4} + \frac{20x}{4} - \frac{504}{4} = \frac{0}{4}$$

$x^2 + 5x - 126 = 0$

$a=1,\ b = 5,\ c = -126$

From the general equation, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-126)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 504}}{2}$$

$$x = \frac{-5 \pm \sqrt{529}}{2}$$

$$x = \frac{-5 \pm 23}{2}$$

$x = \frac{-5+23}{2}$ and $x = \frac{-5\ -23}{2}$

$X = \frac{18}{2}$ and $x = \frac{-28}{2}$

x = 9 or -14.

The present age of the son is 9

2.Find the consecutive numbers such that the sum of their squares is equal to 145

Solution:

Let x be the first number and x + 1 be the second number

Sum of $x^2 + (x + 1)^2 = 145$ their squares

Now, $x^2 + (x + 1)^2 = 145$

$x^2 + x^2 + 2x + 1 = 145$

$2x^2 + 2x - 144 = 0$

Divide by 2 both sides , then $x^2 + x - 72$

a =1, b =1, c = -72

From the general equation, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-72)}}{2\,(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 288}}{2}$$

$$x = \frac{-1 \pm \sqrt{289}}{2}$$

$$x = \frac{-1 \pm 17}{2}$$

$x = \frac{-1 + 17}{2}$ and $x = \frac{-1 \;- 17}{2}$

$x = \frac{16}{2}$ and $x = \frac{-18}{2}$

x = 8 and 9

or x = -9 and -8

The two consecutive numbers are 8 and 9 or -9 and -8.

LOGARITHMS

LOGARITHMS

STANDARD NOTATIONS

Standard notation form is written in form of A x 10^n whereby 1≤ A< 10 and n is any integers

Example

Write the following in standard form
(i) 2380

Solution:
$2380 = 2.38 \times 10^3$

(ii) 97

Solution:
$97 = 9.7 \times 10^1$

(iii) 100000

Solution:
$100000 = 1 \times 10^5$

(iv) 8
Solution:
$8 = 8 \times 10^0$

Example

Write the following in standard form

(i) 0.00056
$= 5.6 \times 10^{-4}$

(ii) 0.001
$= 1 \times 10^{-3}$

(iii) 0.34
$= 3.4 \times 10^{-1}$

(iv) 2. 0001
$= 2.0001 \times 10^0$

EXERCISE 1:

i). Write the following in standard form

17000
$= 1.7 \times 10^4$

ii) 0.00998
= 9.98×10^{-3}

iii). Write in standard form

0.000625
= 6.25×10^{-4}

8/300 correct to four significant figure

8/300 = 0.02666

Now 2.667×10^{-2}

2.667×10^{-2}

iv) If a = br and a = 8.4×10^4 , b = 7.0×10^2 Find r.

solution:

a = 84 000

b = 700

Now

br = a

(700) (r) = 84000

$$r = \frac{84000}{700}$$

r=120
r = 1.2×10^2

DEFINITION OF LOGARITHMS

Consider

3 x 3 x 3 x 3 then

3 x 3 x 3 x 3 = 3^4 = 81, the number 3 is the base ,and 4 is the exponent.

Now we say;

Logarithm of 81 to base 3 is equal to exponent 4

$\log_3 81 = 4$
In short $b^n = a$

$\log_b a = n$

Example 1.
Write the following in logarithmic form

i) $a^5 = 10$
$\log_a 10 = 5$

ii) $10^{-3} = 0.001$
$10^{-3} = 0.001$
$\log_{10} 0.001 = -3$

iii) $2^{-1} = ½$
$\log_2 1/2 = -1$

iv) $3 = 9^{1/2}$
$\log_3 9 = 1/2$

Example 2

Write the following in exponential form
(i) $\log_3 729 = 6$
$3^6 = 729$

(ii) $\log_3 1/3 = -1$
$3^{-1} = 1/3$

(iii) $\log_{10} 0.01 = -2$
$10^{-2} = 0.01$

(iv) $1/2 = \log_4 2$
$4^{(1/2)} = 2$

Example 3

If $\log_{10} 0.01 = y$. Find y

Solution:

$\log_{10} 0.01 = y$

$10^y = 0.01$

$10^y = 1\times10^{-2}$

$10^y = 10^0\times10^{-2}$

$10^y = 10^{-2}$

<u>$y = -2$</u>

If $\log_{10}x = -3$ find x

Solution:
$\log_{10}x = -3$

$10^{-3} = x$

<u>$x = 0.001$</u>

EXERCISE 1
1. Write in standard form
   i) 405.06
   ii) 0.912

<u>Solution:</u>
i) $405.06 = 4.0506 \times 10^2$
ii) $0.912 = 9.12 \times 10^{-1}$

2. Write in logarithimic form
i) $5^{-1} = 1/5$
ii) $0.0001 = 1 \times 10^{-4}$

<u>Solution:</u>
i) $5^{-1} = 1/5$
$\log_5(1/5) = -1$

ii) $0.0001 = 10^{-4}$
$\log_{10}0.0001 = -4$

3. Write in exponential form
i) $\log_a x = n$
ii) $-3 = \log_{10}0.001$
iii) $\log_2(1/64) = -6$

<u>Solution:</u>
i) $\log_a x = n$
$a^n = x$

ii)$-3 = \log_{10} 0.001$
$10^{-3} = 0.001$

iii) $\log_2(1/64) = -6$
$2^{-6} = 1/64$

4. To solve for x
i) $\log_6 x = 4$
$6^4 = x$
$x = 1296$

ii) $x = \log_3 6561$
$3^x = 6561$
$x = 8$

iii) $\log_x 10 = 1$
$x^1 = 10$
$x = 10$

iv) $\log_4 2 = x$
$4^x = 2$
$2^{2x} = 2^1$
$2x = 1$
$x = 1/2$

BASE TEN LOGARITHM

- Is an logarithm of a number to base 10. Also known as common logarithm
  example i) $\log_{10} 5 = \log 5$
  ii) $\log_{10} 75 = \log 75$
  iii) $\log_{10} p = \log p$

SPECIAL CASES

(1). $\log_a a = x$
$a^x = a^1$
$x = 1$

Generally $\log_a a = 1$

Generally $\log_a a = 1$

Example

i) $\log_6 6 = 1$

ii) $\log 10 = 1$

(2) $\log_a(a^n) = x$

$a^x = a^n$

$x = n$

Generally $\log_a(a^n) = n$

Example i) $\log_4(4^5) = 5$

ii) $\log 10^{-3} = -3$

Example 1

If $\log_5 5 = \log_2 m$ Find m

Solution:

$\log_5 5 = \log_2 m$

But $\log_5 5 = 1$

$1 = \log_2 m$

$2^1 = m^1$

$m = 2$

Example 2

Given $\log_5 25 + \log_4 x = 6$, Find x

Solution:

$\log_5 25 + \log_4 x = 6$

$\log_5(5^2) + \log_4 x = 6$

$2\log_5 5+\log_4 x = 6$

$2 +\log_4 x = 6$

$\log_4 x = 4$
$x= 4^4$

$\underline{x = 256}$

EXERCISE 2.

Evaluate

i) $\log_2 4096$

ii) $\log 0.0001$

<u>solution</u>

i) $\log_2 4096$

let $x = \log_2 4096$
$2^x = 4096$

$2^x = 2^{12}$

$x = 12$

$\therefore \log_2 4096=12$

ii) $\log 0.0001$

<u>Solution:</u>

Let $x = \log 0.0001$
$10^x = 1/10000$
$10^x = 1/(10^4)$
$10^x = 10^{-4}$
$x = -4$

$\therefore \log 0001=1$

2) If $\log_k 81 - \log_2 32 = -1$

Solution:

$\log_k 81 - 5\log_2 2 = -1$

$\log_k 81 = -1 + 5$

$\log_k 81 = 4$

$k^4 = 81$

$k^4 = 3^4$

$k = 3$

3. Given $\log_6 y = \log_7 343$. Find y

Solution:

$\log_6 y = 3\log_7 7$

$\log_6 y = 3$

$6^3 = y$

$216 = y$

$y = 216$

4) Solve for m

i) $\log_8 1 = m$

$8^m = 1$ since $a^o = 1$ then

$8^m = 8^0$

$m = 0$

ii) $\log_5 m + \log_3 27 = 8$

$\log_5 m + \log_3 3^3 = 8$

$\log_5 m + 3 = 8$

$\log_5 m = 5$

$m = 5^5$

$m = 3125$

LAWS OF LOGARITHMS

MULTIPLICATION LAW

Suppose, $\log_a x = p$ and $\log_a y = q$ then
$\log_a x = p$….(i)
$\log_a y = q$….(ii)
Write equation (i) and (ii) into exponential form.
$a^p = x$………(iii)
$a^q = y$……..(iv)
Multiply equation (iii) and (iv)

$xy = a^p \times a^q$
$xy = a^{(p+q)}$ …….(v)

In equation (v) apply $\log_a$ both sides
$\log_a (xy) = \log_a a^{(p+q)}$
$\log_a xy = (p + q) \log_a a$
$\log_a xy = p + q$
But $p = \log_a x$
$q = \log_a y$

Generally $\log_a xy = \log_a x + \log_a y$

Example

i) $\log_6(8 \times 12) = \log_8 8 + \log_6 12$
ii) $\log_4 9 + \log_4 3 = \log_4(9 \times 3)$

Example 1

i) Find x , If $\log_3 x = \log_3 15 + \log_3 12$

Solution:

$\log_3 x = \log_3 15 + \log_3 12$
$\log_3 x = \log_3(15 \times 12)$
$\log_3 x = \log_3 180$
$\therefore x = 180$

Example 2

Given $\log_5 20 = \log_5 4 + \log_5 x$ .Find x

Solution:

$\log_5 20 = \log_5 4 + \log_5 x$

$\log_5 20 = \log_5(4 \times x)$
$\log_5 20 = \log_5 4x$
$\therefore 20 = 4x$

$X = 5$

Example 3

If $\log_8 0.01 = \log_8(m \times 2)$. Find m

solution

$\log_8 0.01 = \log_8(2m)$

$\therefore 0.01 = 2m$

$m = 0.01/2$

$m = 0.005$

QUOTIENT LAW

Suppose, $\log_a x = p$ and
$\log_a y = q$ then
$\log_a x = p$……..(i)
$\log_a y = q$……..(ii)

Write equation (i) and (ii) into exponent form
$a^p = x$……(iii)
$a^q = y$…..(iv)

Divide equation (iii) and (iv)
$x/y = a^p/a^q$
$x/y = a^{(p-q)}$ ….. (v)

In equation (v) apply log a both sides
$\log_a(x/y) = \log_a a^{(p-q)}$
$\log_a(x/y) = (p - q) \log_a a$
But $\log_a a = 1$
$\log_a(x/y) = p - q$,where $p = \log_a x$ and $q = \log_a y$

Generally $\log_a(x/y) = \log_a x - \log_a y$

i) $\log_6(8/12) = \log_6 8 - \log_6 12$
ii) $\log_4 9 - \log_4 3 = \log_4(9/3)$

Example

If $\log_2 20 = \log_2 x - \log_2 8$. Find x

Solution:

$\log_2 20 = \log_2 x - \log_2 8$
$\log_2 20 = \log_2(x/8)$
Now, $20 = x/8$
$X = 20 \times 8$
$X = 160$

EXERCISE 3
1. Evaluate

i) $\log_6 3 + \log_6 2$

Solution:

$= \log_6 3 + \log_6 2$
$= \log_6(2 \times 3) = \log_6 6$
$= 1$

ii) $\log 40 + \log 5 + \log 40$

Solution:

$= \log_{10} 40 + \log_{10} 5 + \log_{10} 40$
$= \log_{10}(40 \times 5 \times 40)$
$= \log_{10} 8000$

iii) $\log_{10} 25 - \log_{10} 9 + \log_{10} 360$

Solution:

$\log_{10} 25 - \log_{10} 9 + \log_{10} 360$
$\log_{10}((25 \times 360)/9)$
$= \log_{10} 1000$

$=\log_{10}10^3$
$=3\log_{10}10$
$=3$

2. If log$_{5a}$â• ¡x = log$_{5a}$â• ¡9 + log$_{5a}$â• ¡12. Find x

<u>Solution:</u>

log$_{5a}$â• ¡x = log$_{5a}$â• ¡9 + log$_{5}$â• ¡a12
log$_{5a}$â• ¡x = log$_{5a}$â• ¡(9 ×12)
log$_{5a}$â• ¡x = log$_{5a}$â• ¡108
x = 108

3. If log$_{2a}$â• ¡5 = log$_{2a}$y + log$_{2a}$â• ¡0.001.Find Y

<u>Solution:</u>

i) log$_{2a}$â• ¡5 = log$_{2a}$â• ¡(y×0.001)

5â• ¡ = â• ¡0.001y

y= 5/0.001

Y = 5000

ii)Find y if log$_{6}$â• ¡100 = log$_{6}$â• ¡5 + log$_{6}$â• ¡80 - log$_{6}$â• ¡y

<u>Solution:</u>

log$_{6}$â• ¡100 = log$_{6}$ (5 × 80)/ y

100 = â• ¡400/y

y = 4

4. If log a = 0.9031, log b = 1.0792 and log c = 0.6990. Find logâ□¡ ac⁄b

<u>Solution</u>

log â□¡ac⁄b =log$_{10}$â• ¡ a + log$_{10}$â• ¡c - log$_{10}$â• ¡b
=0.9031 + 1.0792 -0.6990

∴ log â□¡ac⁄b = 1.2833

LOGARITHM OF POWER

If $\log_a$â• ¡x = p then
$X = a^p$
Multiply by power in both sides $x^n = a^{np}$

Apply $\log_a$ both sides
$\log_a x^n = \log_a a^{np}$
$\log_a x^n = np$

But $p = \log_a x$

$\therefore \log_a x^n = n\log_a x$

Generally $\log_a x^n = n \log_a x$

Example(1)

Evaluate

i) $\log_2 (128)^6$
ii) $\log_7 (343)^8$

Solution
i) $\log_2 (128)^6 = 6\log_2 2^7$

$= (7 \times 6) \log_2 2$

$= 42 \times 1$

$= 42$

ii) $\text{Log}_7 (343)^8$
Solution:
$\log_7 343^8 = 8\log_7 343$
$= 8\log_7 7^3$
$= (8 \times 3) \log_7 7$
$= 24$

Example (2)

If $\log_5 625^y = \log_3 729^2$ .Find y.

Solution:

$\log_5 625^y = \log_3 729^2$

$\log_5 625^y = 2\log_3 729$

$y\log_5 5^4 = 2\log_3 3^6$

$(y \times 4) \log_5 5 = (2 \times 6) \log_3 3$

$4y \log_5 5 = 12 \log_3 3$

$4y = 12$

$y=2/4$

$y = 3$

LOGARITHM OF ROOTS

From $\sqrt[n]{x} = x^{1/n}$

Generally

$$\log_a \sqrt[n]{x} = 1/n\log_a x$$

Example (1)

i) $\log_2 \sqrt{8} = \log_2 8^{1/2}$

$= 1/2\log_2 8$

$= 1/2\log_2 2^3$

$= (3 \times ½) \log_2 2$

$= 3/2 \times 1$

$= 3/2$

ii) $\text{Log}_2 \sqrt{512} = \log_2 (512)^{1/2}$

$= \log_2 2^9$

$=9 \log_2 2$

$=9$

$\therefore \text{Log}2 \sqrt{512} = 9$

EXERCISE 4:

1. Evaluate

i) log 60 + log 40 – log 0.3

ii) $\log_3 \sqrt{(1/27)}$

<u>Solution:</u>

i) Log60 + log40 – log0.3

$\log_{10} 60 + \log_{10} 40 - \log_{10} 0.3$

$\log_{10} (60 \times 40/0.3) = \log_{10} (2400/0.3)$

$= \log_{10} 8000$

$=3.9031$

ii) $\text{Log}_3 \sqrt{1/27} = \log_3 (1/27)^{1/2}$

$= 1/2 \log_3 1/27$

$= 1/2 \log_3 27^{-1}$

$= 1/2 \log_3 3^{-3}$

$== 1/2 \times -3\log_3 3$

$=-3/2$

2. If $\log_3 6561^6 = \log_2 512^k$. Find k

<u>Solution:</u>

$\log_3 6561^6 = \log_2 512^k$

$6 \log_3 6561 = k\log_2 512$

$6 \log_3 3^8 = k\log_2 2^9$

$6 \times 8 \log_3 3 = k \times 9\log_2 2$

$48 = 9k$

$k=48/9$

3. Given $\log_2 x = 1 - \log_2 3$. Find x

<u>Solution:</u>

$\log_2 x = 1 - \log_2 3$

$\log_2 x = \log_2 2 - \log_2 3$

$\log_2 x = \log_2 (2/3)$

$x = 2/3$

4. Simplify

i) 2log5 + log36 – log9
ii) (logâ•¡8-logâ•¡4)/(log 4-log2)

<u>Solution:</u>

i) 2log5 + log36 – log9

$\log 5^2 + \log 36 - \log 9$

$\log_{10}25 + \log_{10}36 - \log_{10}9$

$= \log_{10} (25 \times 36)/9$

$= \log_{10} (900/9)$

$= \log_{10}100$

$=\log_{10} 10^2$

$=2 \log_{10} 10$

=2

ii) (logâ•¡8-logâ•¡4)/(log 4-log2)

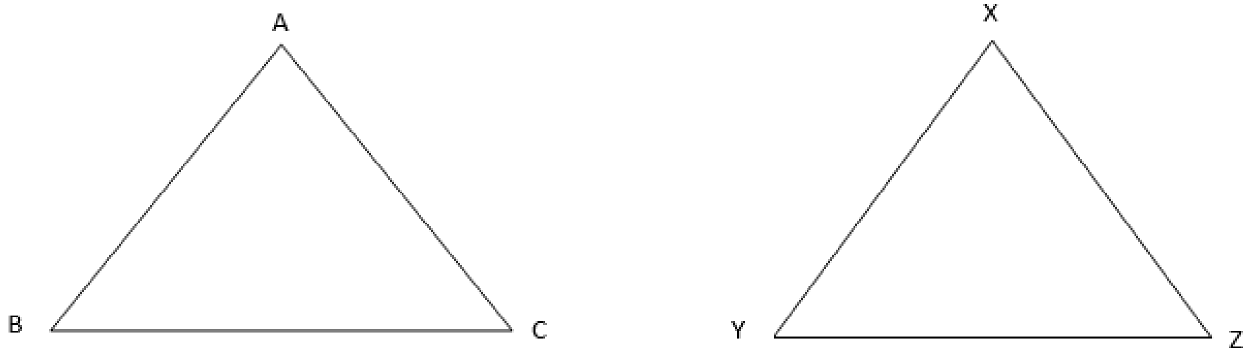
<u>Solution:</u>

(logâ•¡8-logâ•¡4)/(log 4-log2)

$= \log_{10} (8/4) \div \log_{10} (4/2)$

$= \log_{10}2 \div \log_{10}2$
= 1

CONGRUENCE OF SIMPLE POLYGON



The triangles above are drawn such that

C$\hat{A}$B= Z$\hat{X}$Y

A$\hat{B}$C=X$\hat{Y}$Z

B$\hat{C}$A=Y$\hat{Z}$X

Corresponding sides in the triangles are those sides which are opposite to the equal angles i.e.

$\overline{AB}$ corresponds to $\overline{XY}$

$\overline{AC}$ corresponds to $\overline{XZ}$

$\overline{BC}$ corresponds to $\overline{YZ}$

If the corresponding sides are equal i.e.

$\overline{AB} = \overline{XY}$

$\overline{AC} = \overline{XZ}$

$\overline{BC} = \overline{YZ}$

Then ΔABC fits exactly on Δ XYZ.

In other words ΔABC is an exact copy of ΔXYZ. These triangles are said to be congruent.

In general, polygons are congruent if corresponding sides and corresponding angles are equal.

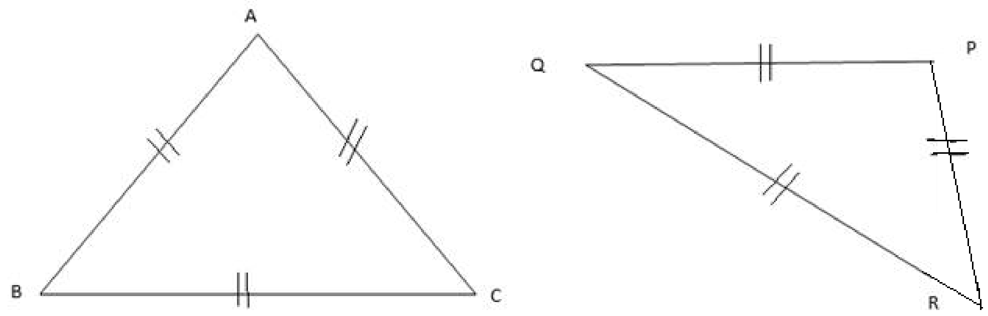
The symbol for congruence is $\equiv$

Congruence of triangles

Case 1: Given three sides

Two triangles are congruent if the three pairs of corresponding sides are such that the sides in each pair are equal.

Consider the triangles below:



Proof;

$\overline{AB} = \overline{PQ}$- given

$\overline{BC} = \overline{QR}$- given

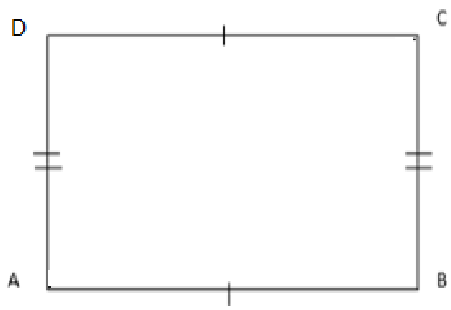
$\overline{AC} = \overline{PR}$- given

Therefore $\triangle ABC \equiv \triangle PQR$ - [SSS] Theorem

Note: SSS- is an abbreviation of side- side- side

<u>Examples :</u>

1. In the figure below prove that $\Delta ABC \equiv \Delta CDA$ and deduce that $D\bar{C}A = B\bar{A}C$

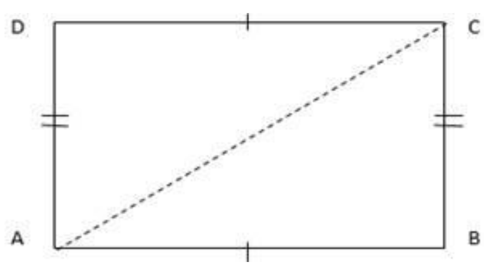


Solution

Given figure ABCD, $\overline{AB} = \overline{CD}$, $\overline{AD} = \overline{BC}$

Try to prove $\Delta ABC = \Delta CDA$

Construction of A is joined C



$D\hat{A}C = B\hat{A}C$

$\overline{DC} = \overline{AB}$ – given

$\overline{DA} = \overline{CB}$ - given

AC -common

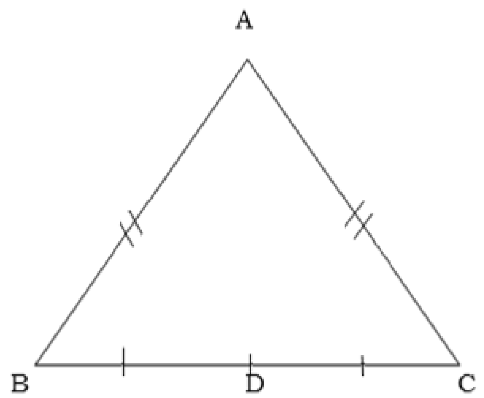
Therefore

$\triangle ABC \equiv \triangle CDA$-[SSS]

(b) $D\hat{C}A = B\hat{A}C$ (Definition of congruence of triangles)

2. ABC is an isosceles triangle in which AB and AC are equal.

If D is the midpoint of BC, prove that $\hat{ABD} \equiv \hat{ADC}$

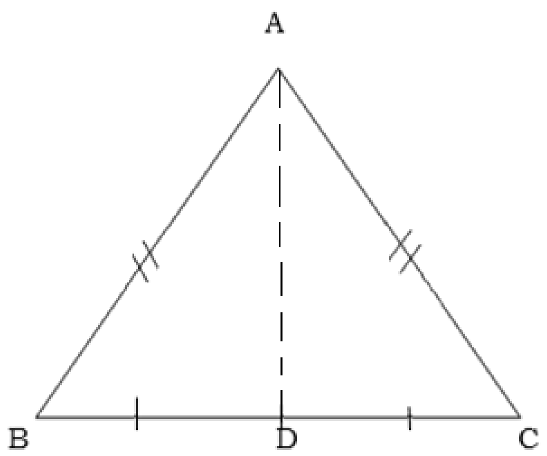


Solution

Given $\Delta ABC, \overline{AB} = \overline{AC}$, D midpoint of $\overline{BC}$

Required to prove $\Delta ABD \equiv \Delta ACD$

Construction; A joined to D



$\overline{AB}=\overline{AC}$ (Given)

$\overline{BD}=\overline{DC}$ (D is the midpoint of $\overline{BC}$)

$\overline{AD}$ (common)

Therefore

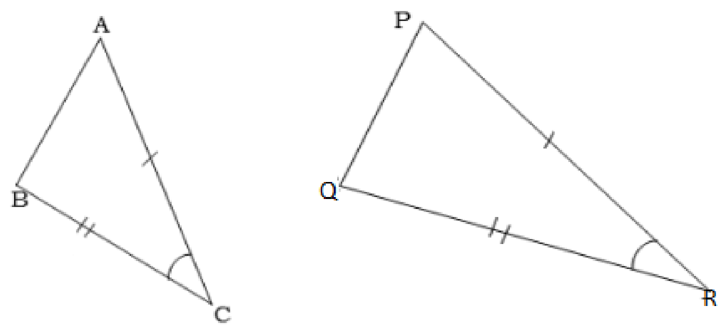
$\triangle ABD \equiv \triangle ACD$ [SSS]

Case 2; Given two sides and the included angle (SAS)

Two triangles are congruent if two pairs of corresponding sides are such that the sides in each pair are equal and the angles included between the given sides in each triangle are equal.

Examples

1. Prove that $\triangle ABC \equiv \triangle PQR$



$\overline{BC}=\overline{QR}$ (Given)

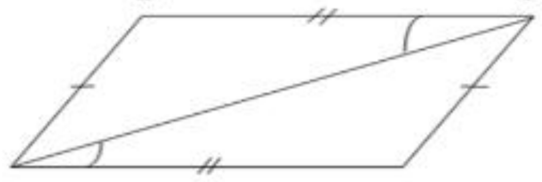
$\overline{AC}=\overline{PR}$ (Given)

$A\hat{C}B=P\hat{R}Q$ (Given)

Therefore

$\triangle ABC \equiv \triangle PQR$-[SAS]

2. Use the following figure to prove that $\triangle DAC \equiv \triangle ABC$



Solution

Given a quadrilateral ABCD

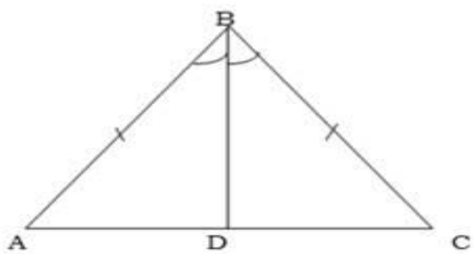
$\overline{AC}$ is common

$\overline{DA} = \overline{BC}$ given

$C\hat{A}B = D\hat{C}A$ - given

<u>Therefore $\triangle DAC \equiv \triangle BAC$</u> [SAS]

3. Use the figure below to prove that $\overline{AD} = \overline{DC}$



Given $\triangle ABC$

$\overline{BA} = \overline{DC}$ (Given)

$A\hat{B}C = C\hat{B}D$ (Given)

Required to prove $\overline{AD} = \overline{DC}$

$\overline{AB} = \overline{BC}$ - given

$A\hat{B}D = C\hat{B}D$ Given

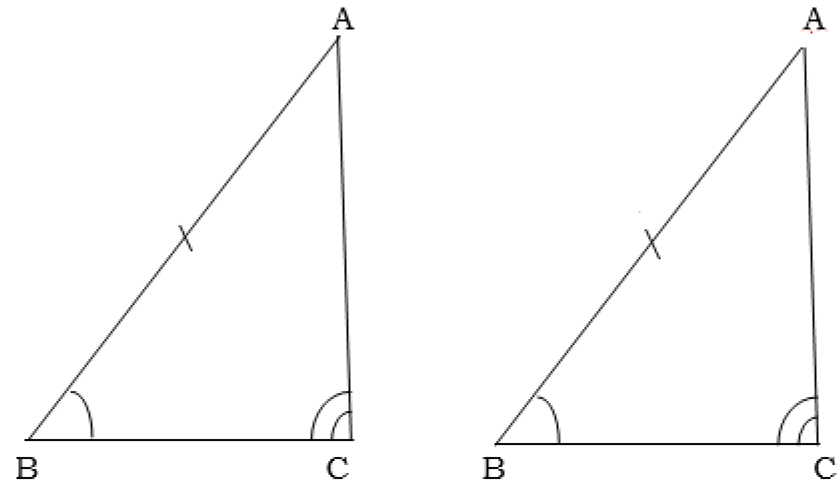
BD -common

Thus $\triangle ABD \equiv \triangle CBD$ (SAS Theorem)

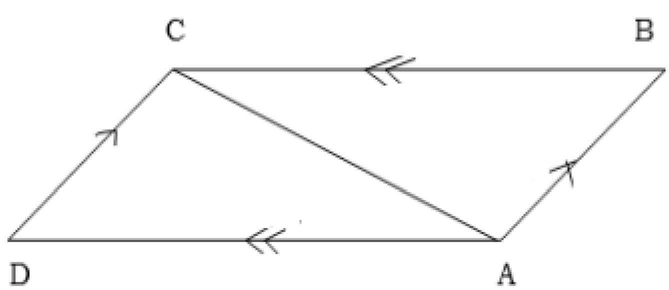
Therefore $\overline{AD} = \overline{DC}$(definition of congruence of SSA)

Case 3; Given two angles and a corresponding side

Two triangles are congruent if two pairs of corresponding angles are such that the angles in each triangle are equal.



Example



Given parallelogram ABCD required to prove that △ABC ≡ △ CDA

Solution

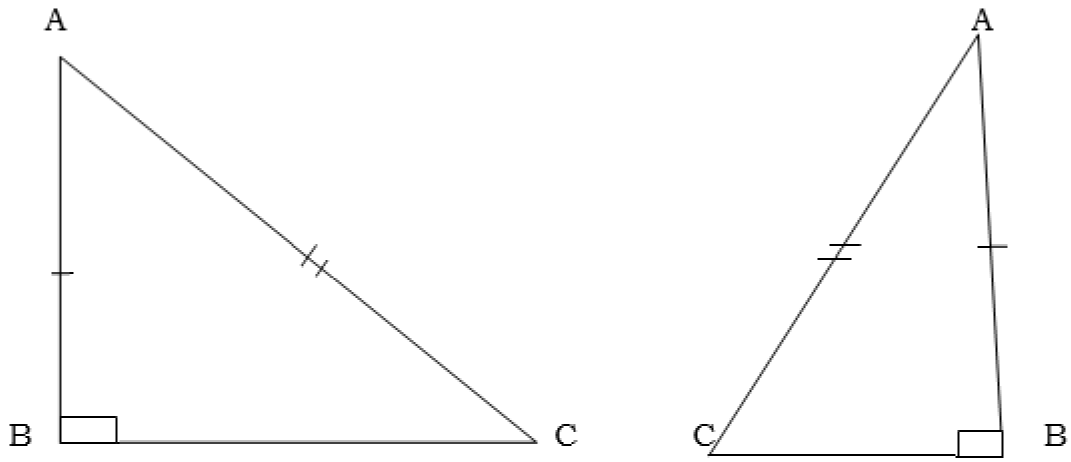
$D\hat{A}C = B\hat{C}A$-alternate interior a $\overline{DC} // \overline{AB}$

AC-common

$A\hat{C}D = B\hat{A}C$ -alternate

Case 4: Given that a right angle hypotenuse and one side (RHS)

The right angled triangles are congruent if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and side of another triangle

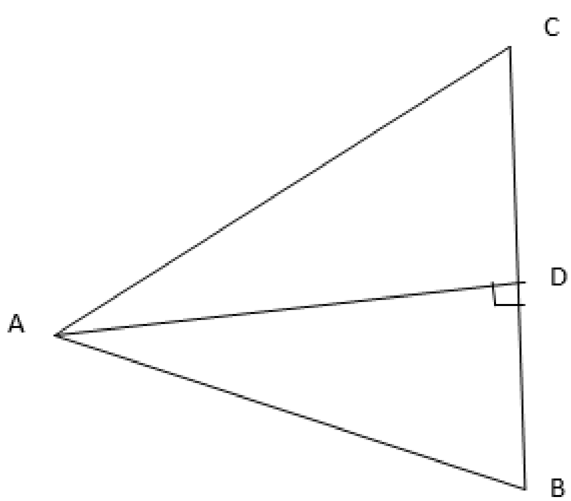


Example:

Use the figure below to prove that

$\triangle ABD \equiv \triangle ADC$

$\overline{DB} = \overline{DC}$



Solution

Given that $\triangle$ ABC, $\overline{AD}$ is lie to $\overline{BC}$ and

$\overline{AB}$ = $\overline{AC}$ required to prove $\triangle$ ABD $\equiv$ $\triangle$ ADC

$\overline{AC}$=$\overline{AB}$ given

$\overline{AD}$- common

$A\hat{D}C = A\hat{D}B$ -right angles

Therefore

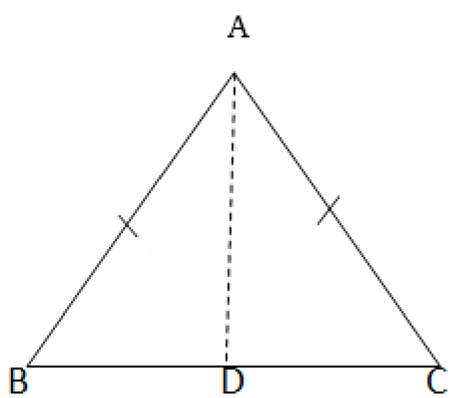
$\triangle$ ABD $\equiv$ $\triangle$ADC- R.H.S Theorem

Note:

R.H.S - Right angle hypotenuse side

Isosceles triangle theorem

The base angles of an isosceles triangle are equal



Given an isosceles triangle ABC

$\overline{AB}$ = $\overline{AC}$

Required to prove: $A\hat{B}C = A\hat{C}B$

Construction:-

An angle bisector of $B\hat{A}C$ is drawn to D

Proof:

In $\triangle ABD$, $\triangle ACD$

$\overline{AB}=\overline{AC}$ - given

$\overline{AD}$ -common

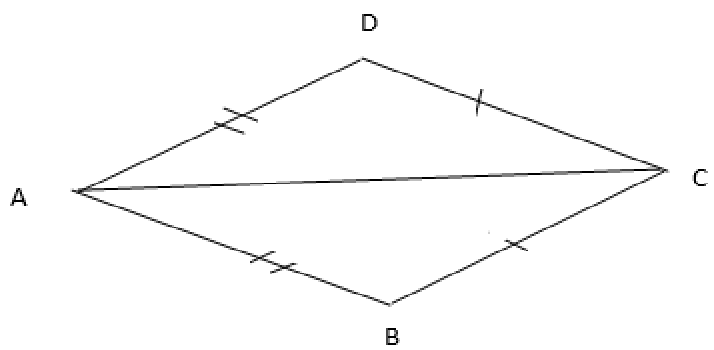
$B\hat{A}D = C\hat{A}D$- $\overline{AD}$ bisector $B\hat{A}C$

$\triangle BAD \equiv \triangle ADC$- SAS

Therefore; $A\hat{B}C=A\hat{C}B$- definition of congruence of triangle

Exercise 1.

In the figure below prove that $\triangle ACD \equiv \triangle ABC$



SOLUTION

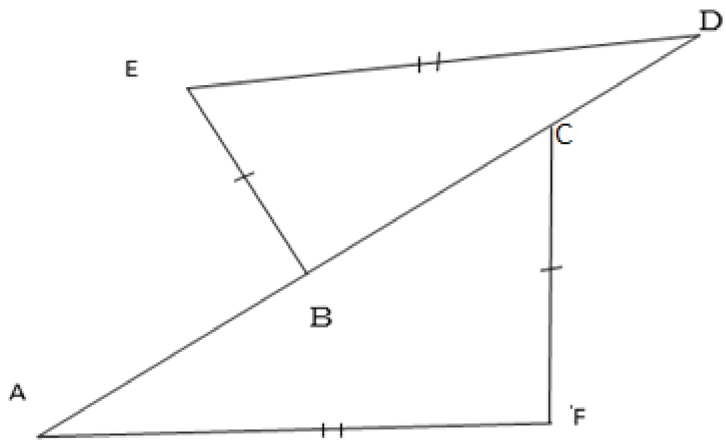
$\overline{AD}$ – common

$\overline{AD}=\overline{AB}$ given

$\overline{BC}=\overline{DC}$- given

Therefore $\triangle ADC \equiv \triangle ABC$ (SSS)

2.If $\overline{AB}=\overline{CD}$ and ABCD is a straight line prove that $B\hat{A}F = C\hat{D}E$



Solution

$\overline{ED}=\overline{AF}$ – Given

$\overline{EB}=\overline{CF}$- Given

ABCD = Common line

$\therefore \Delta ACF \equiv \Delta BED$ (SSS)

$B\hat{A}F=C\hat{D}E$

Thus $A\hat{F}C= D\hat{E}B$

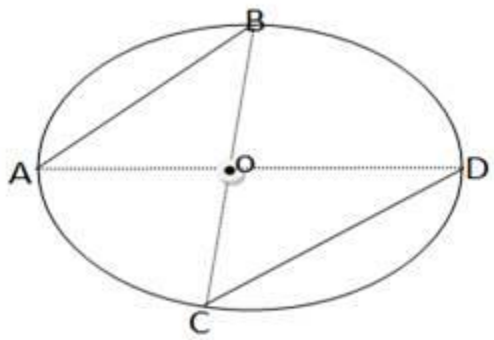
Therefore

$B\hat{A}F =$ CDE and AFC= $D\hat{E}B$

They are alternate interior angle

3 . $\overline{AB}$ and $\overline{CD}$ are two equal chords of all circles with center 0. Prove that

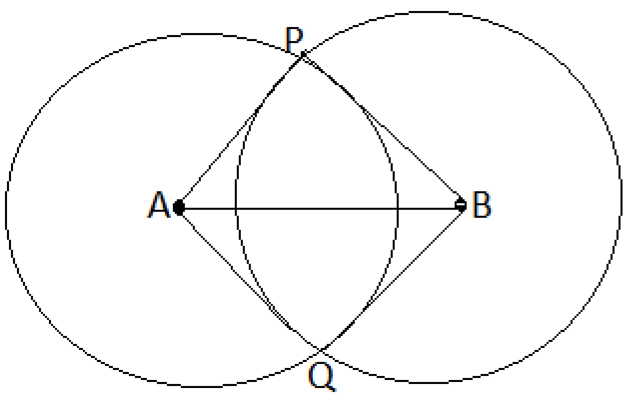
$A\hat{O}B= C\hat{O}D$



AB=CD given

BC =AD given

4 Two circles with centers A and B intersect at points P and Q prove that AB bisect $P\hat{A}Q$ [hint use triangles APB and AQB]



SOLUTION

BÂP= BÂQ

AB Common

PB =PA – circle radius

AQ= BQ - circle radius

AQB= APB

Therefore

AQB = APB

Therefore PQA = QAP

5. Two line segments $\overline{DC}$ and $\overline{AB}$ are drawn apart such that $\overline{AB}$= $\overline{DC}$ and ΔABD=ΔBDC prove that Δ DAB ≡ ΔDCB

SOLUTION

$\overline{DC}$=$\overline{AB}$

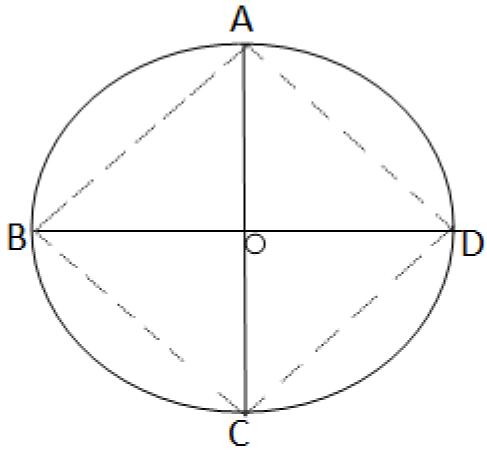
DB- common

ABD= BDC – given

Therefore

DAB = BCD

6. O is the center of the circle ABCD, if AC and BD and diameter of the circle and the line segments AD, AB and CB are drawn prove that $\overline{\overline{AD} = \overline{BC}}$



Solution

$A\hat{O}B = B\hat{O}C$ – Right angles

$\overline{AO}=\overline{OC}$- definition of congruence

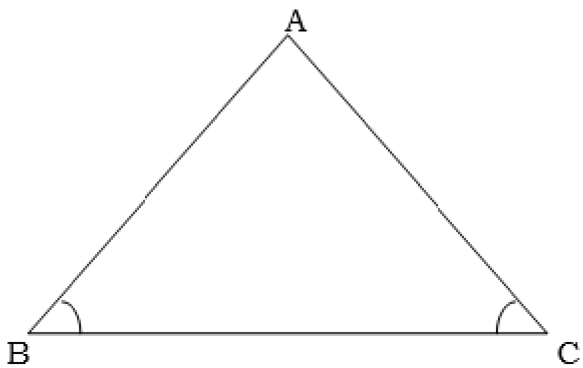
$\overline{BO}=\overline{OD}$

Therefore

$\overline{AD}=\overline{BC}$ –right angle hypotenuse side (RHS)

CONVERSE THE ISOSCELES TRIANGLE THEOREM

If two angles of a triangle are equal then sides opposite those angles are equal



Given that $\Delta ABC, A\hat{B}C = A\hat{C}B$

Required to prove $\overline{AB} = \overline{AC}$

Construction A and D are joined such that

$\overline{AD}$ is a bisector of $B\hat{A}C$

$AB\hat{D} = A\hat{C}D$ - given

AD- common

$B\hat{A}D = D\hat{A}C$ [construction]

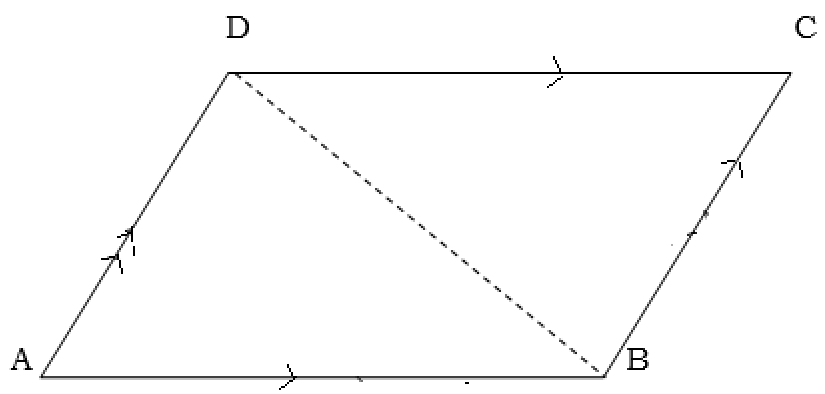
$\triangle ABD \equiv \triangle ACD$ - [AAS]

Therefore

$\overline{AB} = \overline{AC}$- [definition of congruence of triangles]

THEOREMS OF PARALLELOGRAMS

1) The opposite sides of the parallelogram are equal



Given a parallelogram ABCD

Required to prove

$\overline{AD} = \overline{BC}$

$\overline{AB} = \overline{DC}$

Construction:D is formed to B

$A\hat{D}B = C\hat{B}D$ -is interior angles AB//DC

$A\hat{B}D = B\hat{D}C$ -is interior angles AB//DC

$\overline{BD}$= common

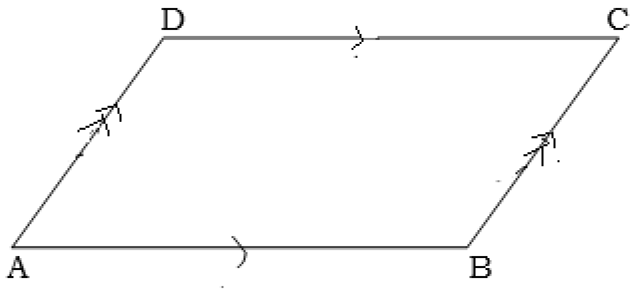
$\triangle ABD \cong \triangle BDC$- [AAS]

Therefore

$\overline{DC} = \overline{AB}$= definition of congruence of triangle

$\overline{AD} = \overline{BC}$ definition of congruence triangles.

2. The opposite angles of the parallelogram are equal



$D\hat{A}B = D\hat{C}B$

$A\hat{D}C + D\hat{A}B = 180^\circ$ Interior angle of the same side of $\overline{AD}, \overline{AB}$ // $\overline{DC}$

$A\hat{B}C + D\hat{A}B = 180^\circ$ interior angles on side of $\overline{AB}, \overline{AD}$ // $\overline{BC}$

Therefore

$A\hat{D}C + D\hat{A}B = ABC + D\hat{A}B$

$D\hat{A}B$- Is common

$A\hat{D}C = A\hat{B}C$

Similarly

$D\hat{A}B + A\hat{B}C = 180^\circ$ interior angles the same side of $\overline{AB}, \overline{AD}$ // $\overline{BC}$

$B\hat{C}D + A\hat{B}C = 180^\circ$ interior angles the same side of $\overline{BC}$, $\overline{AD}$ // $\overline{BC}$

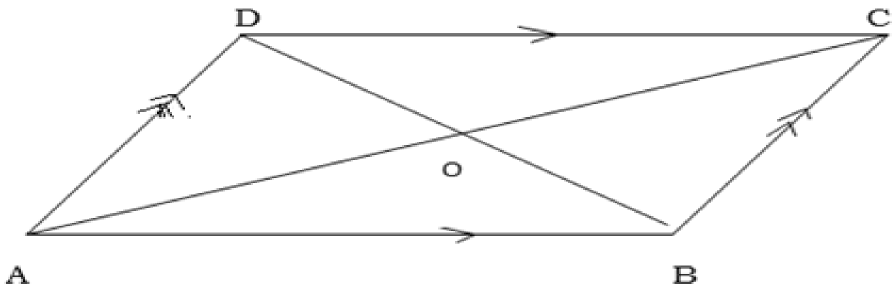
Therefore

$D\hat{A}B + ABC = B\hat{C}D + A\hat{B}C$

$D\hat{A}B = B\hat{C}D$

Hence opposite angles of a parallelogram are equal.

3.The diagonals of a parallelogram bisect each other



Given the parallelogram ABCD diagonal $\overline{AC}$ and $\overline{DB}$ intersecting at O

Required to prove $\overline{AO} = \overline{OC}$, $\overline{DO} = \overline{OB}$ in $\triangle AOB$ and $\triangle DOC$

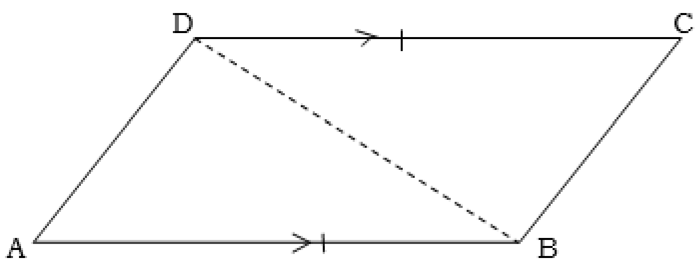
$D\overline{C}A = B\overline{A}C$- alternate interior angle $\overline{AB} // \overline{DC}$

$O\overline{D}C = O\overline{B}A$ - alternate interior angle $\overline{AB} // \overline{DC}$

AB=DC opposite of a parallelogram $\triangle DOC \equiv \triangle AOB$-AAS

4. The diagonals of a parallelogram intersect each other

If one pair of the opposite sides of a quadrilateral are equal and parallel then the other pair of the opposite side are equal and parallel.



Given a quadrilateral ABCD, $\overline{AB} = \overline{OC}$

$\overline{AB} // \overline{DC}$

Required to prove $\overline{AD} = \overline{BC}$, $\overline{AD} // \overline{BC}$

Construction: D and B are joined

In $\triangle$ABD and $\triangle$BCD

$C\hat{D}B = A\hat{B}D$ - alternate interior angle AB//DC

DB - common

$\overline{AB} = \overline{DC}$ - given

$\triangle$ ABD $\equiv$ $\triangle$BDC side side angles

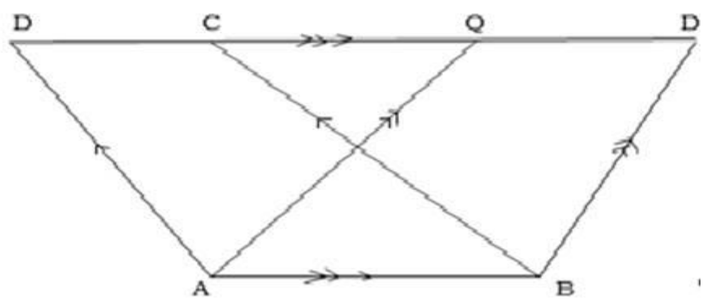
$\overline{AD} = \overline{CB}$ definition of congruence of triangles

$A\hat{D}B = D\hat{B}C$ definition of congruence triangles

Since $A\hat{D}B = D\hat{B}C$, they are alternate interior angle $\overline{AD} // \overline{BC}$ hence the other pair of opposite sides are equal and parallel

Example

In the figure below if $\overline{CQ} = \overline{AB}$ prove that $\overline{DP} = 3\overline{AB}$



Given the figure above $\overline{CQ} = \overline{AB}$

Required to prove $\overline{DP} = 3\overline{AB}$

$\overline{AB} = \overline{CQ}$ - given

$\overline{AB} = \overline{DC}$ - opposite sides of a parallelogram

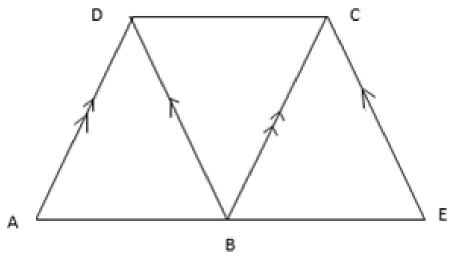
$\overline{AB} = \overline{QP}$ - opposite sides of a parallelogram

Since $\overline{DP} = \overline{DC} + \overline{CQ} + \overline{QP}$

Then $\overline{AB} = \overline{AB} + \overline{AB} + \overline{AB}$

$\overline{DP} = 3\overline{AB}$- hence shown

Using figure below prove that $\overline{EB} = \overline{AB}$ (A, B, E lie on a straight line).



Given the figure ABCDE

Required to prove $\overline{EB} = \overline{AB}$

$\overline{AB} = \overline{DC}$- opposite sides of parallelogram

$\overline{BC} = \overline{DC}$ - opposite sides of parallelogram

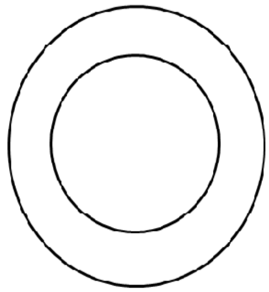
Therefore $\overline{EB} = \overline{AB}$

SIMILARITY AND ENLARGEMENT

Similar figures:

Two polygons are said to be similar if they have the same shape but not necessarily the same size.

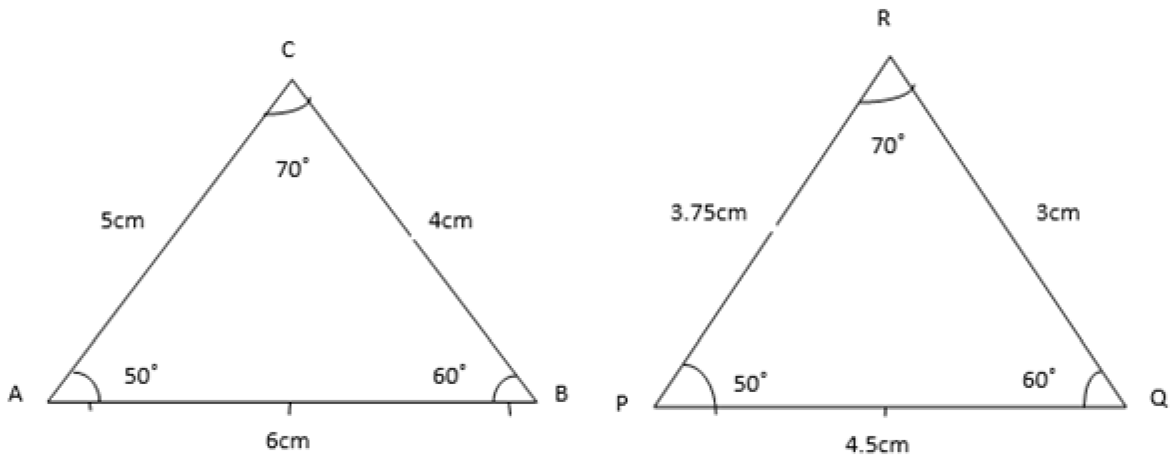
When two figures are similar to each other the corresponding angles are equal and the ratios of corresponding sides are equal.



SIMILAR CIRCLES

SIMILAR TRIANGLE

Triangle are similar when their corresponding angles are equal or corresponding sides proportional consider the figure below :



$\hat{CAB}$ is corresponding to $\hat{RPQ}$ each is 50^0

$\hat{ABC}$ is corresponding to $\hat{PQR}$ each is 60^0

$\hat{ACB}$ is corresponding to $\hat{PRQ}$ each is 70^0

Since corresponding angles are equal then the two triangles are similar

Also:

$\overline{AC}$ corresponding $\overline{PR}$; $\frac{\overline{AC}}{\overline{PR}} = \frac{5}{3.75} = \frac{4}{3}$

$\overline{AB}$ Corresponding $\overline{PQ}$; $\frac{\overline{AB}}{\overline{PQ}} = \frac{6}{4.5} = \frac{12}{9} = \frac{4}{3}$

Since the ratio of corresponding sides are equal then the two triangles are similar

Note

($\sim$) is a sign of similarity, from above ΔABC $\sim$ Δ PQR

Examples

1. Given that Δ SLK $\sim\Delta$NFR, identify all the corresponding angles and corresponding sides

Solution:

$S\hat{L}K$ corresponds $N\hat{F}R$

$S\hat{K}L$ corresponds $N\hat{R}F$

$K\hat{S}L$ corresponds $R\hat{N}P$

Corresponding sides;

$\overline{SK}$ corresponds $\overline{NR}$

$\overline{LK}$ corresponds $\overline{RF}$

$\overline{SL}$ corresponds $\overline{NF}$

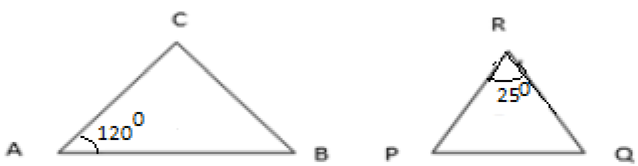
2. given that Δ ABC~Δ PQR, find $A\hat{B}C$

When

a) $B\hat{A}C = 120^{o}$ and $P\hat{R}Q = 25^{o}$

b) $Q\hat{P}R + B\hat{C}A = 145^{o}$

Solution:



Since $P\hat{R}Q = 25^{o}$ then $A\hat{C}B = 25^{o}$

Then $A\hat{B}C = 180^{o} - (25^{o} + 120^{0})$

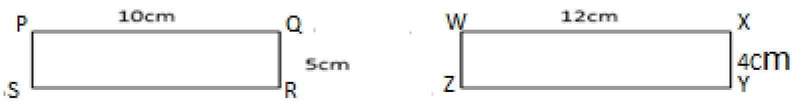
$= 180 - 145^{o}$

$= 35^{o}$

<u>Therefore $A\hat{B}C = 35^{o}$</u>

3. One rectangle has length 10cm and width 5cm. The second rectangle has length 12cm and width 4cm. Are the two rectangles similar? Explain

Solution:



PSR Correspond to $W\hat{Z}Y$

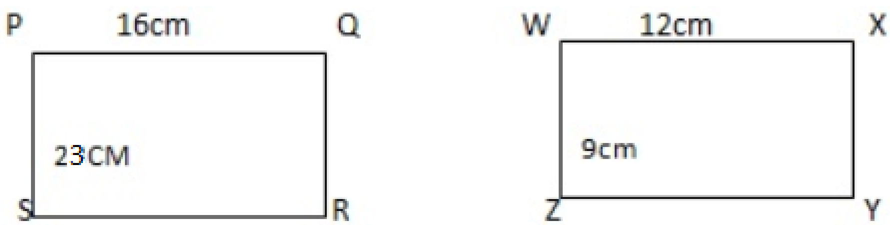
$\overline{SR}$ Correspond to $\overline{ZY}$ = $\frac{\overline{SR}}{\overline{ZY}}=\frac{10}{12}=\frac{5}{6}$

$\overline{SP}$ Corresponding to $\overline{ZW}$=$\frac{\overline{SP}}{\overline{zw}}=\frac{5}{4}=\frac{5}{4}$

<u>**Therefore;** the two rectangles are not similar because the ratio of corresponding sides are not proportional</u>

4. A rectangle has length 16cm and width 23cm, A second rectangle has length 12cm and width 9cm. Are the two rectangles similar? Explain

Solution:



$\overline{PS}$ Corresponds to $\overline{WZ}$ = $\frac{\overline{PS}}{\overline{WZ}}=\frac{23}{9}$

$\overline{PQ}$ Corresponds to $\overline{WX}$=$\frac{\overline{PQ}}{\overline{WX}}=\frac{16}{12}=\frac{4}{3}$

<u>Therefore;The rectangles are not similar because the ratio of corresponding sides are not proportional</u>

Conditions for two triangles to be similar;

1. Corresponding angles are equal or corresponding sides proportional

For other polygons

- Corresponding angles equal and corresponding sides proportional

QUESTIONS:

a) Given that Δ PQR $\sim$ Δ LMN and that Δ PQR $\sim$ Δ ABC identify the corresponding angles and sides between Δ ABC and Δ LMN.

solution

$A\hat{B}C$ corresponds to $L\hat{M}N$

$A\hat{C}B$ corresponds to $L\hat{N}M$

$B\hat{A}C$ corresponds to $M\hat{L}N$

$\overline{AB}$ corresponds to $\overline{LM}$

$\overline{BC}$ corresponds to $\overline{MN}$

$\overline{AC}$ corresponds to $\overline{LN}$

Exercise

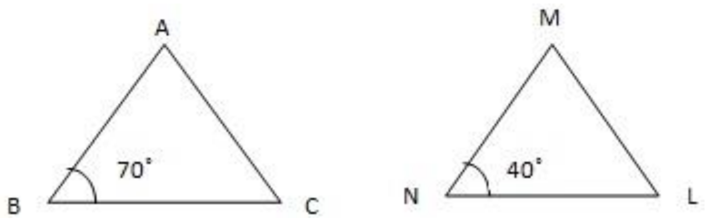
3.Given that ΔABC and ΔLMN are similar, find $A\hat{C}B$ when

$A\hat{B}C = 70^0$ and $M\hat{N}L = 40^0$

$A\hat{B}C + M\hat{L}N = 130^0$

Solution:

a) ABC = 70^O ,MNL = 40^0 ,ACB =?



$A\widehat{B}C + L\widehat{M}N = 130^0$

$180^0 – 130^0 = 50^0$

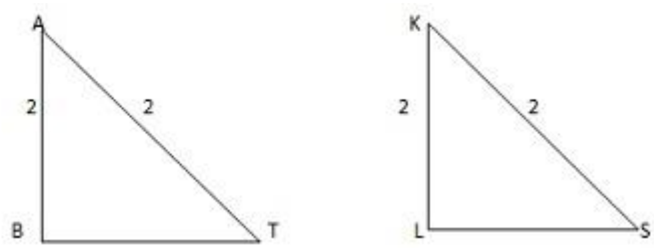
ACB=50°

4. Given that $\frac{\overline{AB}}{\overline{KL}} = 2$, $\frac{\overline{BT}}{\overline{LS}} = 2$ and $\frac{\overline{TA}}{\overline{SL}} = 2$

a) Name the triangles which are similar

b) Identify the corresponds angles

Solution:



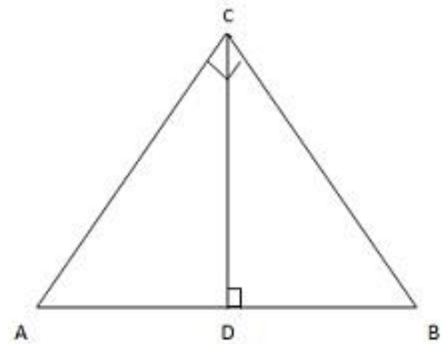
The triangles ABT and KLS are similar

b) $A\hat{B}T$ corresponds to $K\hat{L}S$

$B\hat{T}A$ corresponds to $L\hat{S}K$

$T\hat{A}B$ corresponds to $S\hat{K}L$

8. Name the triangles which are similar to ΔADC



$A\hat{D}C$ Corresponds to $B\hat{D}C$

$D\hat{A}C$ Corresponds to $D\hat{B}C$

$A\hat{C}D$ Corresponds to $B\hat{C}D$

10. Which of the following figures are always similar?

a) circles d) Rhombuses

b) Hexagons e) Rectangles

c) squares f) Congruent polygons

Solution:

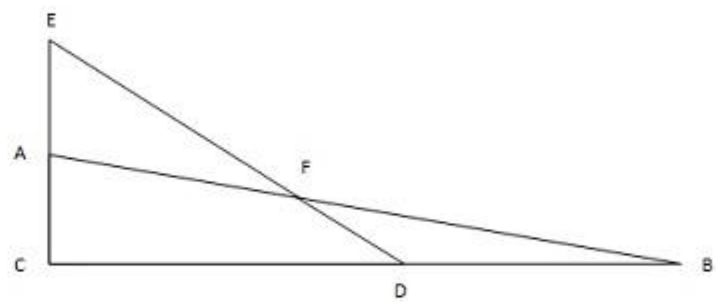
The figures which are always similar

a) circles

b) squares

Exercise 1

1. On the figure given below, $\overline{AC} = \overline{AE}$ and $M < AEF = 42^0$ find $m < AFE$



$M < AEF = 42^0$

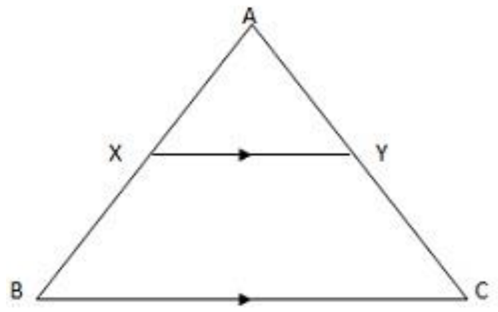
$M < AFE = ?$

$90^0 - 42^0 = 48^0$

$M < AFE = 48^0$

INTERCEPT THEOREM

A line drawn parallel to one side of a triangle divides the other two sides in the same ratio



If $\overline{XY} // \overline{BC}$ Then

$$\frac{\overline{AX}}{\overline{AB}} = \frac{\overline{AY}}{\overline{AC}}$$

OR $\frac{\overline{AB}}{\overline{AX}} = \frac{\overline{AC}}{\overline{AY}}$

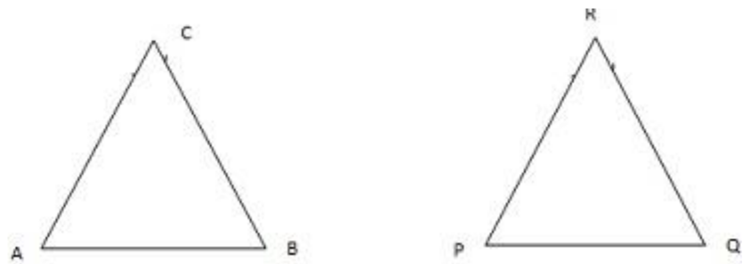
$\Delta AXY \sim \Delta ABC$

The converse of the theorem it is also true that, if it is given that

$$\frac{\overline{AX}}{\overline{AB}} = \frac{\overline{AY}}{\overline{AC}} \ then\ \overline{XY}//\overline{BC}$$

AAA – Similarity theorem

If a correspondence between two triangles is such that two pairs of corresponding angles are equal then the two triangles are similar



$B\hat{A}C = Q\hat{P}R$, $A\hat{C}B = P\hat{R}Q$

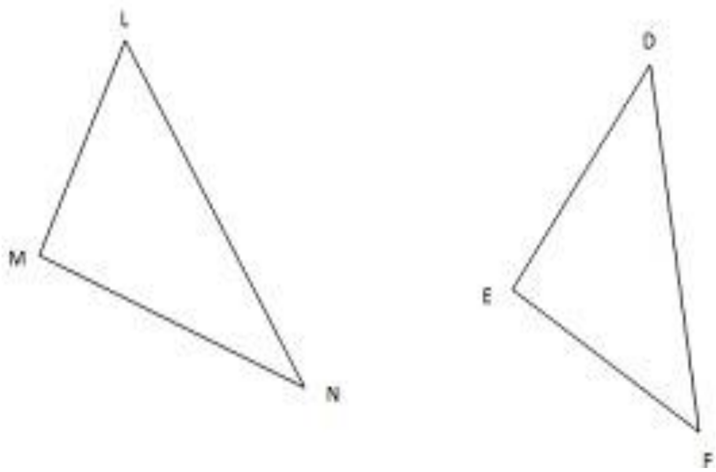
Then $A\hat{B}C = P\hat{Q}R$, Third angles of triangles

Therefore:

$\triangle ABC \sim \triangle PQR$

SSS - similarity Theorem

If the two triangles is such that corresponding sides are proportional, then the triangles are similar



If

$$\frac{\overline{LM}}{\overline{DE}} = \frac{\overline{LN}}{\overline{DF}} = \frac{\overline{MN}}{\overline{EF}}$$

Then $\Delta LMN \sim \Delta DEF$

Hence $M\hat{N}L = E\hat{F}D$

$L\hat{M}N = D\hat{E}F$

And $M\hat{L}N = E\hat{D}F$

SAS – Similarities theorem

If the two triangles is such that two pairs of corresponding sides are proportional and the included angles are congruent then the triangles are similar

If $\frac{\overline{AB}}{\overline{PQ}} = \frac{\overline{AC}}{\overline{PR}}$ and $B\hat{A}C = Q\hat{P}R$

Then $\Delta ABC \sim \Delta PQR$ and $A\hat{B}C = P\hat{Q}R$, $A\hat{C}B = P\hat{R}Q$

PROPERTIES OF SIMILAR TRIANGLES

From the previous discussion, properties of similar triangles can be summarized as:-

1. Corresponding angles of similar triangles are equal

2. Corresponding sides of similar triangles are similar

3. Two triangles are similar if two triangles of one triangle are respectively equal to two corresponding angles of the other

4. Two triangles are similar if an angle of one triangle equals an angle of other and the sides including these angles are proportional.

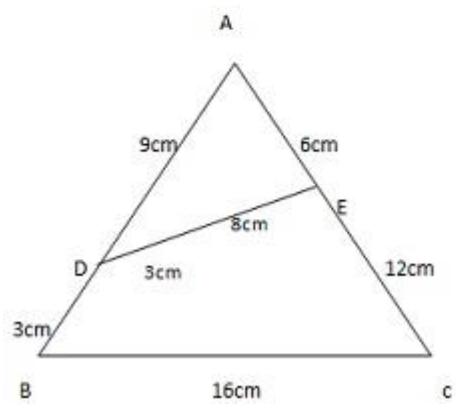
ENLARGEMENT

Scale enlargement

Scale – is a ratio between measurements of a drawing to the actual measurement.

It is normally started in the form 1: in example if a scale o a map is 1: 20000, then 1 unit on the map represents 20000 units on the ground

Scale = $\frac{\text{measurement of drawing}}{\text{Actual drawing /distance}}$



$$\frac{\overline{AD}}{\overline{AB}} = \frac{9}{12} = \frac{3}{4}, \frac{\overline{AE}}{\overline{AC}} = \frac{6}{18} = \frac{2}{6} = \frac{1}{3}$$

Examples of scales

1. Find the length of the drawing that represents

a) 1 stem when the scale is 1:500,000

Solution:

1:500,000 means 1 cm on the drawing represents 500,000 cm on the actual distance

$$\frac{1}{500000} = \frac{x}{1500000}$$

500,000x = 1500,000

$$X = \frac{1500000}{50000}$$

X = 3cm

The drawing length is 3cm

b) 45km when scale is 1cm to 900m

Solution:

Scale = 1: 90000

$$\text{Scale} = \frac{\text{Drawing Measurement}}{\text{Actual Measurement}}$$

$$X = \frac{4500000}{90000}$$

The drawing distance is 50cm

2. Find the actual length represented by

a) 3.5cm metres when the scale is 1: 5000m

Solution:

Scale $= \frac{\text{Drawing Measurement}}{\text{Actual Measurement}}$

$$\frac{1}{5000} = \frac{3.5}{y}$$

y = 5000 x 3.5

y = 17500cm

y = $\frac{17500}{100}$ = 175m

The distance is 175m

b) 1.8mm when the scale is 1cm to 500metres

Solution:

Scale $= \frac{\text{Drawing Measurement}}{\text{Actual Measurement}}$

$$\frac{1}{50000} = \frac{0.18}{v}$$

v= 0.18 x 50000

v = 9000cm

v = 90m

The actual length is 90m

Exercise:

1. Find the length of the drawing that represents

a) 200m when the scale is 1cm to 50meters

Scale = $\frac{\text{Drawing Measurement}}{\text{Actual Measurement}}$

$\frac{1}{20000} = \frac{x}{5000}$

$\frac{20000x}{20000} = \frac{5000}{20000}$

X = 4cm

The length of drawing = 4cm

b) 1.5 when the scale is 1cm to 100metres

$\frac{10000x}{10000} = \frac{150000}{10000}$

x = 15cm

The length of drawing = 15cm

d) 1600km when the scale is 1mm to 1km

$$\frac{1}{100,000} = \frac{x}{160,000,000}$$

$$\frac{100,000x}{100,000} = \frac{160,000,000}{100,000}$$

x = 1600km

The length of drawing is 1.6 mm

e) 10m when the scale is 1: 500

$$\frac{1}{500} = \frac{x}{1000}$$

$$\frac{1000}{500} = \frac{500x}{500}$$

x = 2cm

The length of drawing= 2cm

2. Find the actual length represented by

a) 13.15mm which the scale is 1: 4000

Scale = $\frac{\text{Drawing Measurement}}{\text{Actual Measurement}}$

$$\frac{1}{4000} = \frac{x}{13.15}$$

x = 0.0032875mm

b) 3.78cm when the scale is 1mm to 50km

$\frac{1}{50000000} = \frac{x}{37.8}$

$\frac{3.78}{50000000} = \frac{50000000x}{50000000}$

x = 0.0000000756

3. On a scale drawing the length of a ship is 42cm. If the actual length of the ship is 84cm, what is a scale if width of the ship is 23cm, what is the corresponding width of the drawing?

Solution:

Scale = $\frac{\text{Drawing Measurement}}{\text{Actual Measurement}}$

$\frac{x}{1} = \frac{42}{8400}$

$^{8400x}/_{8400} = {}^{42}/_{8400}$

x = 1:200

Scale = 1:200

$^{1}/_{200} = {}^{4x}/_{2300}$

$^{200x}/_{200} = {}^{2300}/_{200}$

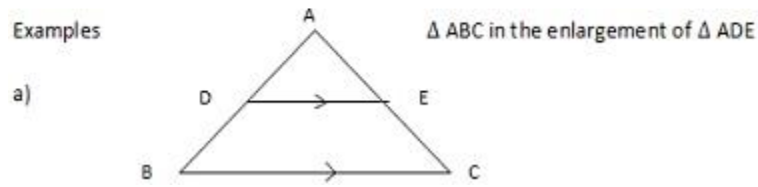
x = 11.5cm

The corresponding width of drawing = 11.5cm

ENLARGEMENT

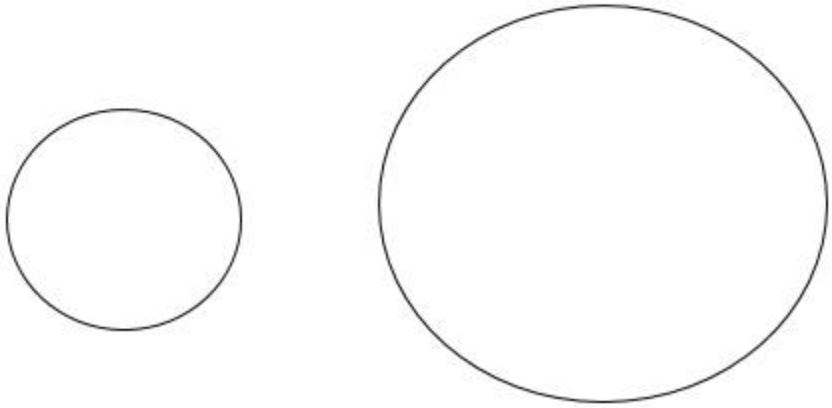
When two figures are similar, one can be considered the enlargement of the other

(a)



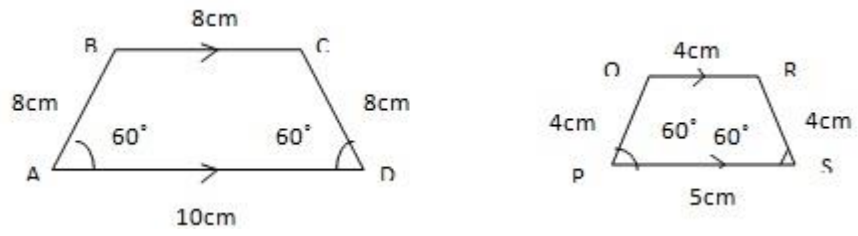
b) Square ABCD is the enlargement of PQRS

c)The larger circle is the enlargement of smaller circle



Example

1. State whether ABCD is the enlargement of PQRS



Solution:

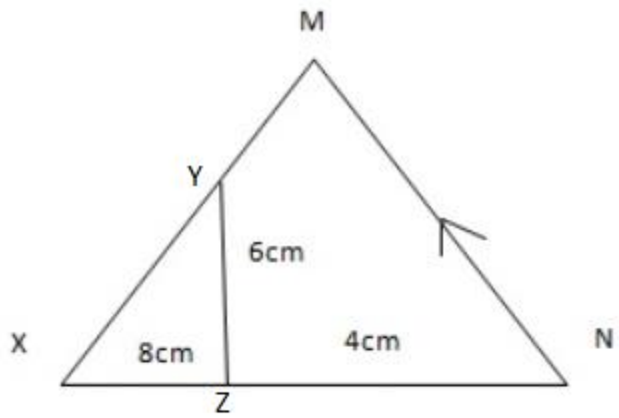
Since the correspond side are in the ratio 0f 2:1 and corresponding equal then ABCD $\sim$ PQRS

Scale factor:

If two polygons are similar and the ratio of their corresponding sides is 5:3, then the enlargement scale is 5/3

Example

Find the scale of enlargement hence calculate



a) $\overline{MN}$ b) $\overline{XY}$

Solution:

$\Delta_{MXN} \sim \Delta_{YXZ}$ – AA – similarity theorem scale of enlargement

$\frac{\overline{XN}}{\overline{XZ}} = \frac{12}{8} = \frac{3}{2}$

a) $\overline{MN} = \frac{3}{2} \times \overline{YZ}$

$\frac{3}{2}$ X $6 = \frac{18}{2} = 9$

b) $\overline{XY} = \frac{2}{3} \times \overline{MX}$ But $\overline{MX} = \overline{MN}$

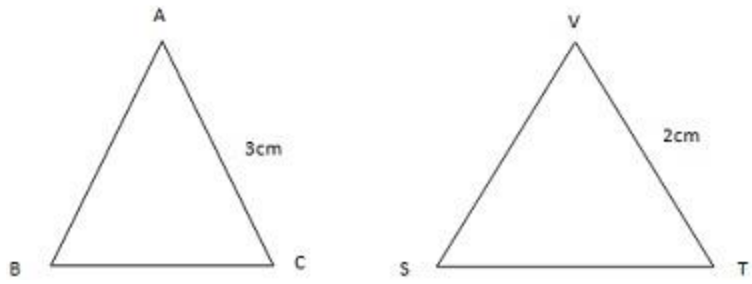
$\frac{2}{3}$ X $9 = 6$

Scale factor for areas

If two polygons have a scale factor of K then the ratio of the areas is K^2

Example

If Δ ABS $\sim\Delta$VST and the area of Δ STV is 6 square cm. find the area of ΔABC



solution

$\frac{A1}{A2} = K^2$ But $K = \frac{3}{2}$

$$\frac{A1}{6} = \frac{9}{4}$$

$$\frac{4A1}{4} = \frac{54}{4}$$

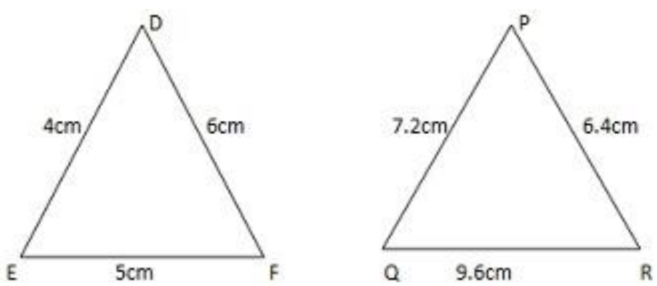
$$Area\ of\ \Delta\ ABC = 13.5cm^2$$

Exercise

1. Two triangle are similar but not congruent. Is one the enlargement of the others one triangle is the enlargement of the other

2. The length of rectangle is twice the length of another rectangle. Is one necessary an enlargement of other. Explain? No, Since the width are not necessarily in the same proportional as the lengths.

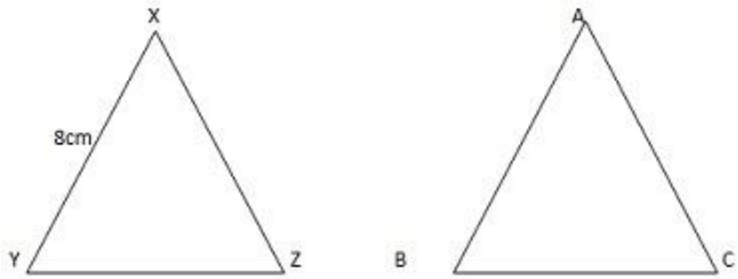
3. In figure below, show that Δ PQR is not an enlargement of Δ DEF



$\frac{ED}{QP} = \frac{4}{7.2} = \frac{5}{9}$, $\frac{EF}{QR} = \frac{5}{9.6} = \frac{25}{48}$

Δ PQR is not enlargement of ΔDEF

5. Triangle XYZ is similar to triangle ABC and XY = 8cm. If the area or the triangle XYZ is $24cm^2$ and the area of the triangle ABC is $96cm^2$, calculate the length of AB.



GEOMETRICAL TRANSFORMATIONS

-A transformation changes the position, size, direction or shape of objects.

-Transformation in a plane is a mapping which moves an object from one position to another within the plane. The new position after a transformation is called an image

Examples of transformations are

1. Reflection

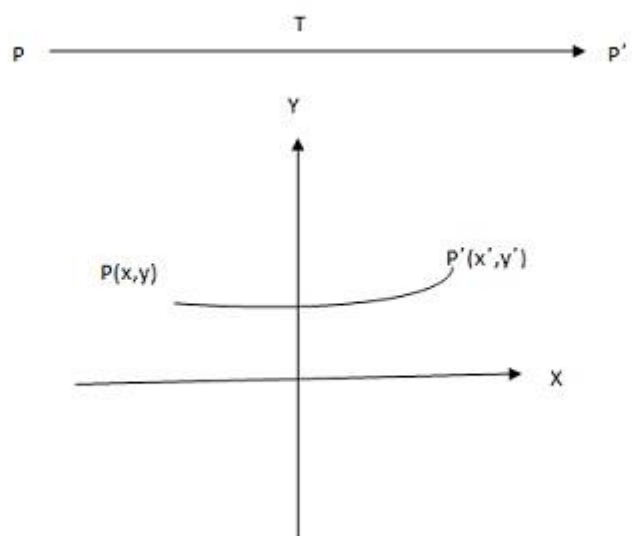
2. Rotation

3. Enlargement and

4. Translation

Suppose a point p[x, y] in the xy plane moves to a point pÎ„ [xÎ„, yÎ„] by a transformation T

P is said to be mapped to PÎ„ by T and may be indicated as



A transformation in which the size of the image is equal to the size of the object is called an Isometric mapping

REFLECTION

-Reflection is an example of an isometric mapping

-Isometric mapping means the distance from the mirror to an object is the same as that from the mirror to the image.

-The plane mirror is the line of symmetry between the object and the image.

-The line joining the object and the image is perpendicular to the mirror.

NOTE

-The symbol/letter for reflection is M.

-The reflection in X- axis and Y- axis are indicated as M_x and M_y respectively.

-The reflections in lines with certain equations are indicated with their equations as subscripts

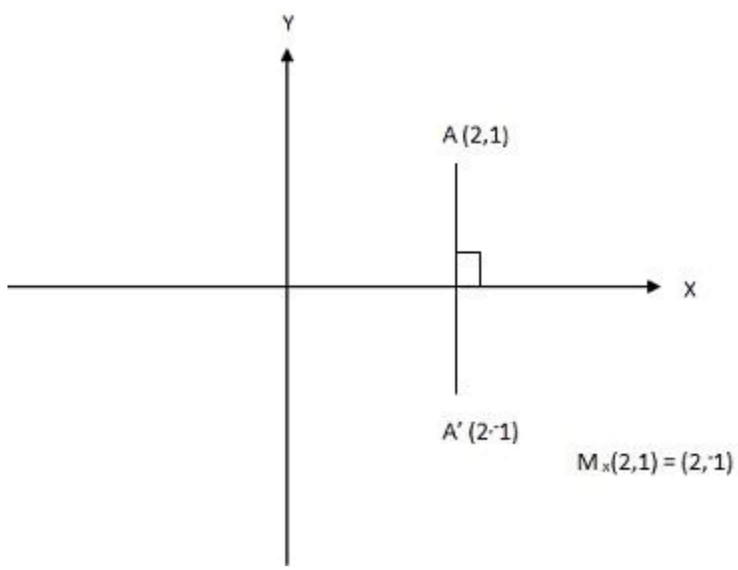
For example: My = x,is given by My = ^{-}x

A) Reflection in the x-axis

Example

1. 1. Find the image of the point A(2,1) after a reflection in the x-axis

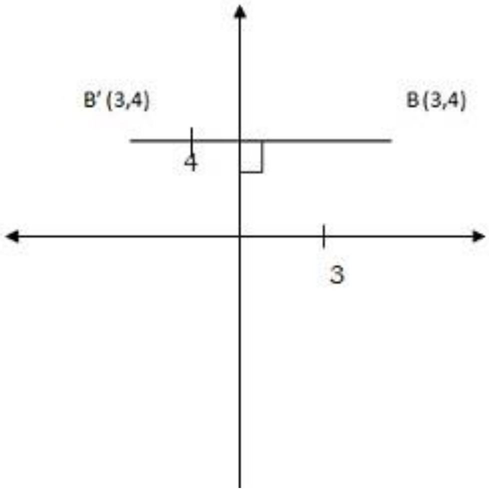
Solution:



(B)Reflection in y-axis

2. Find the image of 0(3,4) under the reflection in the Y-axis

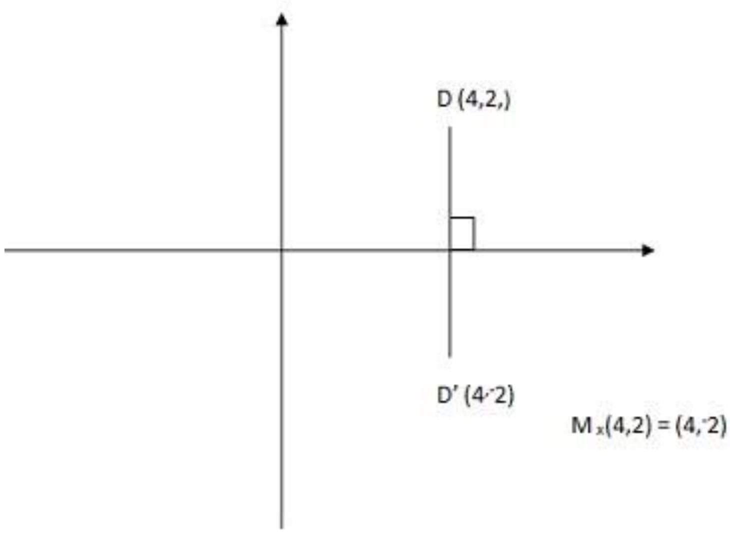
Solution:



Exercise 1

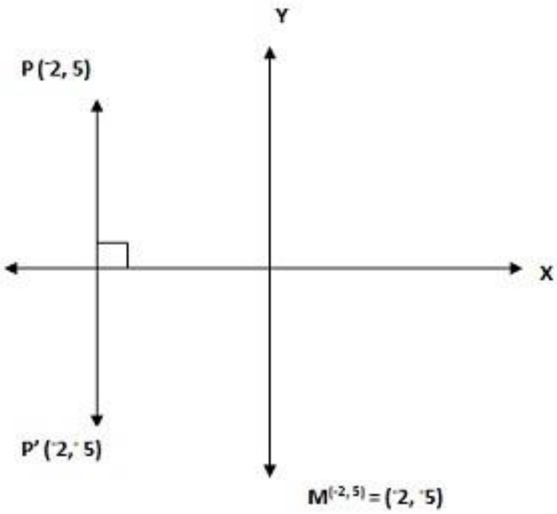
1. Find the image of the point D(4,2) under a reflection in the x-axis

Solution:



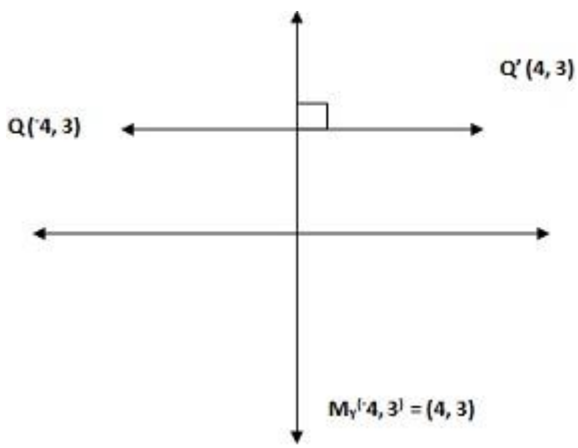
2.Find the image of the point P(-2,5) under the reflection in the x-axis

Solution:



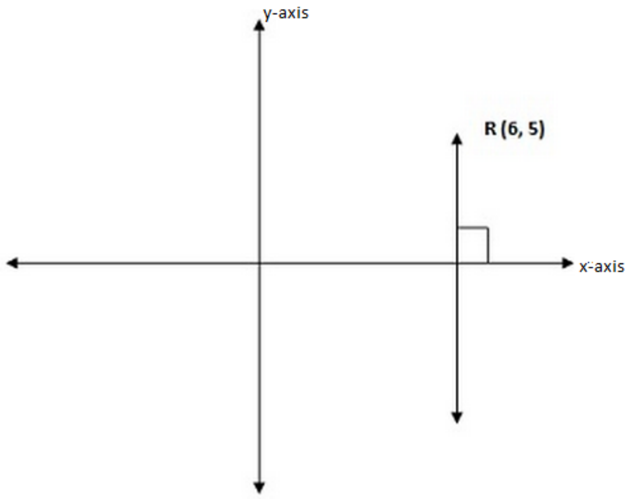
3. Point Q (-4,3) is reflected in the Y- axis

Solution:



4. Point R (6, 5) is reflected in the X-axis.

Find the coordinates of its image



5. The vertices of a triangle PQR are P (6, 2), Q (-2, 8), R (-5, -1). If triangle PQR is reflected in the Y axis, find coordinates of the vertices of its image.

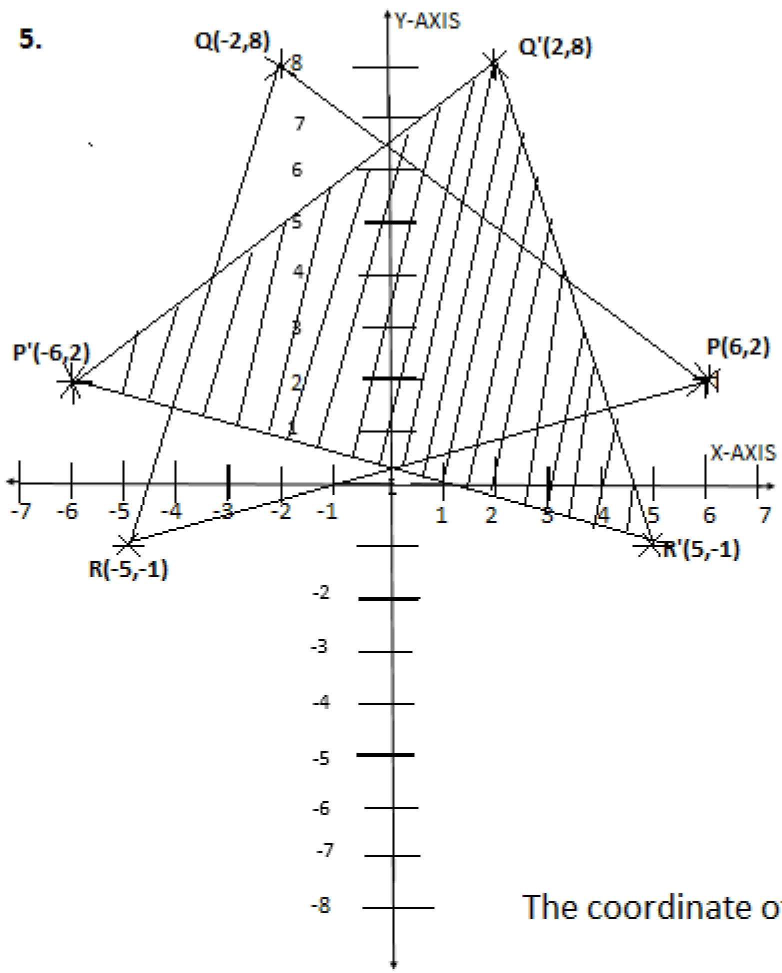
6. The vertices of rectangle area A (2,3), B (2,-4), C (4, -4), D (4,3) rectangle ABCD is reflected in the Y-axis

(a) Find the coordinates of the vertices of its image

(b) Draw a sketch to show the image

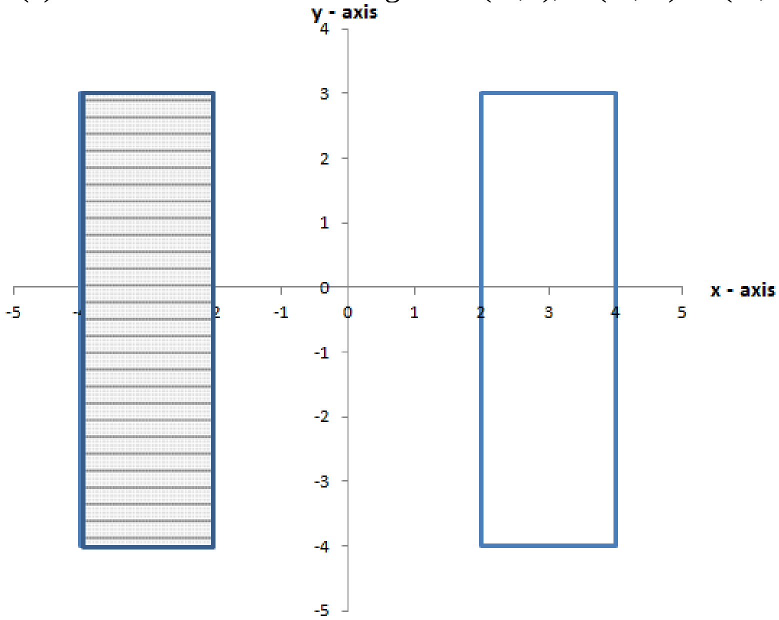
Solution

5.



The coordinate of the image is R'(5,-1),Q(2,8) and P'(-6,2)

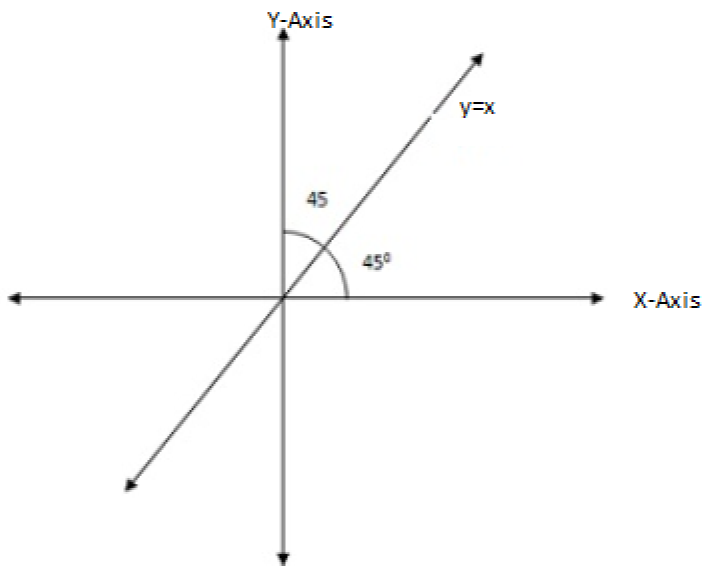
6(a)The coordinate of the image is A'(-2,3), B'(-2,-4). C'(-4,-4) and D'(-4,3)



C) THE REFLECTION IN THE LINE Y = X

The line y = x makes an angle 45° with the x and y axes

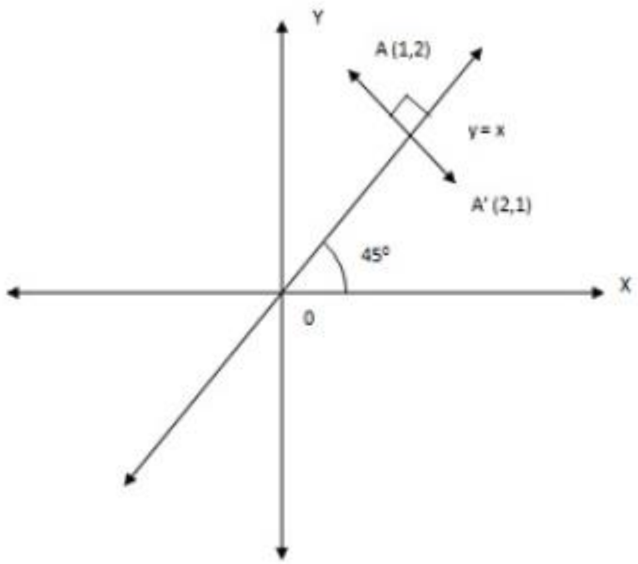
See the diagram



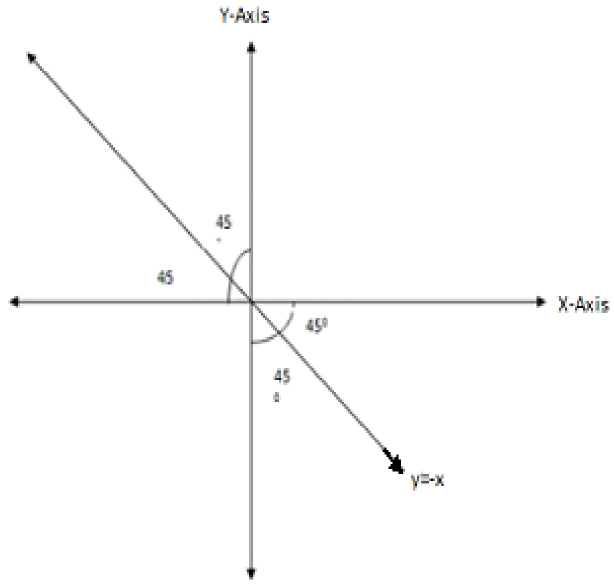
$\therefore M_{y=x}\ (x,y)=(y,x)$

Example

1. Find the image of point A(1,2) after a reflection in the line y=x



D) REFLECTION IN THE LINE Y = -X

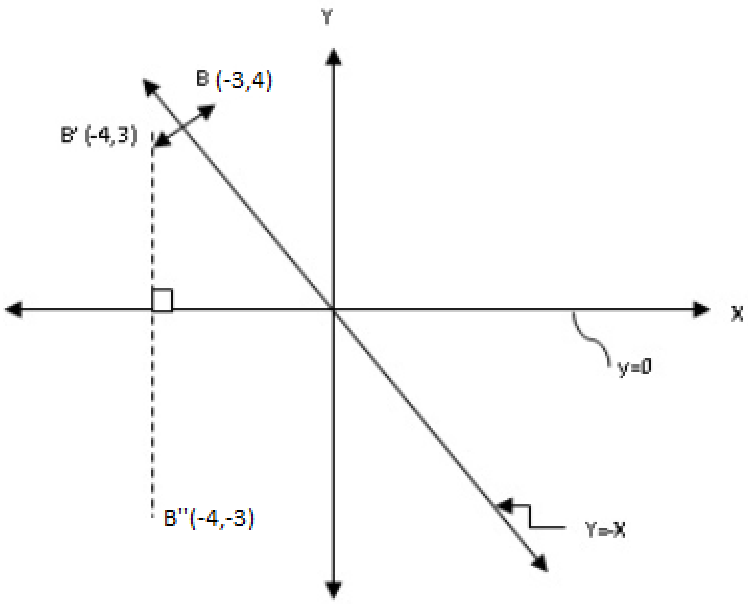


∴ $M_{y=-x}(X,Y)=(-y,-x)$

Example

Find the image of B (-3, 4) after a reflection in the line y=-x followed by another reflection in the line y=0

Solution



The reflection of B (-3, 4) in the line y = -x is B' (-4, 3) and the image of B' (-4, 3) after reflection in the line y = 0 is B' (-4, -3)

NOTE:

If P is the object the reflection of point P(x,y) will be:

1. $M_{x\text{-axis}}$ P (x,y) = P′ (x, -y)
2. M y-axis P (x,y) = P′ (-x, y)
3. M Y=x P (x,y) = P′ (y,x)
4. M y=-x P (x,y) = P′ (-y, -x)

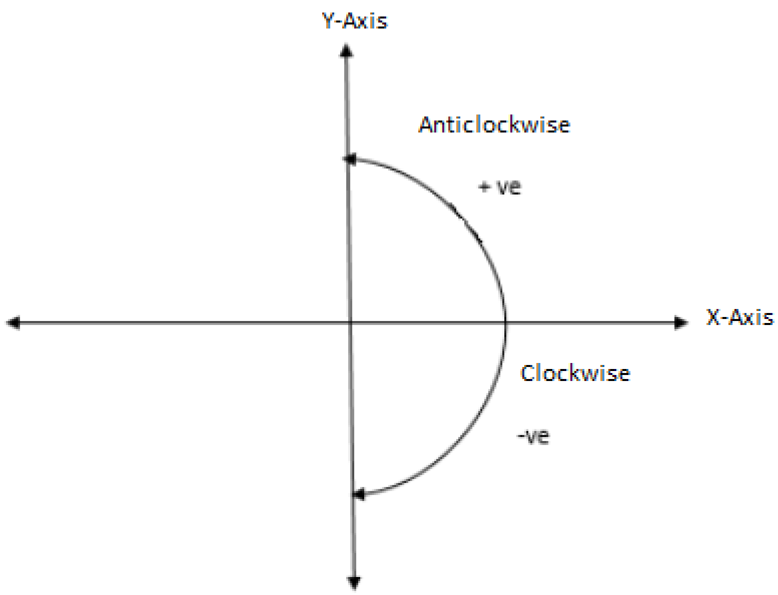
Rotation

- -Rotation is a transformation which moves a point through a given angle.

  -The angle turned through can be either in clockwise or anticlockwise direction.

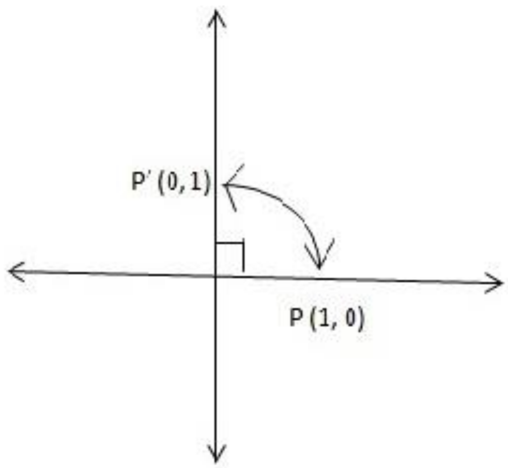
- -Rotation is an isometric mapping and usually denoted as R. R_θ means a rotation through an angle θ

- -In the XY plane when θ is measured in the clockwise direction, the angle is -ve and when measured anticlockwise direction the angle is +ve



Example

1. Find the image of the point P(1,0) after a rotation through 90^0 about the origin in anti-clockwise direction



TRANSLATION

- -Translation is a straight movement without turning.
- -A translation is usually denoted by T. For example T(1,1) = (6,1) means that the point (1,1) has been moved to (6, 1) by a translation T.
- -This translation will move the origin (0,0) to (5,0) and it is written as T = ($^{5}/_{0}$).
-

Examples:

1. A translation takes the origin to (-2, -5) find when it takes (-2, -3)

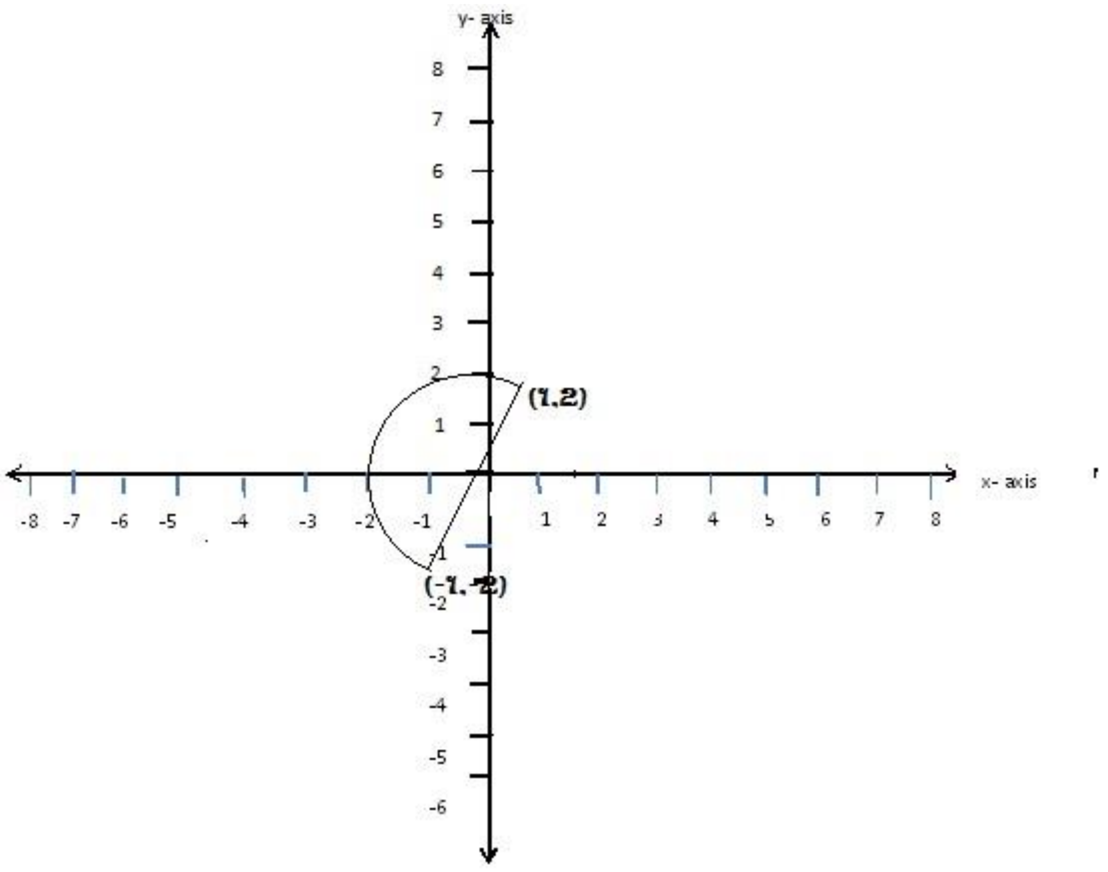
   Solution

   $$T\ (2,\text{-}3) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

   T (2,-3) = (0, -8)

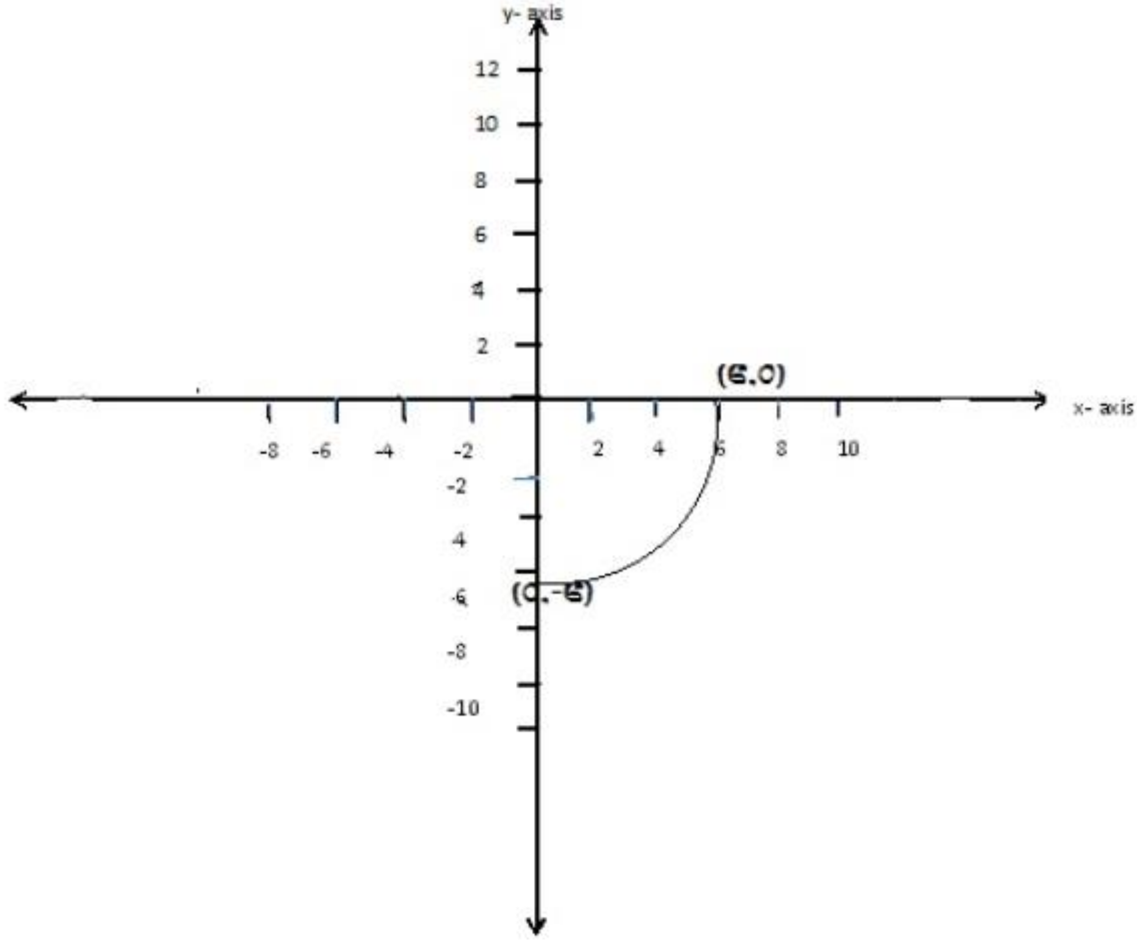
2. Find the image of the point (1,2) under a rotation through 180^0 anti-clockwise about the origin

Solution



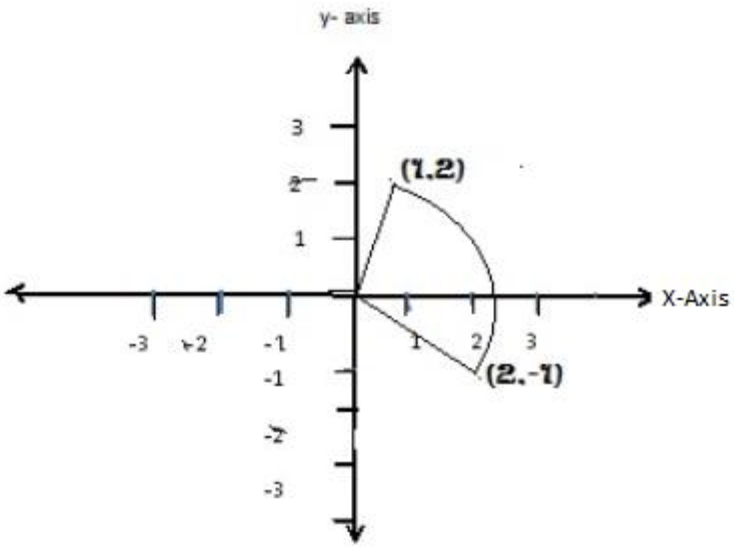
3. Find the rotation of the point (6, 0) under a rotation through 90^0 clockwise about the origin

Solution



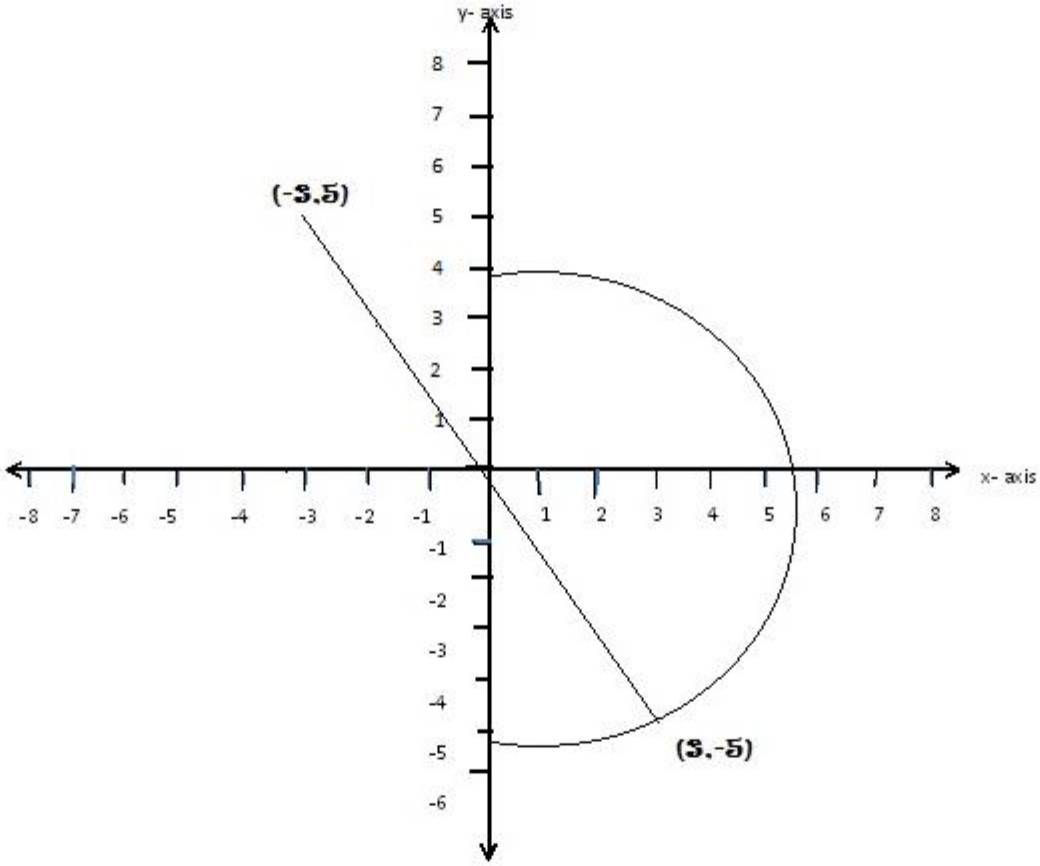
4. Find the image of (1,2) after a rotation of $^{-}90^{0}$ ant –clock wise

Solution



5. Find the image of (-3, 5) after a rotation of $^-180^0$

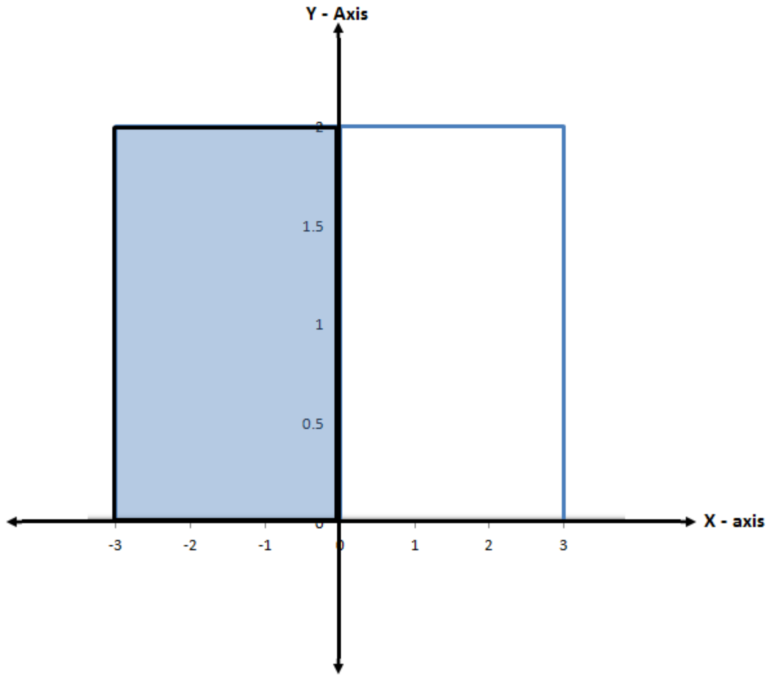
Solution



6. The vertices of rectangle PQRS are P(0,0), Q (3,0), R (3,2), S (0, 2). The rectangle is rotated through 90^0 clockwise about the origin.

(a) Find the co-ordinates of its image

(b) Draw the image



More examples on translation

1. Translation takes the origin to (-2, 5)

Find where it takes

(a) (-6, 6)

(b) (5, 4)

Solution

(a) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix}-2\\5\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$

$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}-2\\5\end{pmatrix} + \begin{pmatrix}-6\\6\end{pmatrix}$

$= \begin{pmatrix}-8\\11\end{pmatrix}$

: The translation takes (-6,6) to $\begin{pmatrix}-8\\11\end{pmatrix}$

(b) $\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}-2\\5\end{pmatrix} + \begin{pmatrix}5\\4\end{pmatrix}$

$= \begin{pmatrix}3\\9\end{pmatrix}$

: The translation takes (5,4) to (3,9)

2. A translation takes every point a distance of 1 unit to the left and 2 units downwards on the xy-plane.

Find where it takes

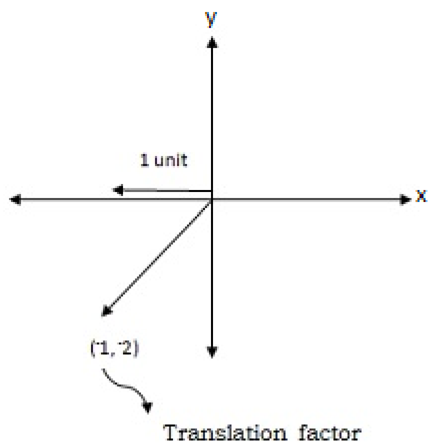
(a) (0,0)

(b) (1,1)

(c) (3,7)

Solution

(a).



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

: . The translation takes the origin to (-1, -2)

(b). $$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

3. 3. A translation moves the origin a distance 2 units along the line y= x upwards.

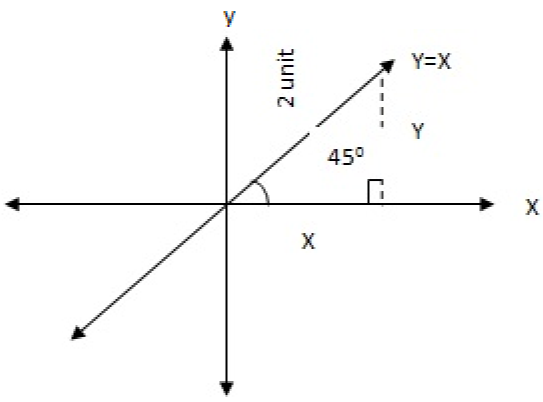
Find where it takes

(a) (0,0)

(b) (2, -1)

(c) (1, 1)

Solution



$\text{Cos } 45° = \frac{Adjacent}{Hypotenuse} = \frac{x}{2}$

$x = 2\cos 45^0 = 2 \times \sqrt{2}/_2 = \sqrt{2}$

Sin $45^0 = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{2}$

$y = 2\sin 45^0 = 2 \times \sqrt{2}/_2 = \sqrt{2}$

Translation factor $(\sqrt{2}, \sqrt{2})$

(a). $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix}$

: . The origin is translated to $(\sqrt{2}, \sqrt{2})$

(b). $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}+2}{\sqrt{2}-1} \end{pmatrix}$

: . (2, $^-$1) is translated to (($\sqrt{2}$ + 2), ($\sqrt{2}$ - 1))

4. A translation takes the point

(3, 2) to (-4, -5), Find where it takes

(0 , 0)

Solution

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$ where $\begin{pmatrix} a \\ b \end{pmatrix}$ is translation factor

$$\begin{pmatrix}-4\\-5\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix}$$

$$\begin{pmatrix}-4\\-5\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$$

$$\begin{pmatrix}-7\\-7\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$$

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}-7\\-7\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix}$$

$$= \begin{pmatrix}-7\\-7\end{pmatrix}$$

ENLARGEMENT

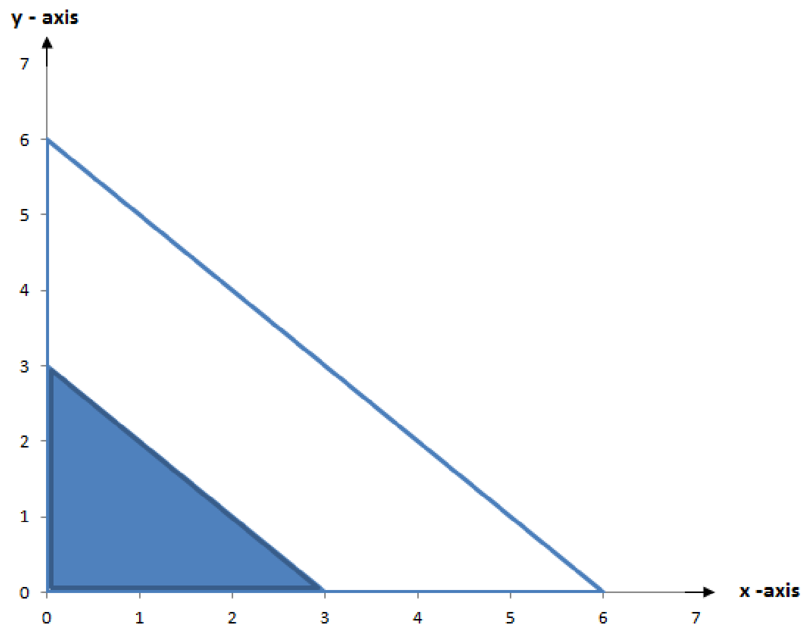
Enlargement is a transformation in which a figure is made larger (magnified) or made smaller (diminished).

- The number that magnifies or diminishes a figure is called the enlargement factor usually denoted by letter K. If K is less than 1 the figure is diminished and if it is greater than 1 the figure is enlarged K times.

  - In case of closed figures if the lengths are enlarged by a factor K then the area is enlarged by K^2

  Examples: -

    1. Draw a triangle PQR with vertices P (0,0), Q (0, 3) and R (3, 0)



P' = 2 (0,0) = (0,0)

Q' = 2 (0,3) = (0,6)

R' = 2 (3,0) = (6,0)

2. From the above question, what is the area of the new (enlarged) triangle?

Solution.

Area of the original triangle

$= \frac{1}{2} \times 3 \times 3$

= 4.5 square units

The area of the new triangle = 4.5 x K^2

= 4.5×2^2

= 18 square units

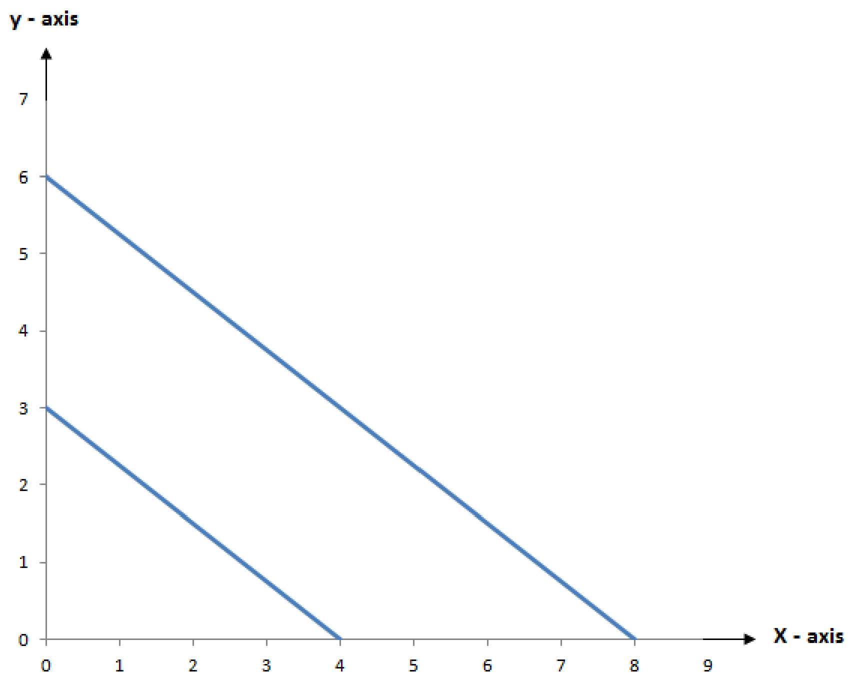
3. The line segment AB with coordinated A (4,0) and B (0,3) enlarge to AÎ„BÎ„ by a factor 2. Find the coordinates for AÎ„ and BÎ„

A' = 2 (4, 0)

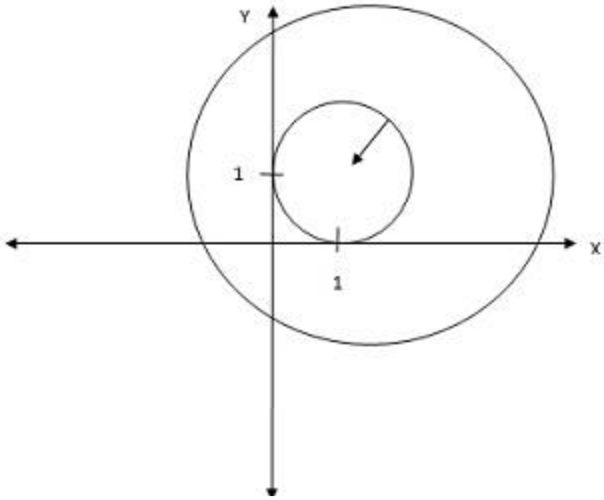
= (8,0)

B'=2(0,3)

=(0,6)



4. Find the image of the circle of radius one unit having its centre at (1,1) under enlargement transformation factor 5



Solution:

= 5(1,1)

= (5,5)

The image of the enlarged circle is (5,5)

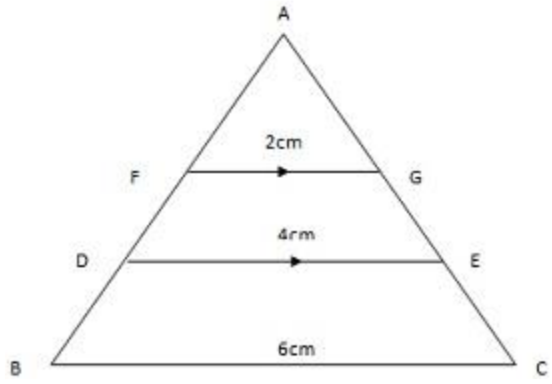
5.

EXERCISE 2

1. In figure below, $\overline{FG}$, $\overline{DE}$, and $\overline{BC}$ are parallel. What is the enlargement factor for transformin

(a) Δ ADE to Δ ABC?

(b) Δ ADE to Δ AFG?



(a) Δ ADE to Δ ABC $= {}^{4cm}/_{6cm}$

$= {}^{2}/_{3}$

(b) Δ ADE to Δ AFG $= {}^{2}/_{4}$

$= {}^{1}/_{2}$

2.The point P(6,2) is enlarged by factor of 4, what is the new end point?

Solution

4 (6,2)

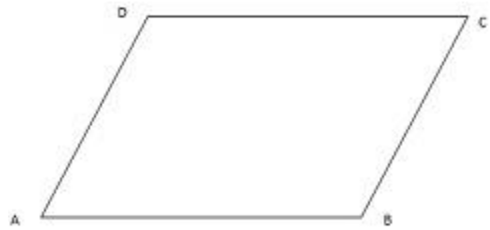
= (24, 8)

:. The point is (24, 8)

2. ABCD is a parallelogram

(a) What is the image of $\overline{BC}$ by the translation $\overline{CD}$

(b) Name the translation that maps $\overline{AB}$ onto $\overline{DC}$



Solution

(a) The image = $\overline{AD}$

(b) The translation = Vertical translation

EXERCISE 3

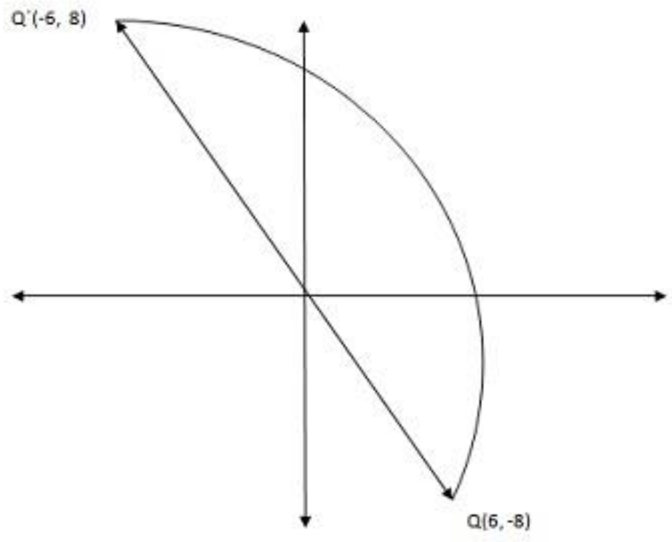
1. List 3 examples of isometric transformation

   - Translation
   - Rotation
   - Reflection

2. Is enlargement an Isometric transformation?

   Enlargement is not an Isometric transformation.

3.Find the image of the point Q (6, $^-8$) after a rotation of 90^0 about the



$R_{90°}(6, {}^{-}8) = ({}^{-}6, 8)$

Draw a parallelogram ABCD with vertices A (2,5), B (5,5) , C (6,8), D (3,8) find and draw the image parallelogram formed by the translation wich moves the origin to (2,4)

Solution

$A = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$A = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$

A = (4,9)

$B = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 9 \end{pmatrix}$

B = (7, 9)

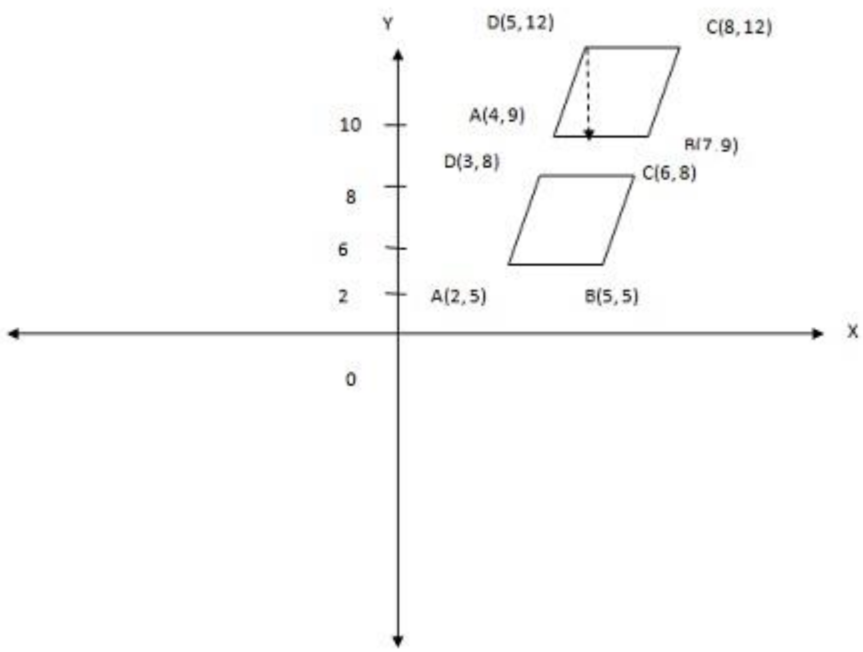
$$C = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 12 \end{pmatrix}$$

C = (8, 12)

$$D = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

D = (5, 12)



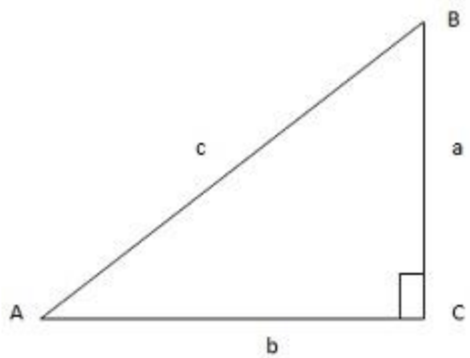
PYTHAGORAS THEOREM

PYTHAGORAS THEOREM

Pythagoras theorem is used to solve problems involving right angled triangles.

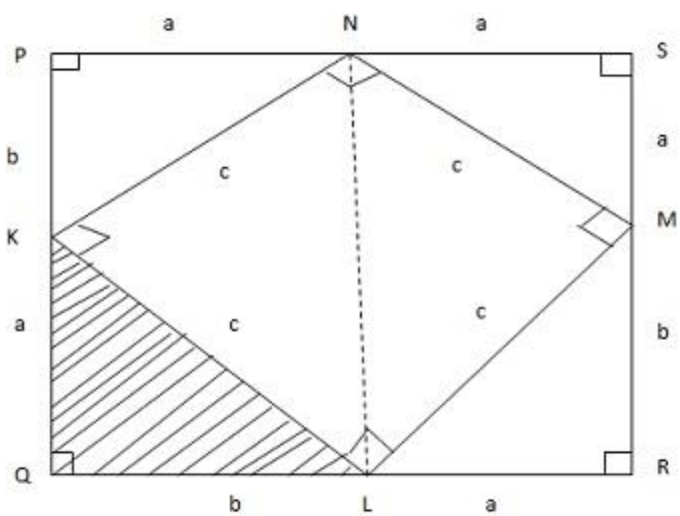
Statement:

In a right- angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.



$$C^2 = a^2 + b^2$$

Shown below



Required to prove: $c^2 = a^2 + b^2$

Construction : Joining L and N. Considering the trapezium PQLN:

Area of the trapezium

but area of trapezium = area Δ PKN + area Δ KQL + area Δ KLN

$\frac{1}{2}(a+b)(a+b) = \frac{1}{2}ab + \frac{1}{2}(c \times c)$

$\frac{1}{2}(a+b)(a+b) = ab + \frac{1}{2}c^2$

$\frac{1}{2}[a^2 + 2ab + b^2] = ab + \frac{1}{2}c^2$

$\frac{1}{2}a^2 + ab + \frac{1}{2}b^2 = ab + \frac{1}{2}c^2$

$a^2 + b^2 = c^2$

$\frac{1}{2}a^2 + \frac{1}{2}b^2 = \frac{1}{2}c^2$

Pythagoras theorem

Examples

1.The side s of a triangle containing the right angle have length of 5cm and 12cm.

Find the length of the hypotenuse

Solution

$C^2 = a^2 + b^2$

$C^2 = 5^2 + 12^2$

$C^2 = 25 + 144$

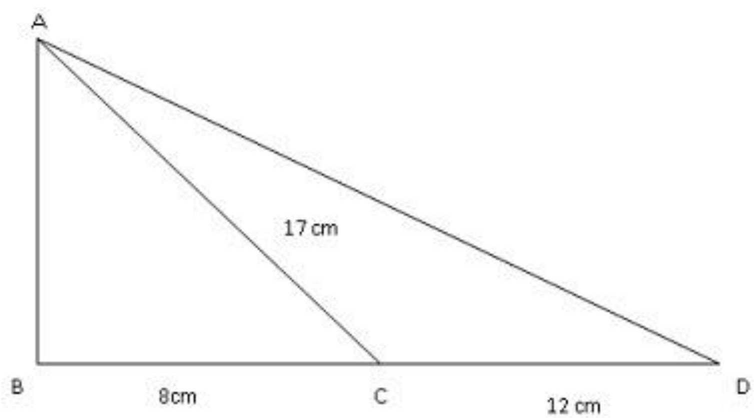
$C^2 = 169$

$C = \sqrt{169}$

$C = 13cm$

∴ The length of the hypotenuse = 13cm.

2. In figure below if AC =17cm, BC = 8cm, and CD = 12cm find AD



Solution:

$\overline{BC}^2 + \overline{AB}^2 = \overline{AC}^2$

$\overline{AB}^2 = \overline{AC}^2 - \overline{BC}^2$

$AB^2 = (17Cm)^2 - (8Cm)^2$

$= (289-64)\ Cm^2$

$\overline{AB}^2 = 225\ Cm^2$

$\overline{AB} = \sqrt{225}\ Cm^2$

$\overline{AB} = 15cm$

So $\overline{AD}^2 = \overline{BD}^2 + \overline{AB}^2$

$\overline{AD} = \sqrt{20^2 + 15^2}$

$= \sqrt{400 + 225}$

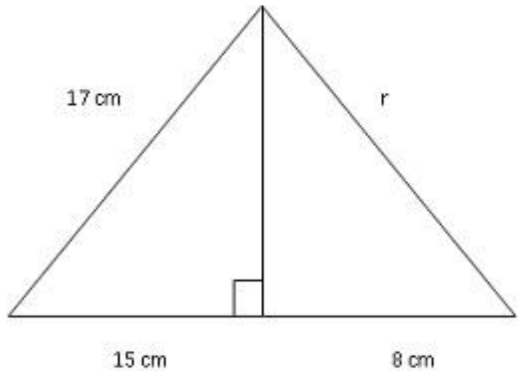
$= \sqrt{625}$

$= 25$

$:.\ \overline{AD} = 25cm$

EXERCISE

1. Calculate the unknown side of the following triangle



SOLUTION:

$17^2 = 15^2 + b^2$

$b^2 = 17^2 - 15^2$

$b^2 = 289 – 225$

$b = \sqrt{64}$

$b = 8cm$

$\therefore r^2 = 8^2 + 8^2$

$r^2 = 64 + 64$

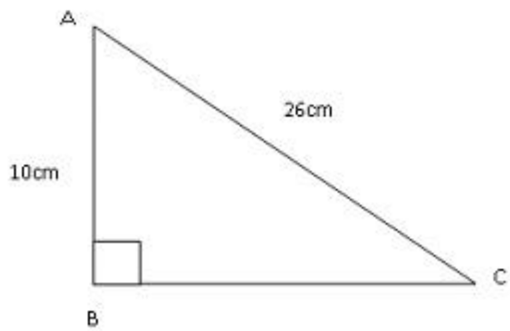
$r^2 = \sqrt{64 + 64}$

$r^2 = \sqrt{128}$

$\therefore$ r =11.31cm

2. Given triangle ABC, where B = 90^0. Find the lengths of the sides which are not given

(a) $\overline{AC}$ = 26cm, $\overline{AB}$ = 10CM



$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$

$10^2 + (\overline{BC})^2 = 26^2$

$100 + (\overline{BC})^2 = 676$

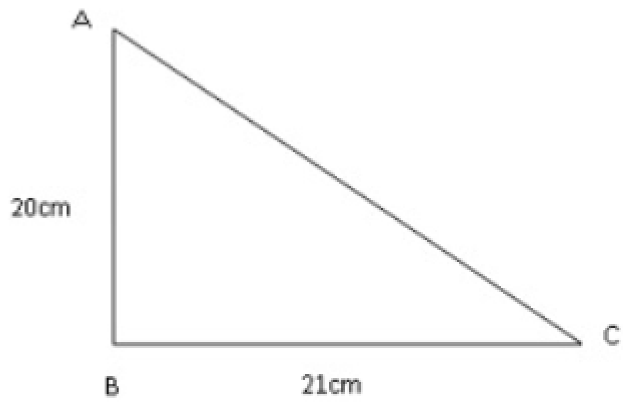
$(\overline{BC})^2 = 576$

$\overline{BC} = \sqrt{576}$

$\overline{BC} = 24$

<u>The length = 24cm</u>

(b) $\overline{AB}$ = 20CM, $\overline{BC}$ = 21CM



$\overline{AC}^2 = 20^2 + 21^2$

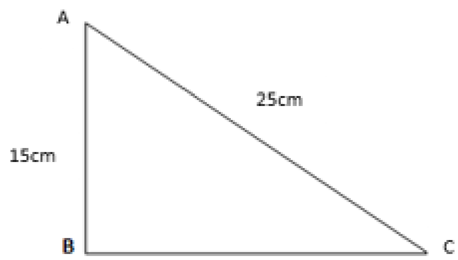
$\overline{AC}^2 = 400 + 441$

$\overline{AC}^2 = \sqrt{841}$

$\therefore \quad \overline{AC} = 29$

$\therefore$The length = 29cm

(c) $\overline{AC}$ = 25cm, $\overline{AB}$ = 15cm



$\overline{BC}^2 = \overline{AC}^2 - \overline{AB}^2$

$\overline{BC}^2 = 25^2 - 15^2$

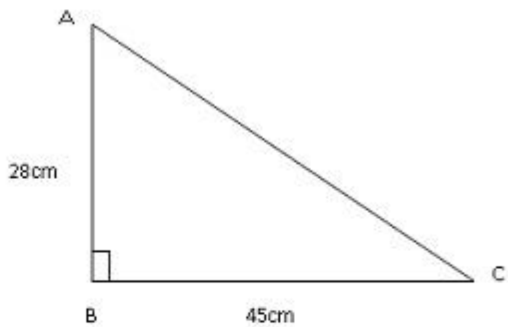
$\overline{BC}^2 = 625 - 225$

$\overline{BC} = \sqrt{400}$

$\overline{BC} = 20Cm$

$\therefore$ The length = 20cm

(d) $\overline{AB} = 28cm$, $\overline{BC} = 45cm$



$\overline{AB}^2 + \overline{BC}^2 = \overline{AC}^2$

$(45)^2 + (28)^2 = (\overline{AB})^2$

$2025 + 784 = (\overline{AB})^2$

$2809 = (\overline{AB})^2$

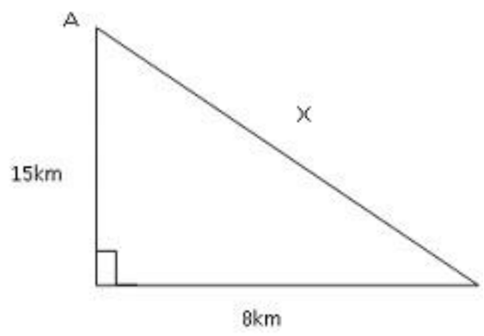
$(\overline{AB})^2 = \sqrt{2809}$

$\overline{AB} = 53$

The length = 53cm

3. A man travels 15km due north and then 8km due west. How far is he from his starting point?

Solution:



$X^2 = 15^2 + 8^2$

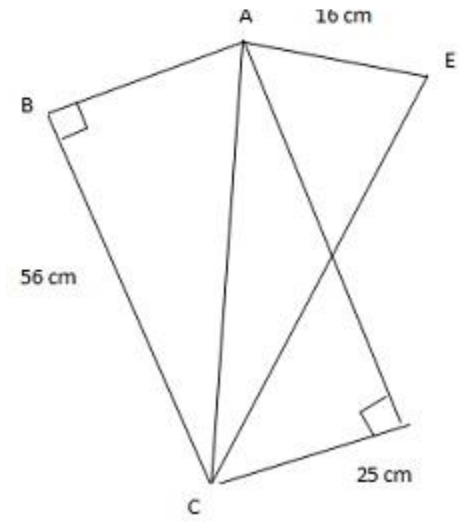
$X^2 = 225 + 64$

$X = \sqrt{289}$

$X = 17km$

$\therefore$ <u>He is 17km from his starting point</u>

4. In a diagram below find the distance $\overline{AC}$, $\overline{EC}$, $\overline{AD}$



(a) $\overline{AB}^2 + \overline{BC}^2 = \overline{AC}^2$

$(25)^2 + (56)^2 = \overline{AC}^2$

$625 + 3136 = \overline{AC}^2$

$3761 = \overline{AC}^2$

$\sqrt{3761} = \sqrt{AC^2}$

$\overline{AC} = \sqrt{3761}$ cm

(b) $\overline{EC}^2 = \overline{AE}^2 + \overline{AC}^2$

$16^2 + (\sqrt{3761})^2 = (\overline{EC})^2$

$256 + 3761 = (\overline{EC})^2$

$\therefore \overline{EC} = \sqrt{4017}$

(c) $\overline{CD}^2 + \overline{AD}^2 = \overline{AC}^2$

$(25)^2 + \overline{AD}^2 = \overline{AC}^2$

$625 + (\overline{AD})^2 = (\sqrt{3761})^2$

$625 + \overline{AD}^2 = 3761$

$\overline{AD}^2 = 3761 - 625$

$\overline{AD}^2 = 3136$

$\sqrt{\overline{AD}^2} = 3136$

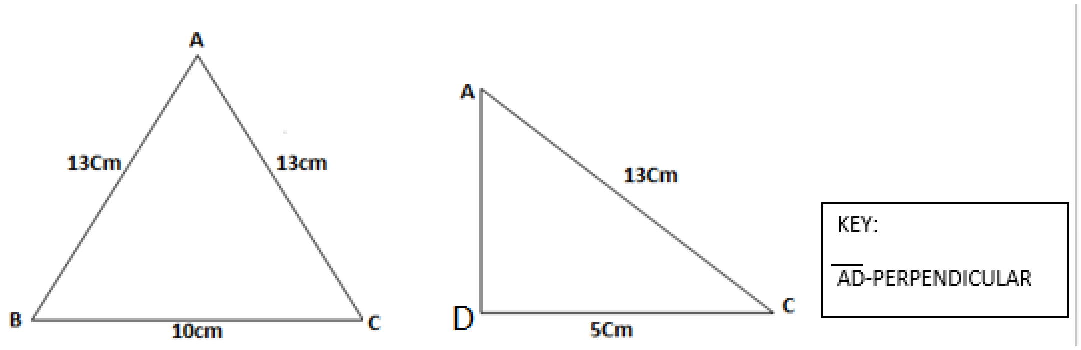
$\overline{AD} = \sqrt{3136}$

$\overline{AD} = 56cm$

4. A triangle ABC, $\overline{AB} = \overline{AC} = 13cm$, $\overline{BC} = 10cm$

Find the area of the triangle and the length of the perpendicular from C to B.

Solution:



$\overline{AD}^2 + \overline{DC}^2 = \overline{AC}^2$

$5^2 + \overline{AD}^2 = 13^2$

$25 + \overline{AD}^2 = 169$

$\overline{AD}^2 = 169 - 25$

$\sqrt{\overline{AD}^2} = \sqrt{144}$

$\overline{AD} = 12 \text{ cm}$

TRIGONOMETRY

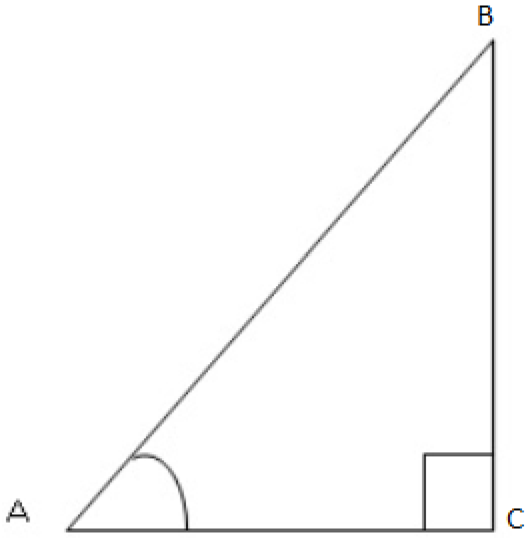
Trigonometric ratio

Introduction: TRI – is the Greek word which means three.

-Trigonometry is the branch of mathematics which deals with measurement.

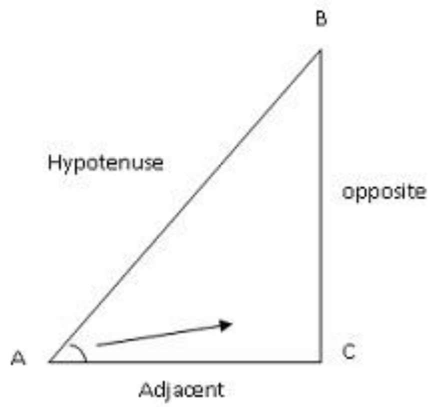
-A Trigonometric ratio consists of three parts that is – Hypotenuse, Adjacent and opposite.

Consider the diagram below which is the right angled triangle

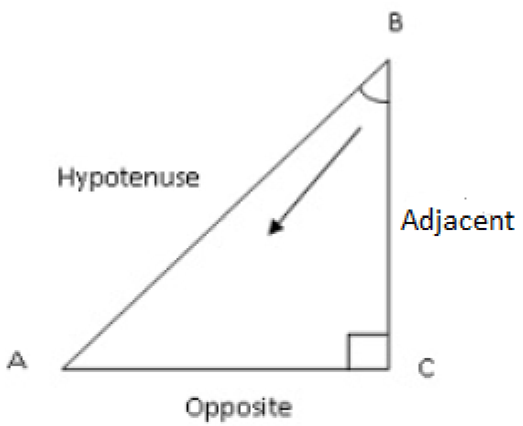


From ΔABC length $\overline{AB}$ is called Hypotenuse (Hy) but for the adjacent and opposite depend on the angle located.

For example

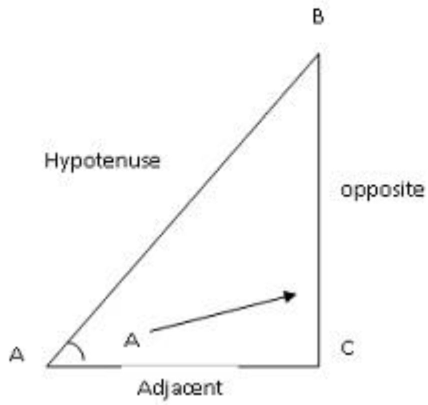


Assume angle B



For trigonometrical ratios we have sine, cosine and tangents which used to find the length of any side and angles.

Refer to the right angled triangle below:-



For sine(sin)

Sine $\hat{A} = \frac{\overline{BC}}{\overline{AB}}$

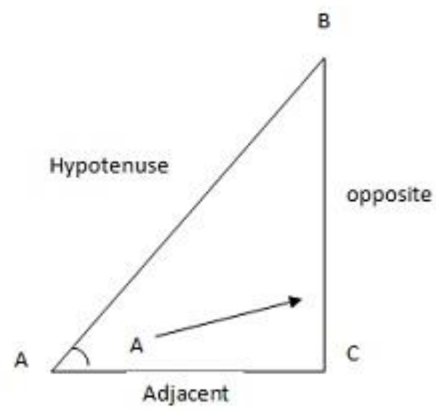
Where by $\overline{BC}$ is length called opposite and $\overline{AB}$ is length called Hypotenuse.

$$\text{Sine } \hat{A} = \frac{\text{opposite}}{\text{Hypotenuse}}$$

$$\text{Sin } \hat{A} = \frac{\text{OPP}}{\text{HYP}}$$

For cosine (cos)

Consider the diagram



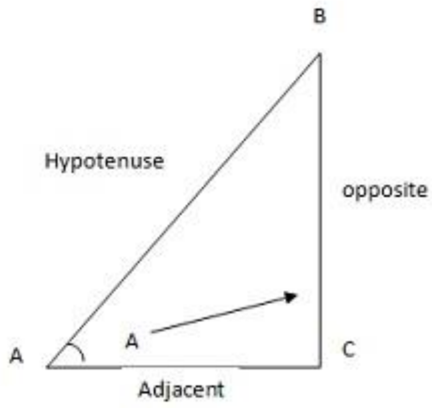
Cos $\hat{A} = \frac{\overline{AC}}{\overline{AB}}$ Where

$\overline{AC}$ is called Adjacent

$\overline{AB}$ is called Hypotenuse

Cos Â = $\frac{\text{Adj}}{\text{Hyp}}$

For Tangents (tan)



Tan Â = $\frac{BC}{AC}$ where

AC is called Adjacent

BC is called Opposite

Tan Â = $\frac{OPP}{ADJ}$

In summary

SO	TO	CA
H	A	H

Where: S – sine, T – tan, C – cos, O –opposite, A – adjacent, H – opposite

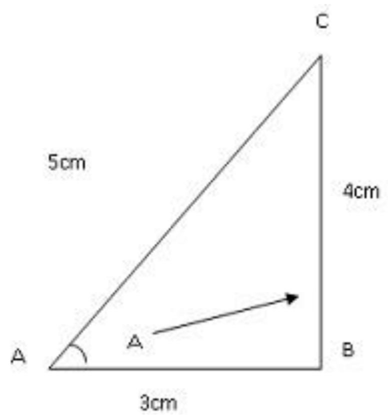
Example 1.

In triangle ABC = $\overline{AB}$ = 3cm $\overline{BC}$ = 4 cm and $\overline{AC}$= 5cm

Find a) Sin Â

b) Cos Â and (c) tan Â

Solution:



a) Sin Â = $\frac{\text{opp}}{\text{hyp}}$

Sin Â = $\frac{4\text{cm}}{5\text{cm}}$

Sin Â = $\frac{4}{5}$

b) Cos Â = $\frac{\text{adj}}{\text{hyp}}$

Cos Â = $\frac{3\text{cm}}{5\text{cm}}$

Cos Â = $\frac{3}{5}$

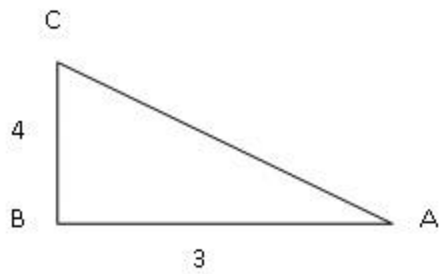
c) Tan Â = $\frac{opp}{adj}$

Tan Â = $\frac{4cm}{3cm}$

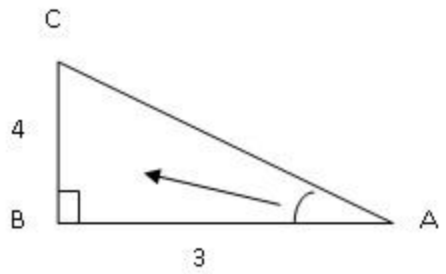
Tan Â = $\frac{4}{3}$

Example 2

Find the value of sin Â, cos Â and tan Â



Solution:



Where Opposite = 4, Adjacent = 3, Hypotenuse =?

Use Pythagoras theorem

$a^2 + b^2 = c^2$

$4^2 + 3^2 = c^2$

$\sqrt{16+9} = \sqrt{c^2}$

$c = 5$

$\sin \hat{A} = \frac{Opp}{hyp} = \frac{4}{5}$

$\cos \hat{A} = \frac{adj}{hyp} = \frac{3}{5}$

$\tan \hat{A} = \frac{Opp}{adj} = \frac{4}{3}$

Example 3

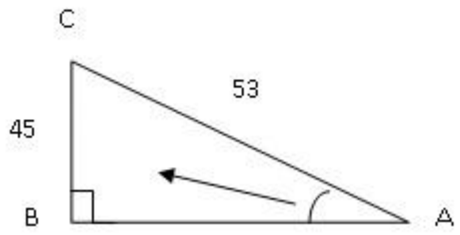
i) If the $\sin \hat{A} = \frac{45}{53}$ find the value of

(a) $\cos \hat{A}$, (b) $\tan \hat{A}$

ii) If $\tan B = \frac{m}{n}$ Find the value of (a) $\sin \hat{B}$ and (b) $\cos \hat{B}$

Solution:

given sin Â = $\frac{45}{53}$ where sin Â = $\frac{opp}{hyp}$



By using Pythagoras theorem

$a^2 + b^2 = c^2$

$(45)^2 + b^2 = (53)^2$

$2025 + b^2 = 2809$

$b^2 = 2809 - 2055$

$\sqrt{b^2} = \sqrt{784}$

b = 28

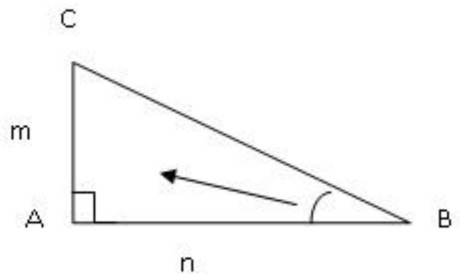
a) Cos Â = $\frac{adjacent}{hypotenuse} = \frac{28}{53}$

b) Tan Â = $\frac{opp}{adj} = \frac{45}{28}$

<u>Solution:</u>

ii) Given Tan $\hat{B}$ = m/n where Tan $\hat{B}$ = $\frac{opp}{adj}$

Find (a) Sin $\hat{B}$ (b) Cos $\hat{B}$



from Pythagoras theorem

$a^2 + b^2 = c^2$

$m^2 + n^2 = c^2$

$(m + n)^2 – 2mn = c^2$

$\sqrt{(m+n)^2 \; - 2mn} = \sqrt{c^2}$

$c = \sqrt{(m+n)^2 \; - 2mn}$

a. $\text{Sin B} = \frac{opp}{hyp} = \frac{m}{\sqrt{(m+n)^2 \; -2mn}}$

b. $\text{Cos } \hat{A} = \frac{adj}{hyp} = \frac{n}{\sqrt{(m+n)^2 \; -2mn}}$

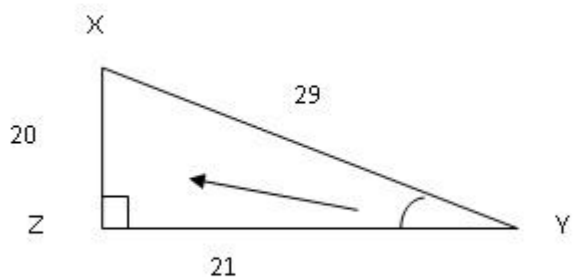
Example 4

If $\cos y = \frac{21}{29}$ Find (a) sin y (b)tan y

Solution:

Given that $\text{Cos } y = \frac{21}{29}$

But $\cos y = \frac{adj}{hyp}$



$a^2 + b^2 = c^2$

$a^2 + (21)^2 = (29)^2$

$a^2 + 441 = 841$

$a^2 = 841 - 441$

$\sqrt{a^2} = \sqrt{400}$

$a = 20$

a. $\text{Sin } y = \frac{\text{opp}}{\text{hyp}} = \frac{20}{29}$

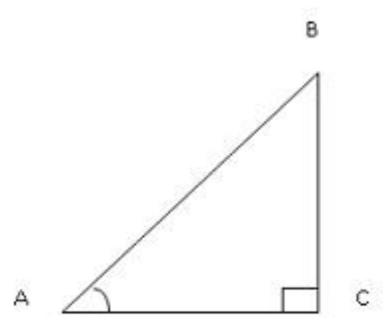
b. $\text{Tan } y = \frac{\text{opp}}{\text{adj}} = \frac{20}{21}$

Special angles

The trigonometrical ratios has the special angles which are 0°, 30°,45°, 60°, and 90°.The special angles does not need table or calculator to find their ratios.

To prove the value of trigonometric ratio for special angles

Consider the diagram below



Note:

when the point B move toward the point c the angle of A = 0° and the length of AB = AC and BC = 0

Sin 0° = $\frac{opp}{hyp}$

Sin = $\frac{BC}{AB}$

Sin 0° = $\frac{0}{AB}$

Sin 0°= 0

Tan 0°= $\frac{opp}{adj}$

Tan 0° = $\frac{0}{AC}$

Tan 0°= 0

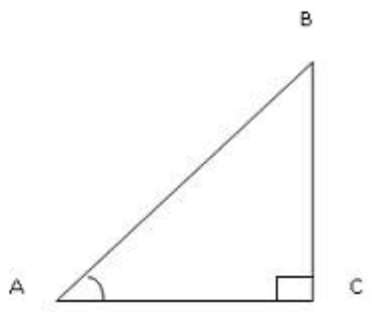
Cos 0° = $\frac{adj}{hyp}$

Cos 0° = $\frac{AC}{AB}$

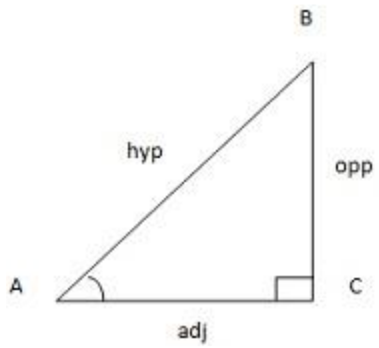
But AC = AB

Cos 0°= 1

Consider the diagram



If the point A moves towards point C the difference from A to C becomes zero. The angle between A and C become 90°



Sin 90° = $\frac{opp}{hyp}$ but opp = hyp

Sin 90° = 1

Cos 90° = $\frac{Adj}{Hyp}$

Cos 90° = $\frac{0}{Hyp}$

Cos 90° = 0

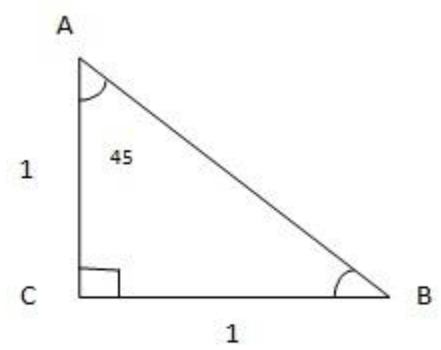
Tan 90° = $\frac{0pp}{Adj}$

Tan 90° = $\frac{0pp}{0}$

Tan 90° = ∞ (undefined)

Example

Find sin 45°, cos 45, and Tan 45° consider a right angled triangle ABC with 1 unit in length



The length Of AC = CB = 1 unit But use Pythagoras theorem to find the length AB.

From Pythagoras theorem

$a^2 + b^2 = c^2$

$1^2 + 1^2 = c^2$

$\sqrt{2} = \sqrt{c^2}$

c= $\sqrt{2}$

Sin 45 ° = $\frac{opp}{hyp}$

Sin 45° = $\frac{1}{\sqrt{2}}$

Rationalize the denominator

Sine 45° = $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

= $\frac{\sqrt{2}}{\sqrt{4}}$ = $\frac{\sqrt{2}}{2}$

Cos 45° = $\frac{\text{adj}}{\text{hyp}}$

Cos 45° = $\frac{1}{\sqrt{2}}$

Cos 45° = $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

= $\frac{\sqrt{2}}{\sqrt{4}}$ = $\frac{\sqrt{2}}{2}$

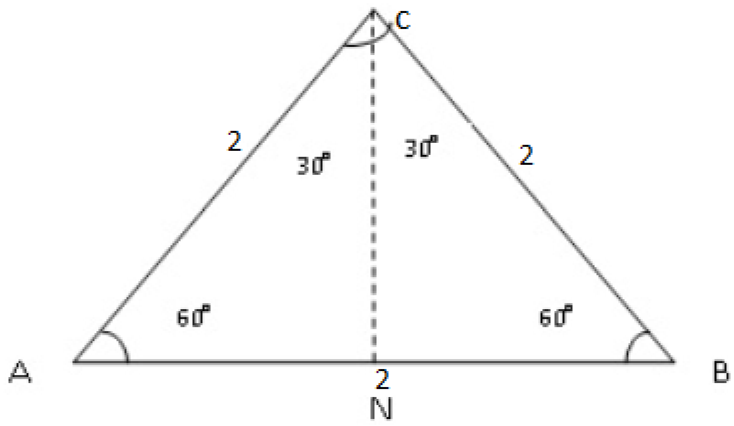
Tan 45° = $\frac{\text{opp}}{\text{adj}}$

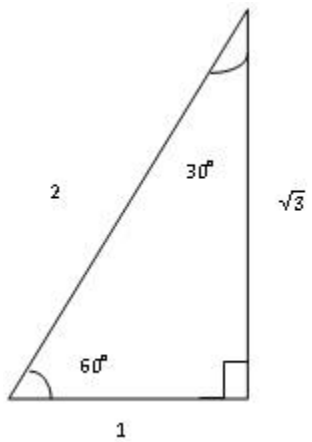
Tan 45° = $\frac{1}{1}$

Tan 45°= 1

Find Sin, Cos and Tan of (30° and 60Ëš)

Consider the equilateral triangle ΔABC





Use Pythagoras theorem

$a^2 + b^2 = c^2$

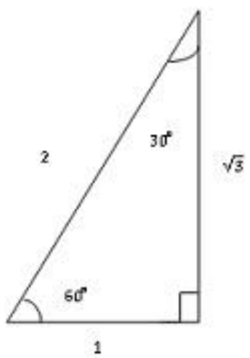
$1^2 + b^2 = 2^2$

$b^2 = 2^2 - 1^2$

$b^2 = 4 - 1$

$\sqrt{b^2} = \sqrt{3}$

$b = \sqrt{3}$



Sin 60 = $\frac{opp}{hyp}$

Sin 60° = $\frac{\sqrt{3}}{2}$

Sin 60° = $\frac{\sqrt{3}}{2}$

Cos 60 = $\frac{adj}{hyp}$

Cos 60° = $^1/_2$ or 0.5

Cos 60° = $^1/_2$ or 0.5

Tan 60° = $\frac{opp}{adj}$

Tan 60° = $\frac{\sqrt{3}}{1}$

Tan 60° = $\sqrt{3}$

Sin 30° = $\frac{opp}{hyp}$

Sin 30° = $^1/_2$ or 0.5

Tan 30° = $\frac{opp}{adj}$

Tan 30°= $\frac{1}{\sqrt{3}}$

= $\frac{\sqrt{3}}{3}$

Tan 30° = $\frac{\sqrt{3}}{3}$

Cos 30° = $\frac{adj}{hyp}$

Cos 30° = $\frac{\sqrt{3}}{3}$

In summary:

	0°	30°	45°	60°	90°
Sin	0	1	2	3	4
Cos	4	3	2	1	0

(√ of each value, divided by 2)

Sin 0 = $\frac{\sqrt{0}}{2}$

Sin 0 = $\frac{0}{2}$

Sin 0 = 0

Example 1.

Find the value of 4 sin 45° + 2 tan 60 without using table

Solution:

4 sin 45° + 2 Tan 60°

= 4 ($\frac{\sqrt{2}}{2}$) + 2 $(\sqrt{3})$

= 2 $\sqrt{2}$ + 2 $\sqrt{3}$

Note:

Trigonometry: Is the branch of mathematics that deals with the properties of angles and sides of right angled triangle

TRIGONOMETRICAL RATIONS FOR SPECIAL ANGLES

Trigonometrical rations for special angles deal with 0°, 30°, 45°, 60°, 90°.

A	0°	30°	45°	60°	90°
Sin A	0	$^1/_2$	$\sqrt{2}/_2$	$\sqrt{3}/_2$	1
Cos A	1	$\sqrt{3}/_2$	$\sqrt{2}/_2$	$^1/_2$	0
Tan A	0	$\sqrt{3}/_3$	1	$\sqrt{3}$	∞

Example 1.

Without using mathematical table evaluate

(i) 4 sin 45° + cos 30°

(ii) 4 tan 60° - Sin 90°

Solution:

i) 4 sin 45° + Cos 30°

$= 4 \times \sqrt{2}/2 + \sqrt{3}/2$

$= 2\sqrt{2} + \sqrt{3}/2$

$= 4 \frac{\sqrt{2}}{2} + \sqrt{3}/2$

(ii) 4 tan 60° – sin 90°

Solution:

4 tan 60° – sin 90° = $4\sqrt{3} - 1$

(iii) 3(cos60° + Tan 60°)

Solution:

$= 3(1/2 + \sqrt{3})$

$= 3 \frac{(1 + 2\sqrt{3})}{2}$

$= \frac{3 \times 1 + 3 \times 2\sqrt{3})}{2}$

$= \frac{3 + 6\sqrt{3})}{2}$

Example 2

Find the value of $\frac{7\cos 30° + \sin 60° \tan 30°}{2\cos 45° + 6\sin 60°}$ without using mathematical table

Solution:

$$\frac{7\sqrt{3}/2 + \sqrt{3}/2 - \sqrt{3}/3}{2(\sqrt{2}/2) + 6(\sqrt{3}/2)}$$

$$\frac{7\sqrt{3}/2 + \sqrt{3}/2 - \sqrt{3}/3}{\sqrt{2} + 3\sqrt{3}}$$

$$\frac{\sqrt{3}(7/2 + ½ - 1/3)}{\sqrt{2} + 3\sqrt{3}}$$

$$\frac{\sqrt{3}(^{11}/_{3})}{\sqrt{2} + 3\sqrt{3}}$$

$$\frac{11^{\sqrt{3}}/_{3}}{\sqrt{2} + 3\sqrt{3}}$$

$$11^{\sqrt{3}}/_{3} \div \sqrt{}_{2 + 3}\sqrt{}_{3}$$

$$11^{\sqrt{3}}/_{3} \times \frac{1}{\sqrt{2} + 3\sqrt{3}}$$

$$\frac{11\sqrt{3}}{3\sqrt{2} + 9\sqrt{3}}$$

EXERCISE

1. Evaluate the following without using mathematical table

(a). $3 \sin 45° + 7\tan 30°$

(b). $\frac{2\tan 45 + \text{Cos}60°}{8\text{Cos}30° - \text{Sin } 60°}$

Solution:

$3\sin 45° + 7\tan 30°$

$= 3 \text{ x } ^{\sqrt{2}}/_{2} + 7 \text{ x } \sqrt{}_{3}$

$= 3 \times \sqrt{2}/2 + 7 \times \sqrt{3}$

$= \frac{3 \times \sqrt{2} + 14 \times \sqrt{3}}{2}$

TRIGONOMETRIC TABLES

- Trigonometric tables deals with readings of the value of angles of sine, cosine and tangent from mathematical table when they are already prepared into four decimal

HOW TO READ THE VALUE OF TRIGONOMETRIC ANGLES FROM MATHEMATICAL TABLES

Example 1

Find the value of the following by using mathematical table

(i) Sin 43°= 0.6820

(ii) Sin 58º = 0.8480

(iii) Sin 24°42′ = 0.4179

(iv) Sin 52°26′ = 0.7923 + 4 = 0.7927

Example 2

By using mathematical tables evaluate the following

(a)Cos 37° =0.7986

(b)Cos82° =0.1392

(c)Cos 71°34′ =0.3162

(d)Tan 20° = 0.3640

(e)Tan 68° =2.4751

(f)Tan 54°22′ = 1.3950

Example3.

Find the value of the following letter from trigonometric rations

(i) Sin p = 0.6820

Sin p = 0.6820

P = Sin^{-1} (0.6820)

P = 43°

ii) Sin Q = 0.7291

Q = Sin^{-1} (0.7291)

Q = 46°48'

iii) Tan R = 5.42°45

R = 5. 42°45

R = tan^{-1} (5.42°45)

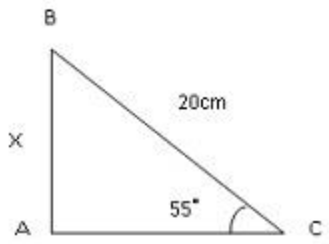
R = 79°33

APPLICATION OF SINE, COSINE AND TANGENT RATIOS IN SOLVING A TRIANGLE

Sine, cosine and triangle of angles are used to solve the length of unknown sides of triangles

Example

Find the value of x in ΔABC



From

SO	TO	CA
H	A	H

Sine C = $\frac{Opposite}{Hypotenus}$

Sin 55Ëš= $\frac{x}{20}$

20 x Sin 55Ëš= x

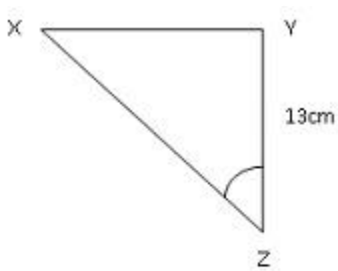
20 x 0.8198 = x

x = 16.4 cm

The value of x = 16.4cm

Example 2

Find the length XY in a XYZ



Solution:

From

SO	TO	CA
H	A	H

tan 62Ëš = $\frac{\text{Opposite}}{\text{Adjacent}}$

tan 62° = $^{xy}/_{13}$

13 x tan 62° = xy

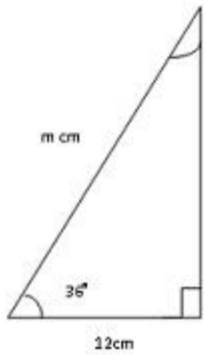
13 x 1.8807 = xy

24.4cm = xy

xy = 24.4cm

Example

Evaluate the value of m and give your answer into 3 decimal places



From

SO	TO	CA
H	A	H

Cos R = $\frac{Adj}{Hyp}$

Cos 36° = $\frac{12}{m}$

m = $\frac{12}{Cos\ 36°}$

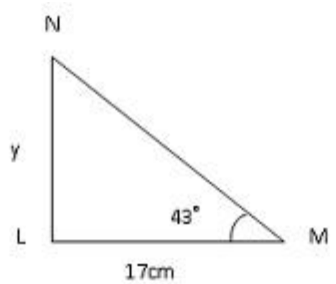
m = $\frac{12}{0.8090}$

m = 14.833 cm

The value of m is 14.833 cm

EXERCISE

Δ Given LMN below. If FM = 17cm and LMN = 43. Find the value of y



Solution:

From

SO	TO	CA
H	A	H

tan 43° = $\frac{opp}{adj}$

tan 43°= $\frac{y}{17}$

17 x tan 43° = y

y = 17 $\times 0.9325$

y =15.85cm

SETS

A set is a group/ collection of things such as a herd of cattle, a pile of books, a collection of trees, a shampoos bees and a fleck of sheep

Description of sets

-A set is described/denoted by Carl brackets { } and named by Capital letters

Examples

If A is a set of books in the library then A is written as

A= {All books in the library} and read as A is a set of all books written in the library

-The things/objects In the set are called Elements or members of the set

Example

1. If John is a student of class B, then John is a member of class B and shortly denoted as £B.

2. If A= {1,2,3} then 1 $\in$A, 2 $\in$A and 3 $\in$A}

The number of elements in a set is denoted by n(A)

Example

1. If A= {a, e, i, o, u} then n (A) =5

Example

If A is a set of even, describe this set by

a) Words

b) Listing

c) Formula

Solution:

a) By words;

A= {even numbers}

b) By Listing

A= {2, 4, 6, 8…}

c) By Formular

A= {x: x= 2n} where n= {1, 2, 3…} and is read as A is a set of all element x such that x is an even number.

2. Describe the following sets by Listing

A= {whole numbers between 1 and 8}

Solution:

A= {2, 3, 4, 5, 6, 7}

3. Write the following sets in words

A= {an integer < 10}

Solution:

A= {integers less than ten} or A is a set of integers less than ten

TYPES OF SETS

Finite set; Is a set where all elements can be counted exhaustively.

Infinite set: An Infinite set is a set that all of its elements cannot be exhaustively counted

Example

B= {2, 4, 6, 8…}

An Empty set; Is a set with no elements. An Empty set is denoted by { } or Ø

Example

If A is an Empty set then can be denoted as A= { } or A= Ø

Exercise

1. List the elements of the named sets

A= {x: x is an odd number < 10}

A= {1, 3, 5, 7, 9}

B= {days of the week which began with letter S}

B= {Saturday, Sunday}

C= {Prime numbers less than 13}

C= [2, 3, 5, 7, 11}

2. Write the named sets in words

B= {x: x is an odd number < 12}

B is a set of x such that x is an odd number less than twelve

E= {x: x is a student in your class}

E is a set of x such that x is a student in your class

3. Write the named sets using the formula methods

A= {all men in Tanzania}

A= {x: x is all men in Tanzania}

B= {all teachers in your school}

B= {x: x is all teachers in your school}

C= {all regional capital in Tanzania}

C= {x: x is all regional capital in Tanzania}

D= {b, c, d, f, g…}

D= [x: x is a consonant}

COMPARISON OF SETS

-SET may be equivalent, equal or one to be a subset of other

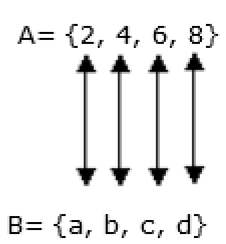
-Equivalent sets are sets whose members (numbers) match exactly

Example

A= {2, 4, 6, 8} and B= {a, b, c, d}

Then A and B are equivalent

The two sets can be matched as



Generally if n(A) = n(B) then A and B are equivalent sets

Example

If A= {1, 2, 3, 4} and B= {1, 2, 3, 4} since n(A) = n(B) and the elements are alike then set A is equal to set B

Subset: Given two sets A and B, B is said to be a subset of A. If all elements of B belongs to A

Example

If A= {a, b, c, d, e} and B= {a, b, c, e}, Set B is a subset of A since all elements of set B belongs to set A. But set B has less elements than set A. Then set B is a proper subset of set A

and A is a super set of B.

Symbolically $B \subset A$

If A= B then either A is an improper subset of B or B is an improper subset of A.

Symbolically written as $A \subseteq B$ or $B \subseteq A$

Note: an empty set is a subset of any set

-The number of subset in a set is found by the formula 2^n where n= number of elements of a set

Example

1. List all subset of A= {a, b}

Solution:

2^n , n = number of element in a set.
So $2^2 = 4$

The number of subset = 4

The subset of A are { }, {a}, {b}, {a, b}

2. How many subset are there in A= {1, 2, 3, 4}

Solution:

The number of subset= $2^4 = 16$

UNIVERSAL SET [U]

-Is a single sets which contains all elements sets under consideration for example the set of integers contains all the elements of sets such as odd numbers, even numbers, counting numbers, and whole numbers. In this case the set of integers is the Universal set.

Exercise

1. Which of the following sets are

a) Finite set

b) Infinite set

c) Empty set

A= {Nairobi, Dar es Salaam}

B= {2, 4, 6…36}

E= {All mango trees in the world}

F= {x: x is all students aged 100 years in your school}

H= {1, 3, 5, 7}

D= {all lions in your school}

I= Ø

Solution:

a) Finite set are

A= {Nairobi, Dar es Salaam}

B= {2, 4, 6…36}

H= {1, 3, 5, 7}

Infinite set

E= {All mango trees in the world}

F= {x: x is all students aged 100 years in your school}

b) Empty sets are

D= {all lions in your school}

I= Ø

2. In each of the following pairs of sets shown by matching whether the pairs are equivalent or not Equivalent are:

A= {a, b, c, d} and B= {b, c, d, e}

Which are not equivalent are:

B= {Rufiji, Ruaha, Malagarasi} and C= {lion, leopard}

B and C are not equivalent.

3. Which of the following sets are equal

A= {a, b, c, d}, B= {d, a, b, c}, C= {a, e, I, o, u}, D= {a, b, c, d}, E= {d, c, b, a}, F= {a, e, b, c, d}

Solution

A, B, D and E are equal

4. List all subsets of each of the following sets

a) A= 1

The number of subset = 2^1= 2

Therefore; The subset of A are { }, {1}

b) B= Ø

Therefore; number of subsets is { }

c) C = {Tito, Juma}

Number of subset = 2^2=4

Therefore; the subsets of C are { }, {Tito}, {Juma}, {Tito, Juma}

5. Name the subsets of each pair by using the symbol ⊂

a) A= {a, b, c, d, e, f, g, h} and B= {d, e, f}

Therefore = B⊂A

b) A= {2, 4} and D= {2, 4, 5} = A⊂D

c) A= {1, 2, 3, 4 …} and B= {2, 4, 6, 8…} = A⊆B

6. Given G = {cities, towns and regions of Tanzania} which of the following sets are the subsets of G?

A= {Nairobi, Dar es Salaam}

B= {Dodoma, Mombasa, Mwanza}

C= { }

D= {Arusha, Iringa, Bagamoyo}

E= {Mbeya, Tunduru, Ruvuma}

Therefore; the subsets of G are C, D, E

7. Which of the following sets are the subsets of K given that K= {p, q, r, s, t, u, v, w}

A= {p, s, t, x}

B= {q, r, d, t}

C= { }

D= {p, q, r, s, t, u, v, w}

E= {a, b, c, d}

F= {s, v, q}

Therefore; the subsets of K is D, C, F

8. What is n(A) if A= { }

n(A) = 0

9. Write in words the universal set of the following sets

a) A= {a, b, c, d}

The universal set of A is a set of alphabets

b) B= {1, 2, 3, 4}

The universal set of set B is the set of natural numbers

OPERATION WITH SETS

UNION

The union of two sets A and B is the one which is formed when the members of two sets are putted together without a repetition. Thus the union is $\cup$, this union of A and B can be denoted as A $\cup$ B is defined as x; X $\in$ A or X $\in$ B

Example

1. If A= {2, 4, 6} and B= {2, 3, 5} then A $\cup$ B= {2, 4, 6} $\cup$ {2, 3, 5}= {2, 3, 4,5, 6}

2. Find A $\cup$ B when A= {a, b, c, d, e, f} and B= {a, e, I, o, u}

Solution:

A $\cup$ B= {a, b, c, d, e, f, I, o, u}

INTERSECTION

The Intersection of two sets A and B is a new set formed by taking common elements. The symbol for intersection is "$\cap$"

Example

1. A= {1, 2, 3, 4, 5}, B= {1, 3, 5} then A $\cap$ B= {1,3,5}
2. Find A $\cap$ B if A= {a, e, i, o, u}, B= {a, b, c, d, e, f} then A $\cap$ B= {a, e}

COMPLEMENT OF A SET

If A is a subset of a universal set, then the members of the universal set which are not in A, form compliment of A denoted by AÎ„

Example

If U= {a, b, c … z} and A= {a, b} then AÎ„= {c, d, e, … z}

Given that U= {15, 45, 135, 275} and A= {15} find AÎ„

Solution:

A'= {45, 135, 275}

JOINT AND DISJOINT SETS

JOINT SETS; Are sets with common elements

E.g. A= {1, 2, 3, 5}, D= {1, 2} then A and D are joint sets since {1, 2} are common elements

DIS JOINT SETS; Are sets with no elements in common

For example A= {a, b, c} and B= {1, 2, 3, 4} then A and B are disjoint sets since they do not have a common element

EXERCISE

1.Find

a) Union

b) Intersection of the named sets

1. A= {5, 10, 15}, B= {15, 20}

a) A ∪ B = {5, 10, 15, 20}

b) A ∩ B= {15}

2. A= { }, B= {14, 16}

a) A ∪ B= { , 14, 16}

b) A ∩ B= { }

3. A= {First five letters of the English alphabet}, B= {a, b, c, d, e}

a) A $\cup$ B= {a, b, c, d, e}

b) A $\cap$ B= {a, b, c, d, e}

4. A= {counting numbers}, B= {prime numbers}

a) A $\cup$ B= {counting numbers}

b) A $\cap$ B= {prime numbers}

5. A = {o, }, B= { }

a) A $\cup$ B = {o, Δ }

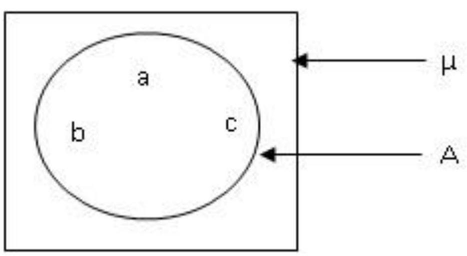
b) A $\cap$ B= { Δ }

VENN DIAGRAM

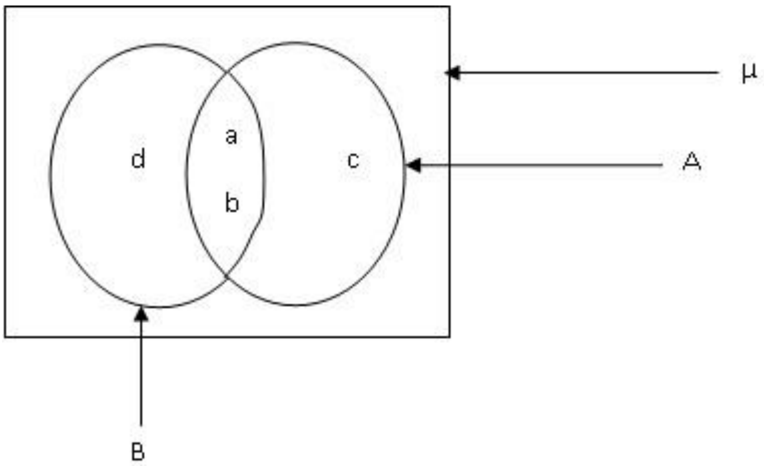
-Are the diagrams (ovals) devised by John Venn for representation of sets

Example

If A= {a, b, c} can be represented as

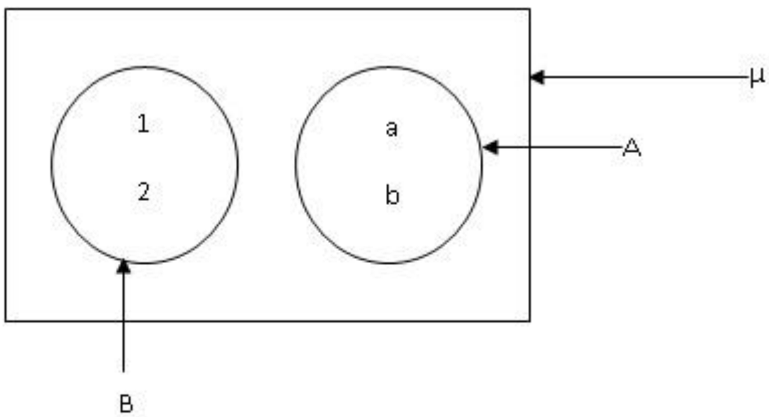


μ is the universal set, in this case is the set of all English alphabets. If the set have any elements in common, the ovals over lap for example, If A= {a, b, c} and B= {a, b, c, d} then it can be represented as



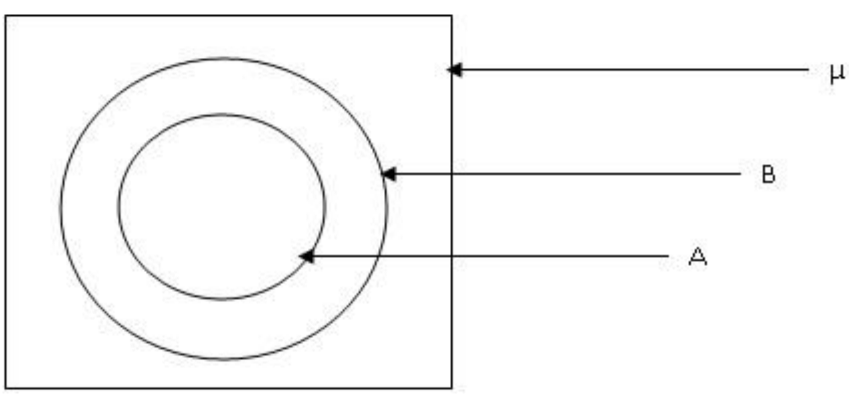
Disjoint sets also can be represented on a Venn diagram

Example: If A= {a, b}, B= {1, 2} the relation A and B is as follows



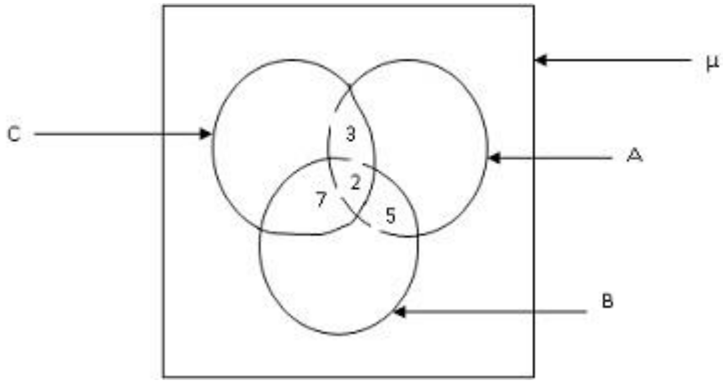
Examples

If A is a subset of B, represent the two sets on a Venn diagram



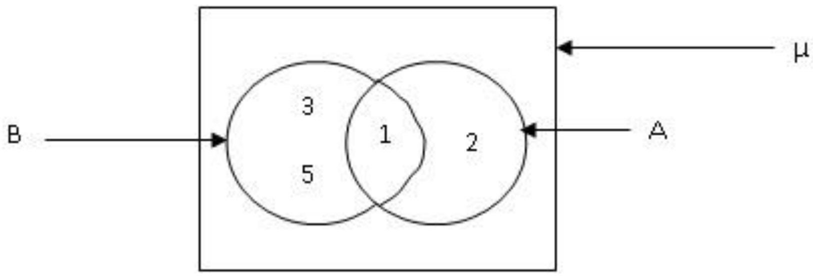
Represent A= {2, 3, 5}, B= {2, 5, 7} C= {2, 3, 7} in a Venn diagram

Solution:



Represent AUB in a Venn diagram given that A= {1, 2}, B= {1, 3, 5}

Solution:

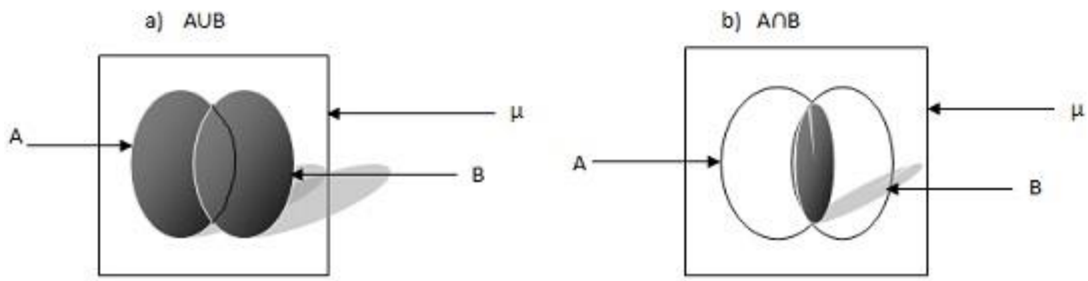


If set A and B have same elements in common, represent the following in a Venn diagram

a) $A \cup B$

b) $A \cap B$

Solution:



a)

In a certain primary school 50 pupils were selected to form three schools teams of football, volleyball and basketball as follows

30 pupils formed a football team

20 pupils formed a volleyball team

25 pupils formed a basketball team

14 play both volleyball and basketball

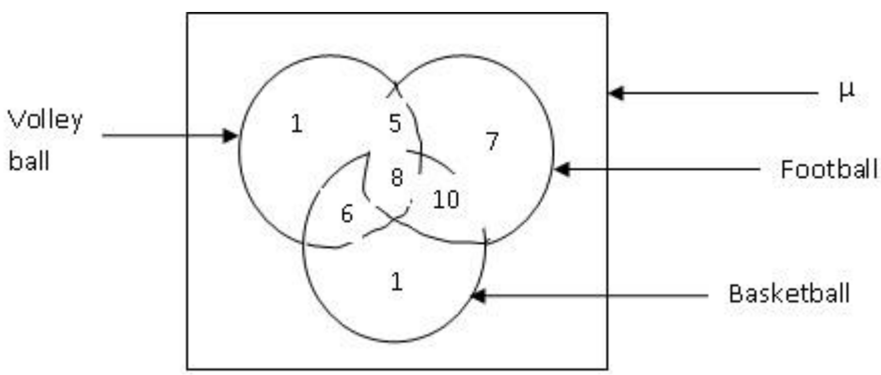
18 pupils play football and basketball

8 pupils play all of the three games

7 pupils play football only

Represent this information in a Venn diagram

Solution:

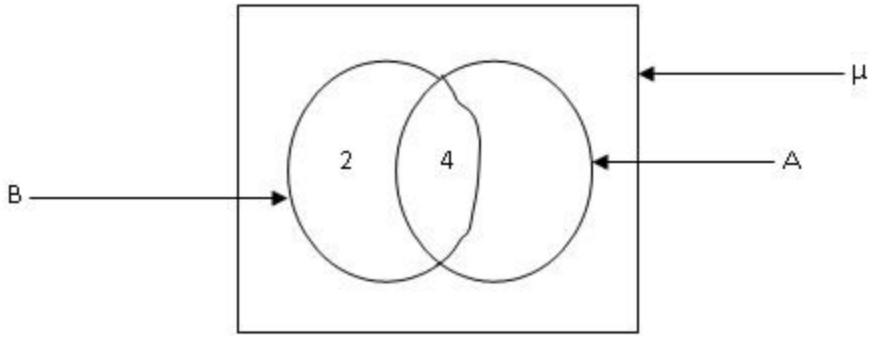


A and B are sets such that $n(A\cap B)=4$ and $n(A\cup B)=6$ if A has 4 elements

a) How many elements are there in B?

b) Which set is the subset of the other

Solution:



(a). 6 elements are in B

(c). A⊂B

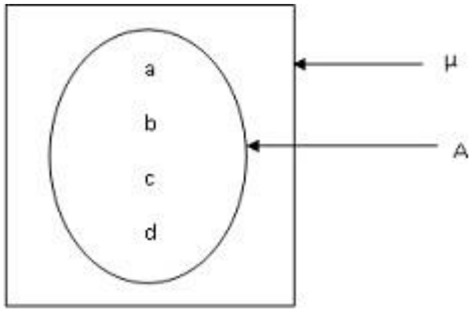
In general the number of elements in two sets is connected by the formula

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

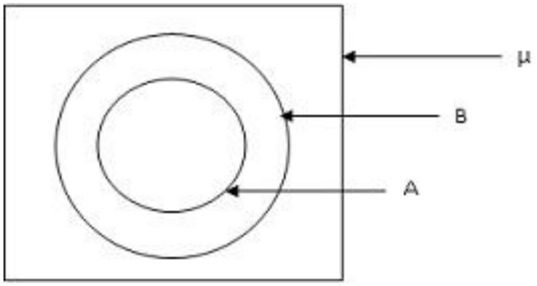
Exercise:

1. Represent the following in Venn diagrams

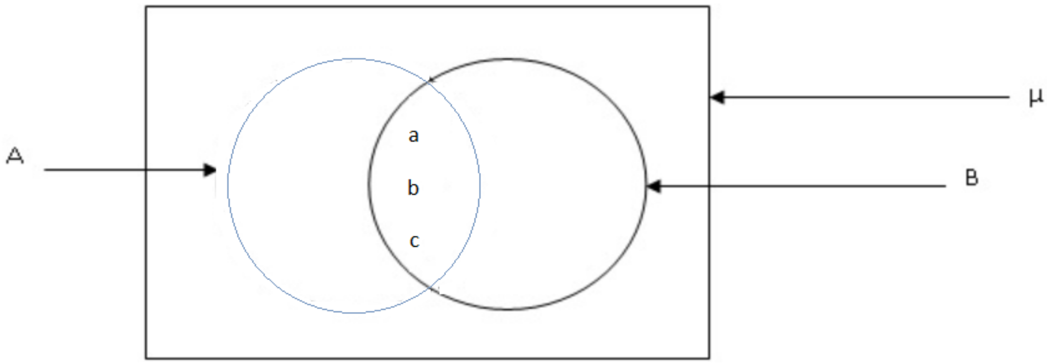
a) A={a, b, c, d}



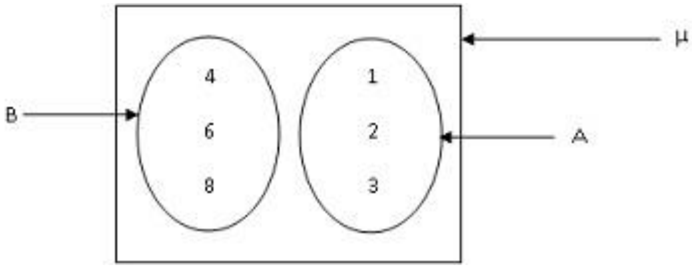
b) A⊂B



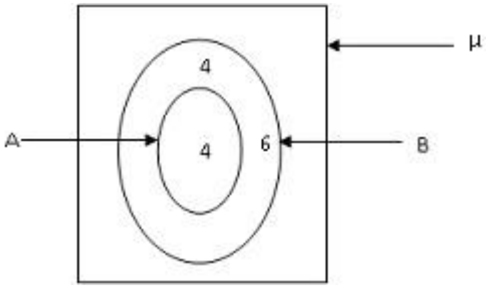
c) A= {a, b, c} and B= {a, b, c}



d) A= {1, 2, 3} and B= {4, 6, 8}



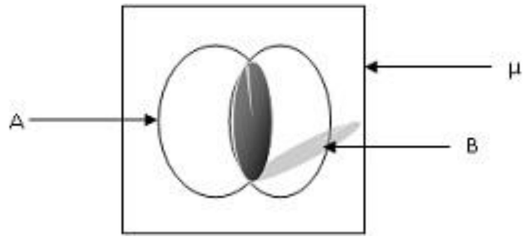
2. Write in words the relationship between the two sets shown in the figure below



-Their relationship is A⊂B

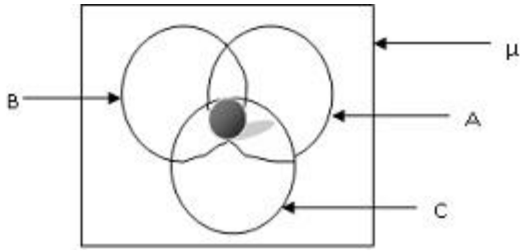
3. Describe in set notation the meaning of the shaded regions in the following Venn diagrams

a) $A \cap B$



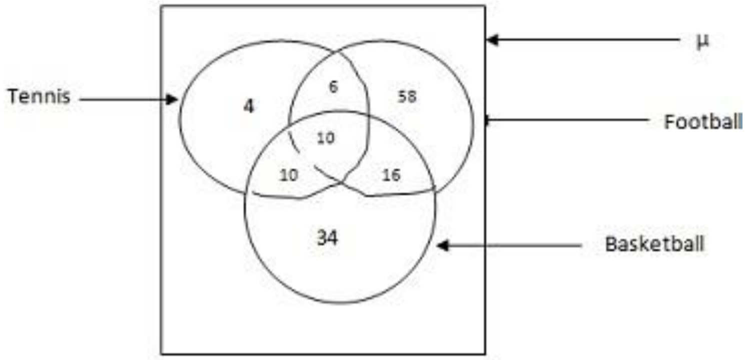
A= {a, b, c}

b) $A \cap B \cap C$



B= {a, b, c}

4. In a boys school of 200 students, 90 play football, 70 play basketball, and 30 play Tennis. 26 play basketball and football, 20 play basketball and Tennis, 16 play football and Tennis, while 10 play all three games. How many students in school play none of the three games



4+10+34+6+10+16+58+N= 200

138+N= 200

N=200-138

N=62

<u>62 students play none of the games</u>

<u>COMPLEMENT OF A SET</u>

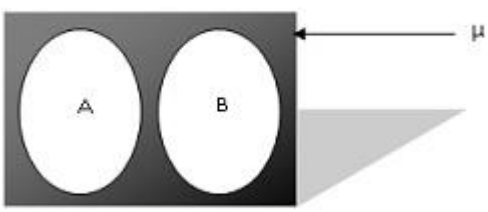
If A is a subset of a universal set, then the compliment of set A may be represented in a Venn diagram

Example

1. Show in a Venn diagram that (AUB)'

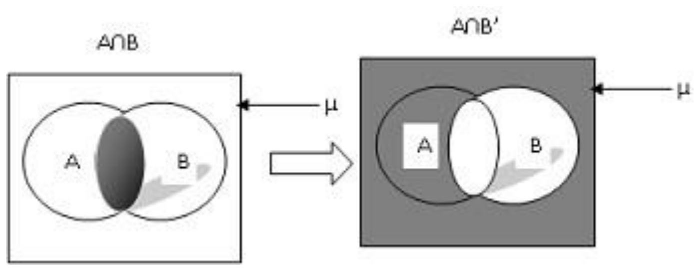
<u>Solution:</u>

(A $\cup$B)Î„

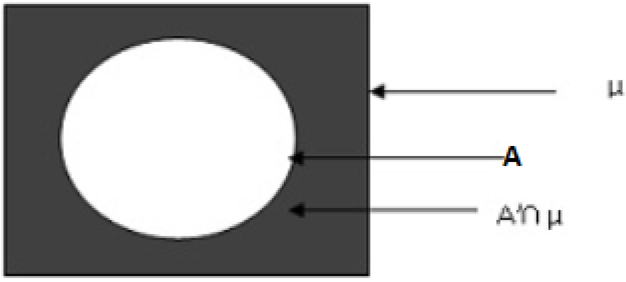


(A $\cup$B)' = members of outside A $\cup$B

2. A $\cap$B'



3. Represent AÎ, $\cap$, µ in a Venn diagram and shade the required region



A is a subset of Universal set

WORD PROBLEMS

Examples

1. In a certain school of 120 students, 40 learn English, 60 learn Kiswahili and 30 learn both Kiswahili and English. How many students learn

a) English only

b) Neither English nor Kiswahili

Solution:

Let μ = {students in a school}

A= {Students learning English}

B= {Students learning Kiswahili}

a) $n(A) - n(A \cap B)$ = number of students learning English only

$40 - 30 = 10$

Therefore the number of students learning English only is 10

b) $= n(\mu) - [n(A) + n(B) - n(A \cap B)]$

$= 120 - [40 + 60 - 30]$

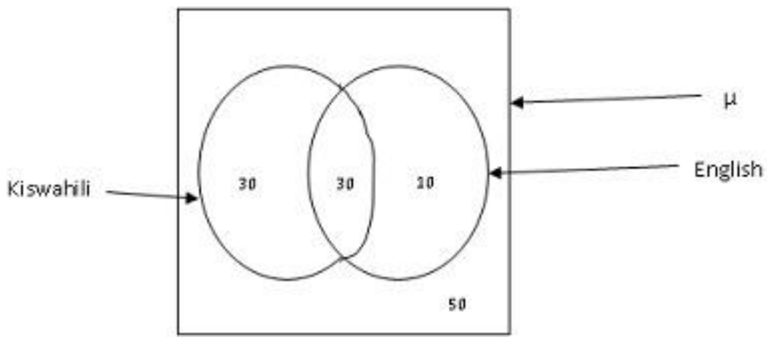
$= 120 - 70$

$= 50$

50 students learn neither English nor Kiswahili

Alternatively

By Venn diagram



a) 10 students learn English only

2. In a certain school 50 students eat meat, 60 eat fish and 25 eat both meat and fish. Assuming that every students eat meat or fish, find the total members of students in a school

Solution:

Let μ= {total number of students}

A= {students eating fish}

B= {students eating meat}

$n(A \cup B)= n(A)+ n(B)- n(A \cap B)$

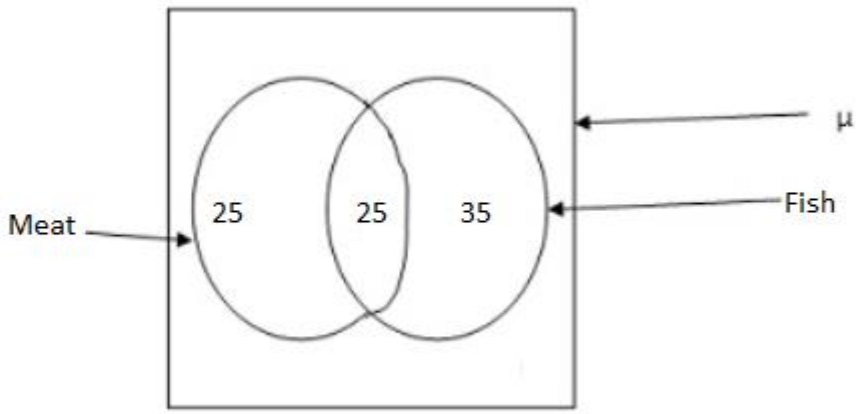
$n(A \cup B) = 50+60-25$

$n(A \cup B) = 85$ students

There are 85 students in a school

Alternatively

By Venn diagram



85 students were in school

3. There are 24 men at a meeting, 12 are farmers, 18 are soldiers, 8 are both farmers and soldiers

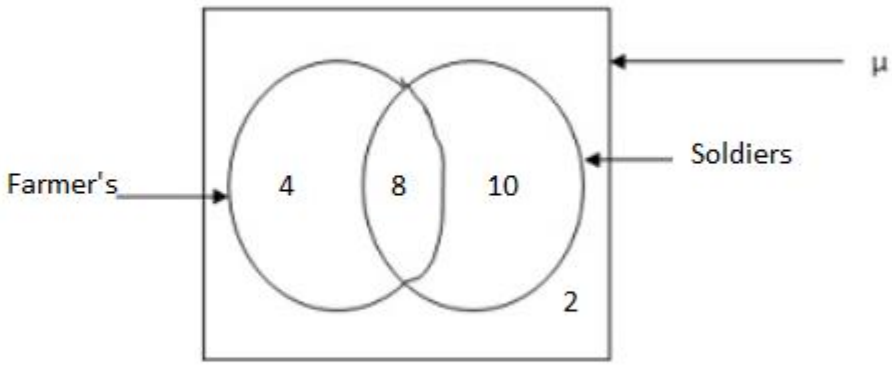
a) How many are farmers or soldiers

b) How many are neither farmers nor soldiers

Solution:

NB; both/and means "intersection"

By Venn diagrams



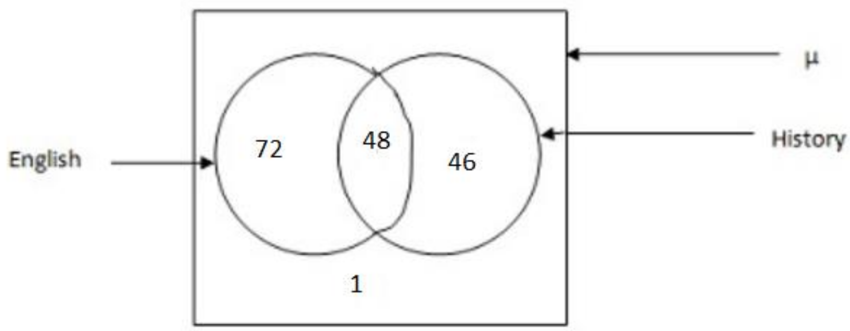
a) 22 men are soldiers or farmers

b) 2 are neither farmers nor soldiers

4. In an examination, 120 candidates offered math, 94 English and 48 offered both math and English. How many candidates offered English but not math assuming that every candidate offered one of the subjects or both math and English

Solution:

By Venn diagram



46 students offered English but not math

EXERCISE

1. A class shows that 15 of the students play basketball, 11 play netball and 6 play both basketball and netball. How many students are there in a class? If every student plays at least one game

Solution:

$n(A \cup B)= n(A)+ n(B)- n(A \cap B)$

$n(A \cup B)=15+11-6$

$n(A \cup B)=20$

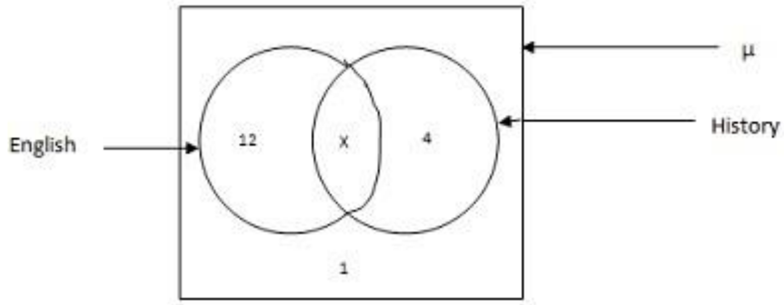
There are 20 students in the class

2. In a class of 20 pupils, 12 pupils study English but not History, 4 study History but not English and 1 who study neither English nor History. How many study History

Solution:

Let A= {pupils who study English}

B= {pupils who study History}



12 + x + 4 = 20

X = 3

History = x + 4 =7

7 pupils study History

3. At a certain meeting 30 people drank Pepsi, 60 drank Coca-Cola, and 25 drank both Pepsi and Coca-Cola. How many people were at the meeting assuming that each person took Pepsi or Coca-Cola

Solution:

Let A= {drank Pepsi}

B= {drank Coca-Cola}

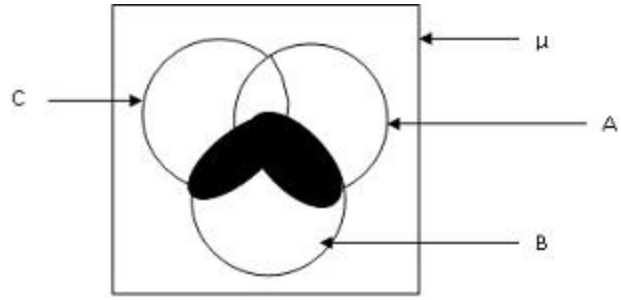
$n(A \cup B) = n(A)+n(B)-n(A \cap B)$

=30+60-25

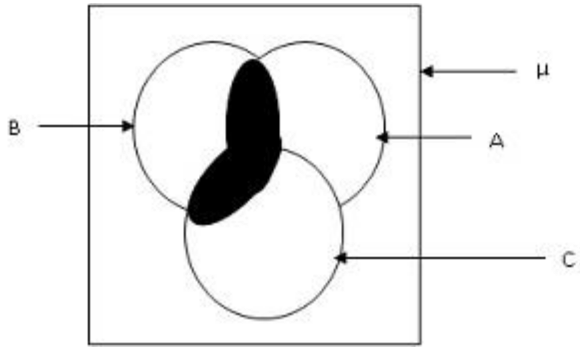
=65

65 people were at the meeting

4. Represent $(A \cap B) \cap (B \cap C)$ on a Venn diagram

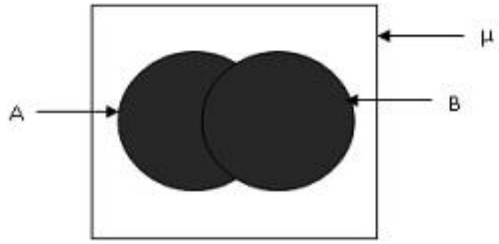


5. Represent $(A \cup B) \cap C$

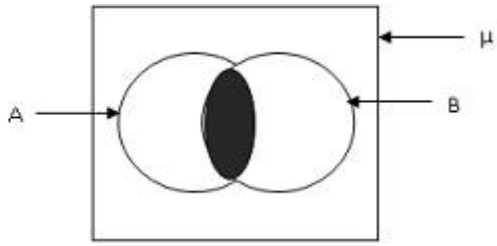


6. If set A and B have the same common elements represent

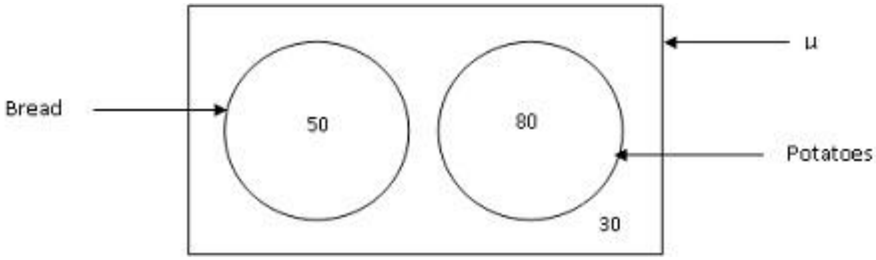
a) $A \cup B$



b) $A \cap B$ in a Venn diagram

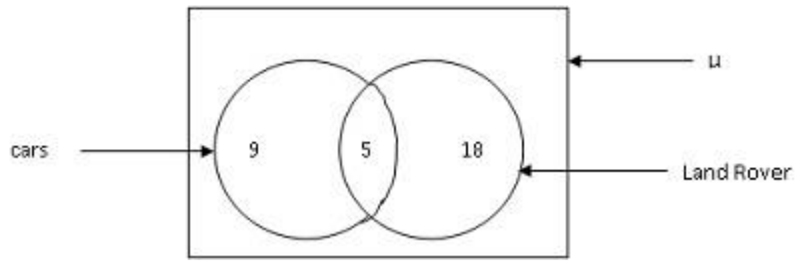


7. In a school of 160 pupils, 50 have bread for breakfast and 80 have sweet potatoes. How many pupils have neither Bread nor potatoes assuming that none take bread and sweet potatoes



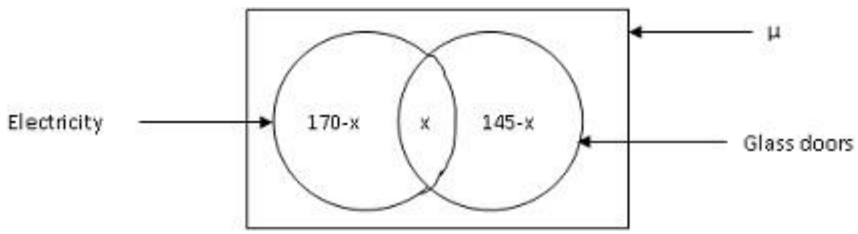
30 pupils have neither Bread nor Potatoes

8. Every Man in a certain club owns a Land Rover or a car. 23 men own Land Rover, 14 own cars and 5 owns both Land Rovers and cars. How many men are in a club?



32 men were in the club

9. In a certain street of 200 houses. 170 have electricity and 145 have glass doors. How many houses have both electricity and glass doors, assuming that each houses has either a glass door or electricity or both



170-x + x + 145-x = 200

170+145+x-x-x = 200

315-x = 200

X=315-200

X=115

115 houses has both electricity and glass doors

REVISION EXERCISE

1. How many subset are there in A= {a, b, c, d, e, f, g}

Solution:

Since n(A) = 7 then

From $2^n=2^7=128$

Set A has 128 subsets

2. List all the subsets of A= {2, 4, 6}

Solution:

n(A)= 3

$2^n=2^3= 8$

The subsets are { }, {2}, {4}, {6}, {2, 4}, {2, 6}, {4, 6}, {2, 4, 6}

3. If µ= {a, b, c, d, e}, B= {e, d}, A= {a, b, c} list the elements of B'

a) B'= {a, b, c}

b) Find AÎ„ ∩BÎ„

Solution:

AÎ„= {d, e}

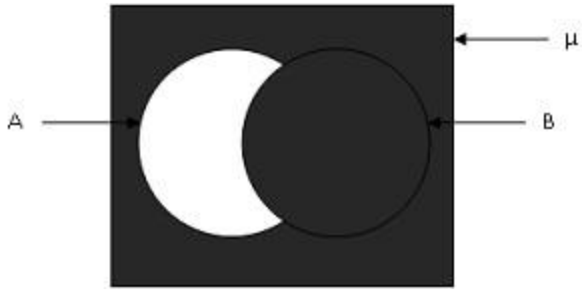
BÎ„= {a, b, c}

AÎ„ ∩BÎ„ = { }

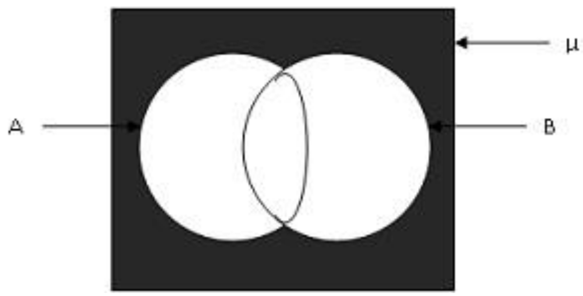
c) A ∪(BÎ„ ∩AÎ„) = {a, b, c}

4. Draw a Venn diagram and shade the required region of the following

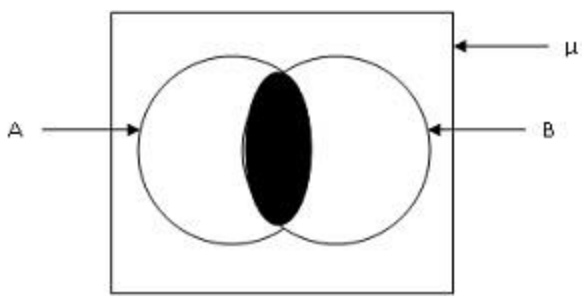
a) AÎ„ ∪B



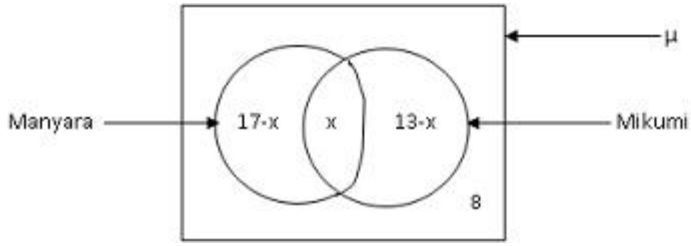
b) BÎ„ ∩AÎ„



c) A ∩B



5. In a group of 29 tourists from different countries, 17 went to Manyara national park, 13 to Mikumi national park and 8 went neither Mikumi nor Manyara national park. How many tourists went to both places



To find x

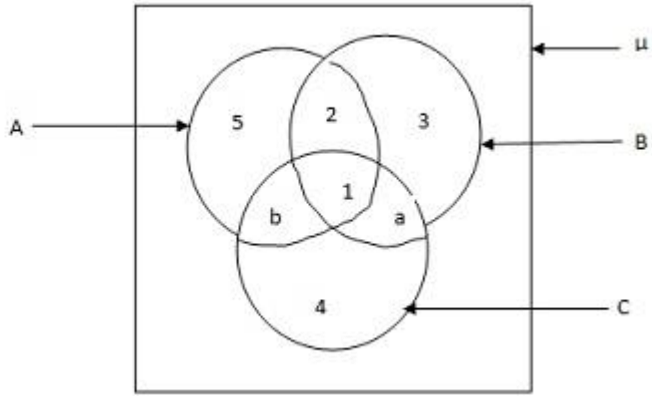
17-x + x + 13-x + 8 = 29

38-x = 29

X=*9*

:. 9 tourists went both places

6. From the figure



a) List the members of set A

A= {1, 2, 5, b}

b) List the members of set C

C= {1, 4, a, b}

STATISTICS

Definition

-is a branch of mathematics dealing with the study of method of collecting,organizing, analyzing, presenting and interpreting numerical details to reach conclusions.

Frequency distribution

Is a number of times each data point

Example 1

1. Make a frequency table from the following data from the followings data of ages 10 students

14, 15, 16. 14, 17, 15, 16, 13,

ages	frequency
17	1
16	2
15	2
14	4
13	1
	n=10

2. In mathematics test the following marks were obtained;

48 , 47,42, 67, 73, 50, 76 ,47, 44, 44, 57, 58, 54, 45, 58, 56 , 66, 67, 45, 43, 71, 48, 64, 52, 42, 54, 62, 32, 49, 34, 35, 46, 89, 37, 47, 54, 45, 60, 64, 44,.

If the class size of the class interval is 8 group the works starting with the interval 32-39 and draw the frequency distribution table.

solution

Mark	Frequency
88-95	1
80-87	0
72-79	2
64-71	6
56-63	6
48-55	8
40-47	13
32-39	4
	n=40

Example

From example below find the class mark of the class interval 88 – 95 and 80 – 87

class mark: $\frac{88+95}{2}$ and $\frac{80+87}{2}$

$\frac{183}{2}$ and $\frac{167}{2}$

= 91.5 and 83.5

Class limit ; example in class interval 88 - 95

88 is the class lower limit

95 is the class upper limit

CLASS REAL LIMITS

class lower real limit – is the number obtained by subtracting 0.5 from a class lower limit e.g. 88-0.5= 87.5

class upper real limit obtained by adding 0.5 to the upper class limit eg, 95 + 0.5 = 95.5

class size- is the value obtained by the difference between the upper real limit and the lower real limit

example .

from class interval 88-95 and 31-35

find the class size

solution:

Lower class real limit = 88 – 0.5 = 87.5

Upper class real limit = 95+ 0.5 = 95.5

Class size = 95.5 – 87.5 = 8

Lower class real limit = 31 – 0.5 = 30.5

Upper class real limit = 35+ 0.5 = 35.5

Class size = 35.5– 30.5= 5

Exercise 1:

(1).In biology class test the following marks when obtained;

54,54,40,55,54,43,73,34, 75, 47, 35, 45,73,46,31,43,47,35,35,60,67,51,44,48,55,45,50,37,51,36

By grouping the marks in class interval 20-29 ,30-39, 40-49, etc construct the the frequency

Solution:

DISTRIBUTION TABLE

Marks	Frequency(f)
20 -29	0
30 –39	7
40 -49	10
50 -59	8
60 -69	2
70 -79	3
	N = 30

(2) The following data represent the masses of 10 people in kg. Construct the frequency distribution table for these people

30 25 35 28 38 40 25 25 40 24

Solution:

Masses	Frequency
24	1
25	3
28	1
30	1
35	1
38	1
40	2

N = 10

(3). The following is a set of marks on a geography examination presents the frequency distribution table with class intervals, real limit, class marks, interval size starting with the interval 8-15 at the bottom

Solution:

class interval	Real limits	class marks	interval	f
88-95	**87.5-95.5**	**91.5**	**8**	**3**
80-87	**79-87.5**	**83.5**	**8**	**3**
72-79	**71.5-79.5**	**75.5**	**8**	**6**
64-71	**63.5-71.5**	**67.5**	**8**	**3**
56-63	**55.5-63.5**	**59.5**	**8**	**6**
48-55	**47.5-55.5**	**51.5**	**8**	**4**
40-47	**39.5-**	**43.5**	**8**	**7**

	47.5			
32-39	31.5-39.5	35.5	8	4
24-31	23.5-31.5	27.5	8	8
16-23	15.5-23.5	19.5	8	2
8-15	7.5-15.5	11.5	8	4
				n=50

(4). Fill in the blank columns

Distribution of 100 math s examination score

class interval	real limit	class marks	interval	f
95-99	94.5-99.5	97	5	3
90-94	89.5-94.5	92	5	7
85-89	84.5-89.5	87	5	9
80-84	79.5-84.5	82	5	13
75-79	74.5-79.5	77	5	20
70-74	69.5-74.5	72	5	23
65-69	64.5-69.5	67	5	17
60-64	59.5-64.5	62	5	8
				N=100

Note:

Class real limits are also known as class boundaries

GRAPHS OF FREQUENCY DISTRIBUTIONS:

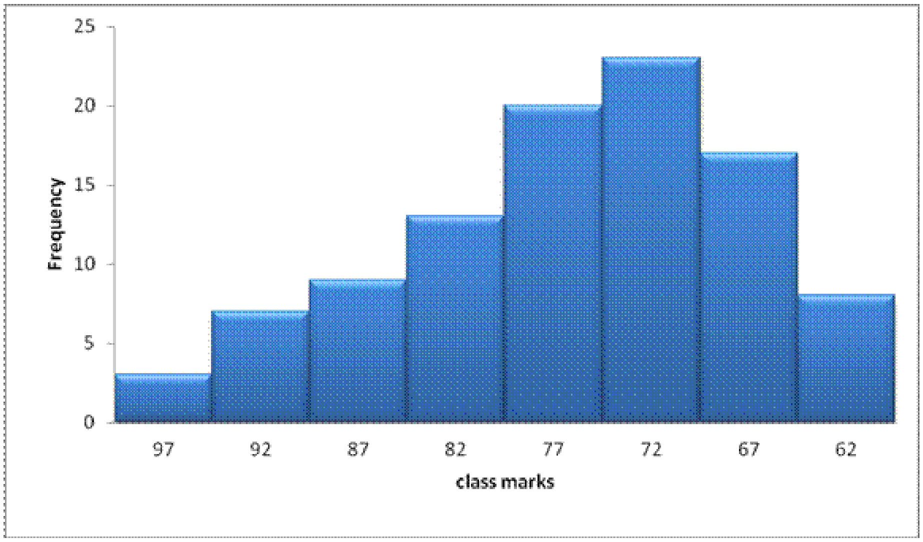
HISTOGRAMS

Histograms of frequency distribution are rectangular figures plotted with class marks against frequency . The width of the histogram equal to the class size.

Example:

1. Draw a histogram of 100 mathematics examination scores in the table below

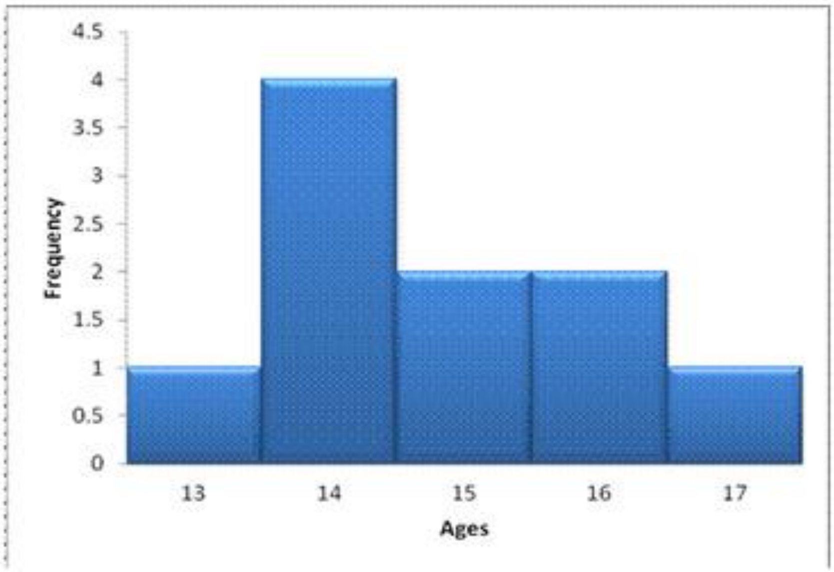
class interval	class mark	frequency
95-99	97	3
90-94	92	7
85-89	87	9
80-84	82	13
75-79	77	20
70-74	72	23
65-69	67	17
60-64	62	8



3. .2. Use the following distribution table below to draw a histogram

age	Frequency
13	1
14	4
15	2
16	2
17	1

Solution



FREQUENCY POLYGON

Is the line graph of class frequency plotted against class marks

Steps ;

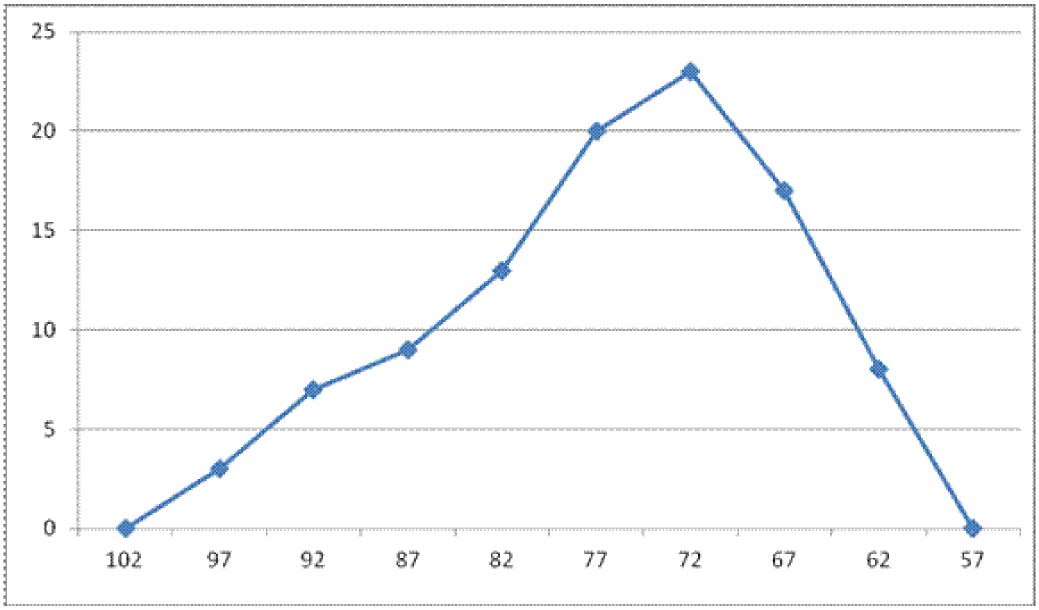
1. 1. Add one interval below the lowest interval and one above the highest interval and assign them as zero frequency.

2. 2. Plot a point and join them by straight lines

Example

1. Draw a frequency polygon from the following data.

c-interval	c-mark	frequency
100-104	102	0
95-99	97	3
90-94	92	7
85-89	87	9
80-84	82	13
75-79	77	20
70-74	72	23
65-69	67	17
60-64	62	8
55-59	57	0



EXERCISE

1. The following table shows female death between 0 and 34 years to the nearest numbers represent this information by

A) Histogram

B) Frequency polygon

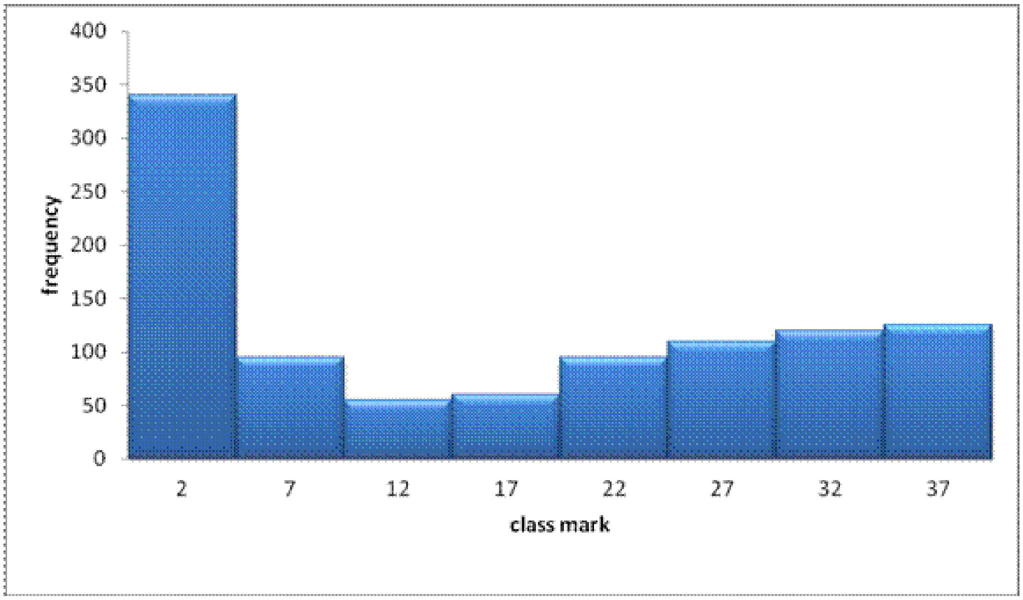
Expected death of female per 100 women

ages	F(death risks)	age
0-4	340	2
5-9	95	7
10-14	55	12
15-19	60	17
20-24	95	22
25-29	110	27
30-34	120	32
35-39	125	37

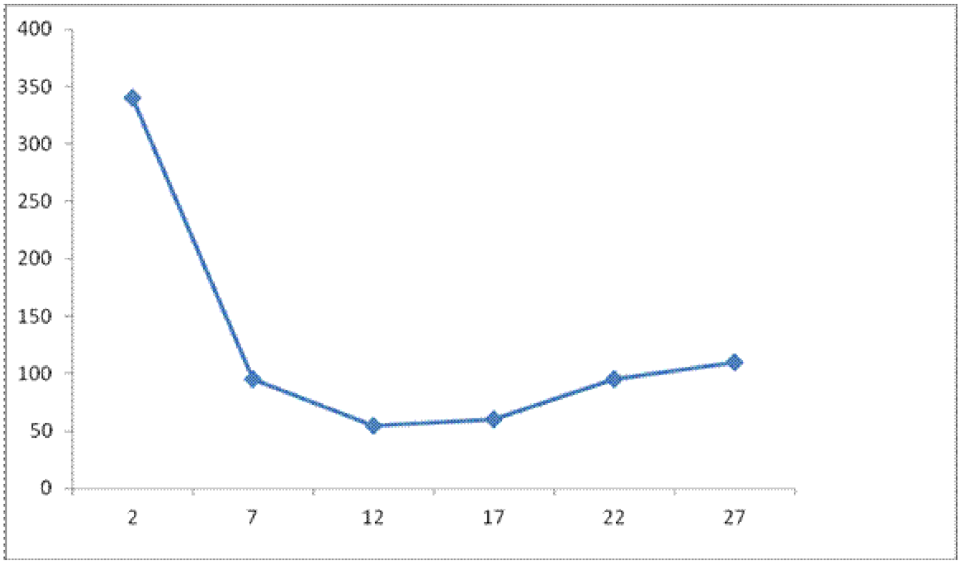
N=1000

Solution:

A) Histogram

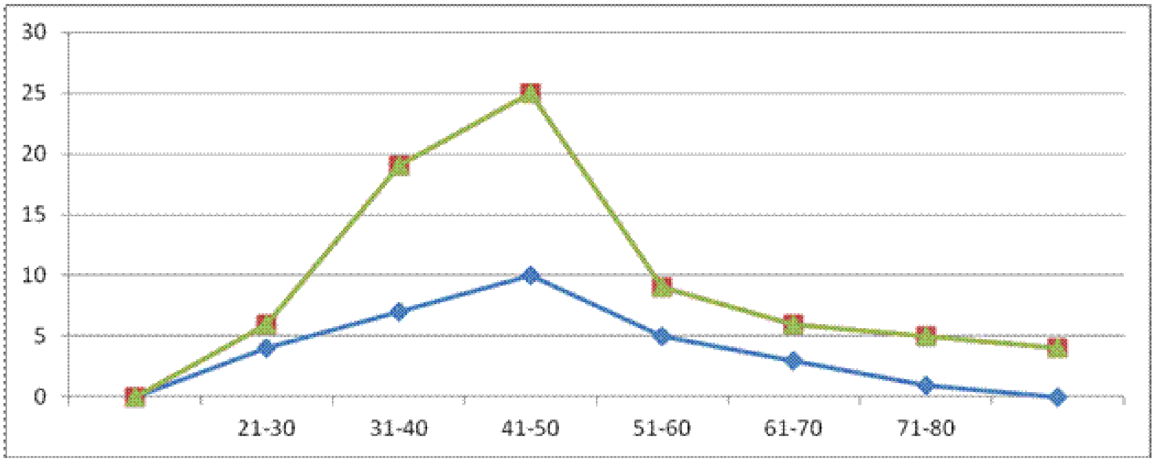


B) FREQUENCY POLYGON



2. Table below show the distribution of marks obtained by 110 students in two different monthly tests. Draw the frequency polygon on the same chart

marks	frequency	marks	frequency
21-30	**4**	**21-30**	**2**
31-40	**7**	**31-40**	**12**
41-50	**10**	**41-50**	**15**
51-60	**5**	**51-60**	**4**
61-70	**3**	**61-70**	**3**
71-80	**1**	**71-80**	**4**
	N=40		N=40



CUMULATIVE FREQUENCY CURVE (ORGIVE)

- Cumulative frequency is the sum of all the frequency less than or equal to a given mark or class interval

- To calculate the cumulative frequency start with the smaller upper real limit

- Add the frequency of the smallest interval to the next interval downwards or up wards depending on whether the data is arranged in descending or ascending order

Note: The last entry in the cumulative frequency is always equal to the total number of observations

- Plot upper real limit against class marks.

- Join adjacent points by a free hand.

EXAMPLES

1. Draw an orgive for the scores data below.

score	f
70-74	**16**
65-69	**12**
60-64	**14**
55-59	**10**
50-54	**8**
45-49	**18**
40-44	**6**
35-39	**4**
30-34	**2**
	N=90

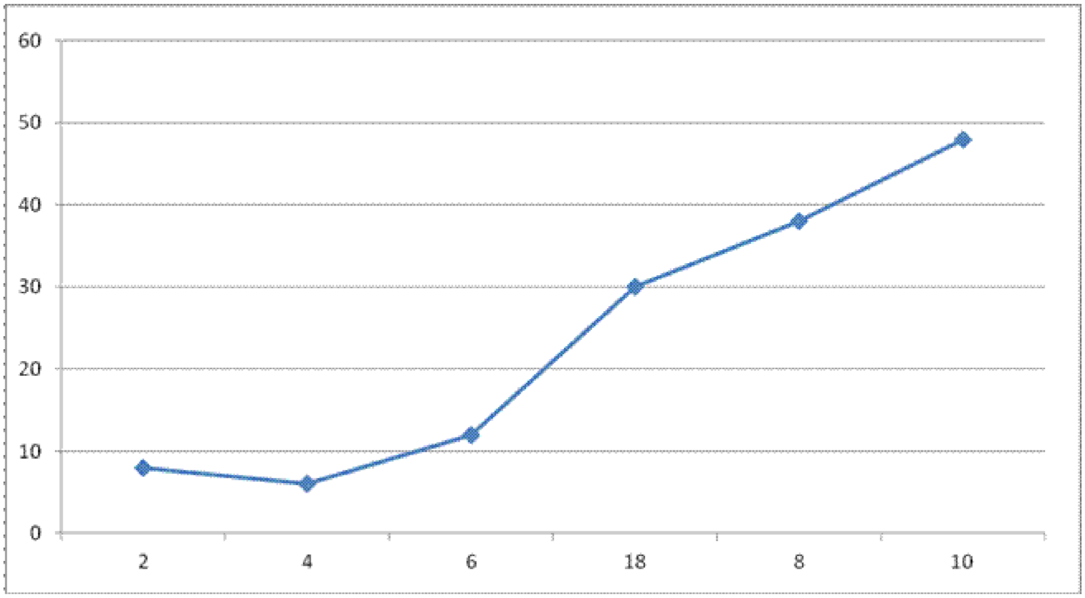
Solution;

THE CUMULATIVE FREQUENCY DISTRIBUTION,

Score	frequency	cumulative frequency
less than 34.5	**2**	**2**
less than39.5	**4**	**6**
less than 44.5	**6**	**12**
less than 49.5	**18**	**30**
less than 54.5	**8**	**38**

less than 59.5	**10**	**48**
less than 64.5	**14**	**62**
less than 69.5	**12**	
less than 74.5	**16**	**90**

N =90

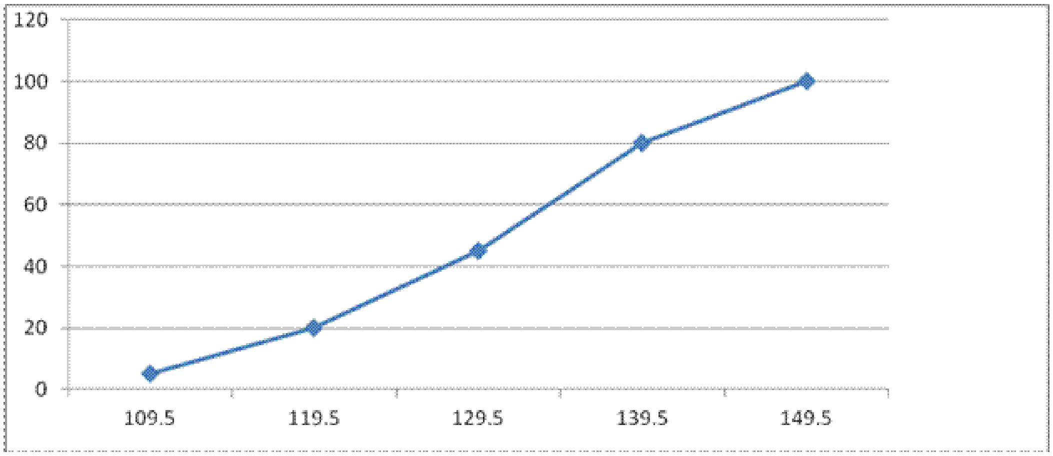


2. Motor vehicle company tested 100 cars to see how far they could travel on 10 litres of petrol. Draw the cumulative frequency curve for this company

distance in km	**100-109**	**110-119**	**120-129**	**130-139**	**140-149**
numbers of car	**5**	**15**	**25**	**35**	**20**

solution

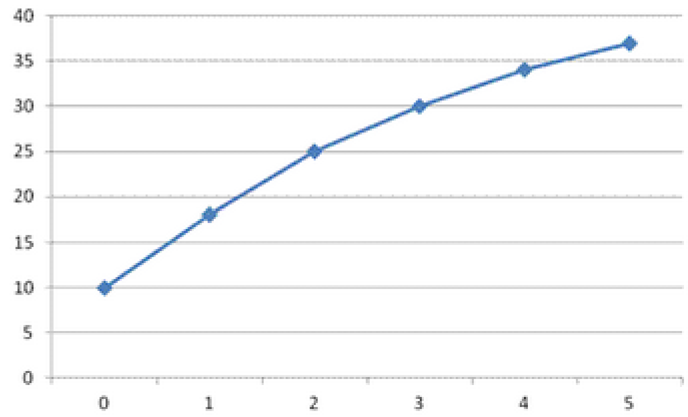
distance in km	f	cum. F
less than 109.5	5	5
less than 119.5	15	20
less than 129.5	25	45
less than 139.5	35	80
less than 149.5	20	100
		N=100



3. Platform in each square metre of a lawn were counted and recorded as follows. Draw an orgive for the platform

no.of plat forms	f	c. Frequency
0	10	10
1	8	18
2	7	25
3	5	30
4	4	34

5	**5**	**37**
	N = 37	



REVISION EXERCISE

1. 1. The ages of the 22 players in a football match were recorded in the following

17 18 15 16 16 16 18 15 18 15 15 18 18 15 16 17 15 16 17 15 15 16 15 18 15

Express the data in a frequency table.

Solution:

AGES	FREQUENCY
15	10
16	5
17	2
18	5
	N = 22

2. 2. The examination marks of 45 students are,

65 58 71 62 64 35 72 32 64 46 59 82 73 76 64 63 75 71 61 36 64 80 61 64 76 64 60 68 48 35 92 73 46 24 35 43 30 50 70 40 46 64 24 28

A)Make a frequency distribution using class interval 21-30, 31-40, 41-50,

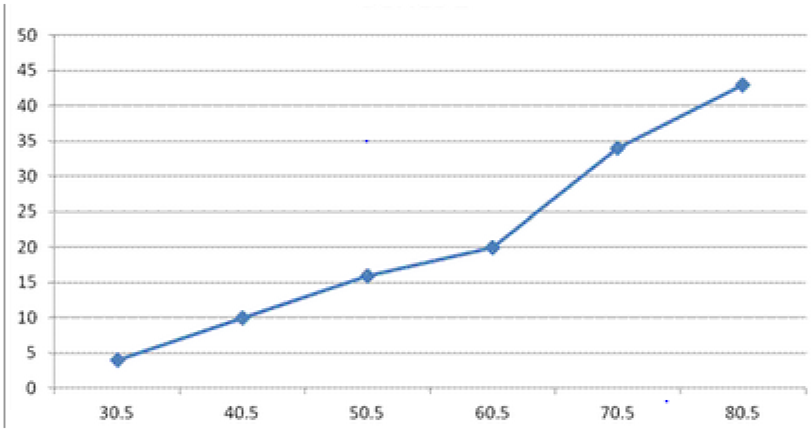
Solution:

c-	frequency

interval	
21-30	4
31-40	6
41-50	6
51-60	4
61-70	14
71-80	9
81-90	2
	n=45

B) Draw cumulative frequency curve

Solution:

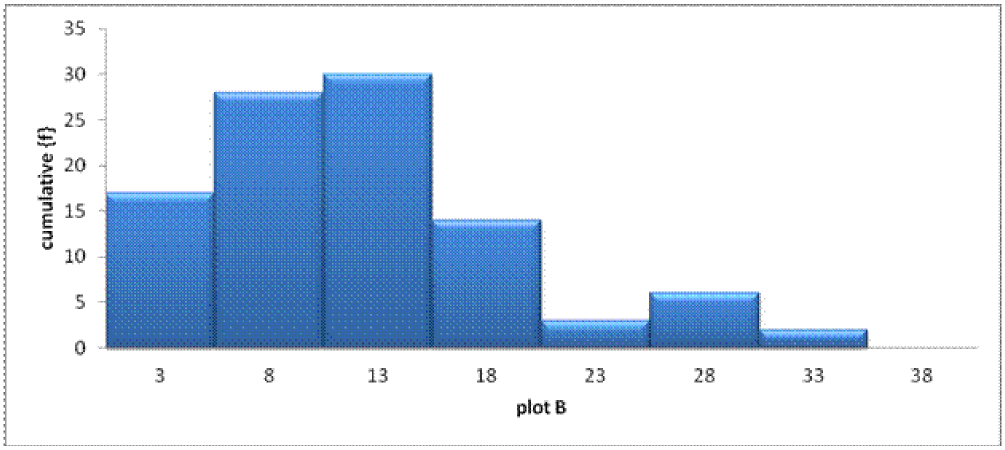


3. 3. Two plot A and B were treated with different families. The frequency number of potatoes on on samples of 100 plants on each plot are shown below

no.of potatoes	3	8	13	18	23	28	33	38
plot A	1	26	28	27	5	8	3	2
plot B	17	28	30	14	3	6	2	0

Draw a histogram for plot B.

Plot B



4. In a certain examination the result were as follows;

3 student got marks between 0and 10

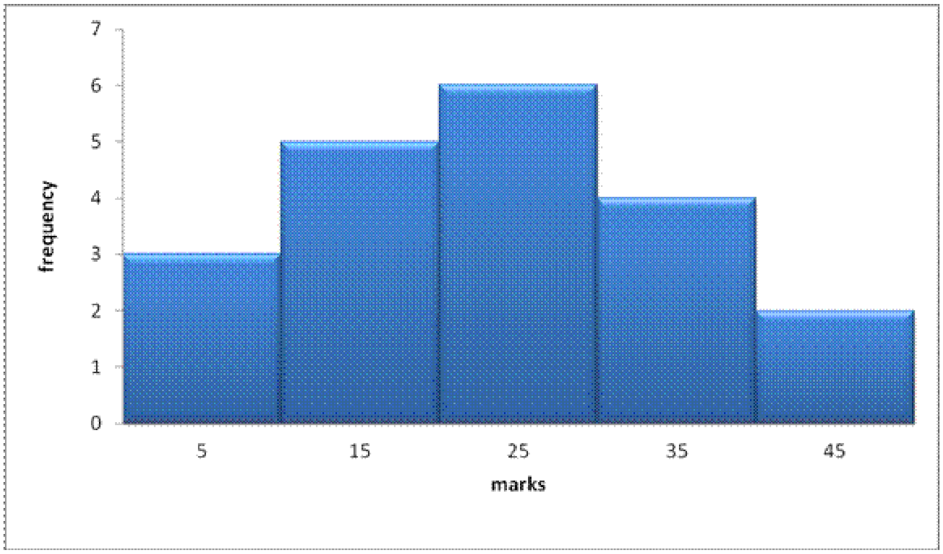
5 students got marks between 10 and 15

6 students got marks between 20 and 40

4 students got marks between 30 and 40

2 students got marks between 40 and 50

Construct a histogram



5] final score of history examination were recorded as shown in table below

score	frequency	c-mark
50-54	1	52
55-57	2	57
60-64	11	62
65-69	10	67
70-74	13	72
75-79	12	77
80-84	21	82
85-89	6	87
90-94	9	92
95-99	4	97

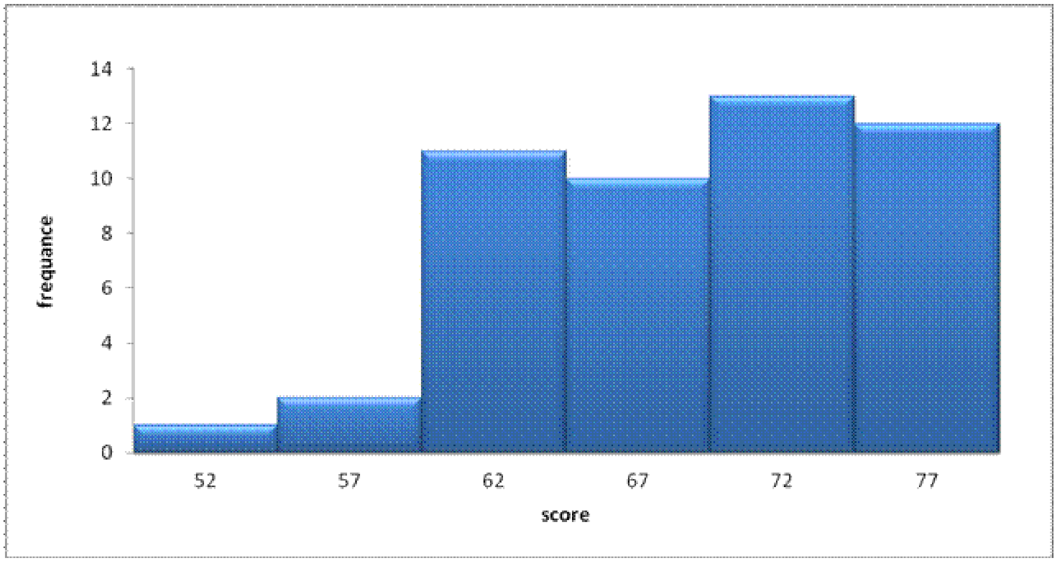
A) What is the size of class intervals?

Solution:

5 is the size of class intervals .

B)Draw a histogram to represent the scores .

Solution:



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