RELATIONS

A relation associates an element of one set with one or more elements of another set.

If **"a"** is an element from set A which associates another element **"b"** from set B, then the elements can be written in an ordered pairs as (a,b)
Thus we can define a relation as a set of ordered pairs.

Some relations are denoted by letter R; in set notation a relation can be written as
R = {(a, b): a is an element of the first set, b is an element of the second set}

Example of a relation

1. 1. Mwajuma is a wife of Juma.
2. 2. Amina is a sister of Joyce.
3. 3. y = 2x + 3
   4. Juma is tall, Anna is short. (Not a relation)

NOTE If the relation R defines the set of all ordered pairs (x,y) such that .

y = 2x + 3 this can be written symbolically as

R = {(x, y): y=2x +3}

PICTORIAL REPRESENTATION OF RELATIONS

Relation can be represented pictorially;

i) Arrow diagram.

ii) Cartesian graph.

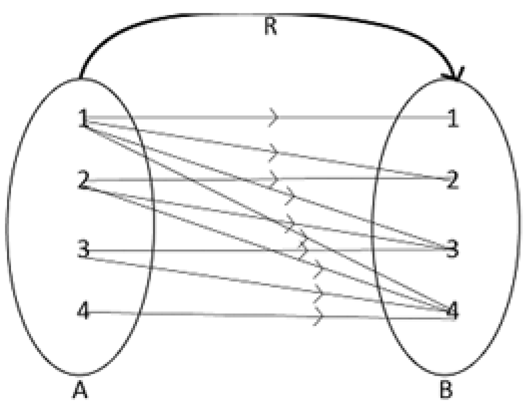
Arrow diagram

An arrow diagram (arrow gram or arrow graph) is a representation of a relation between sets by using the arrows.

Example:

1. Show the relation "is less than or equal to"between the members of the set {1, 2, 3, 4}, by using arrow diagram.

Solution:



R = is less than or equal to

Note: The arrow indicates that one element of one set relates to one or more elements off the other set.

The element of a set which mapped onto another set is called the **Domain of a relation**. The onto set is called the **Range of a relation.**

∴The elements of set A above are called the domains and those of set B are called the range.

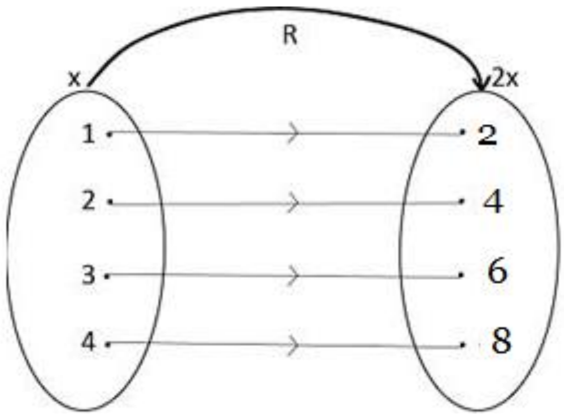
Also we use $A \longrightarrow B$ to mean "set A is mapped onto B"

Example 1

If x ⟶ 2x, We mean"x is mapped onto 2 times x".

When x is known we can select values of x as

x = 1, 2, 3, 4, 5 so the relation can be written as:-



Example 2

Given that $x \in A$ where A = {-1, 0, 2, 3, 4}. Draw a pictorial representation of the relation.

a) R: x ⟶ 3x
b) R: x ⟶ x^2

Solutions

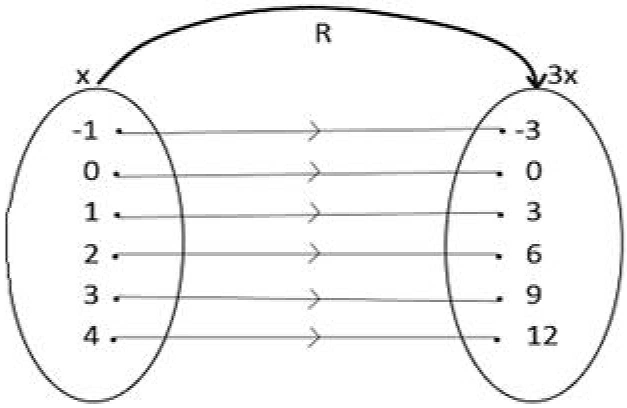
(a) R: x → 3x

Table of values

a)

x	-1	0	1	2	3	4
3x	-3	0	3	6	9	12

Pictorial representation



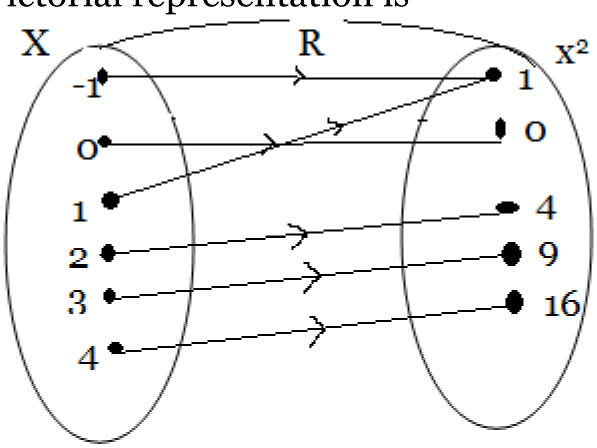
(b)

R: $x \longrightarrow x^2$

Table of values

x	-1	0	1	2	3	4
x^2	1	0	1	4	9	16

Pictorial representation is



Domain and Range of a relation

Consider a relation R which is a set of all ordered pairs (x, y). The Domain and the range of R can be defined as follows.

Domain of R = {x: (x, y) belongs to R for some y}

Range of R = {y: (x, y) belongs to R for some x}

Note: x is called the independent variable.

y is the dependent variable.

Examples

1. 1. Given that the relation R ={(x, y): y is a husband of x},find the domain and range of R

Solution

Domain of R = {all wives}

Range of R = {all husbands}

2. 2. Find domain and range of the relation

R = {(0, 2), (0, 4), (1, 2), (3, 5)}

Solution:

Domain of R = {0, 1, 3}

Range of R = {2, 4, 5}

3. 3. Find the range and domain of relation of

$y = 3x^2 + 2$

Solution:

Domain = {all real numbers x}

To find the range, make x the subject.

$$y = 3x^2 + 2$$

$$3x^2 = y - 2$$

$$x^2 = \frac{y-2}{3}$$

$$x = \sqrt{\frac{y-2}{3}}$$

$$\therefore x \text{ is real only if } \frac{y-2}{3} \geq 0$$

$$y \geq 2$$

$$\therefore Range = \{all\ real\ number\ y \geq 2\}$$

Graphs of a relation

Graph of a relation is another way of representing a relation. T he graph is drawn in the Cartesian plane and can also be called Cartesian graph.

Examples:

1. 1. Draw the sketch of the relation:-

R = {(x, y): y = 2x}, state domain and range

Solutions:-

Table of values

X	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

The graph can be obtained by plotting the ordered pairs in the x-y plane.

Domain of R={ all real numbers}

Range of R= {all real numbers}

2. Draw the graphs of the relations

$R = \{(x,y): x < y, x + y > 0 \text{ and } y < 2\}$ *Then find the domain and range*

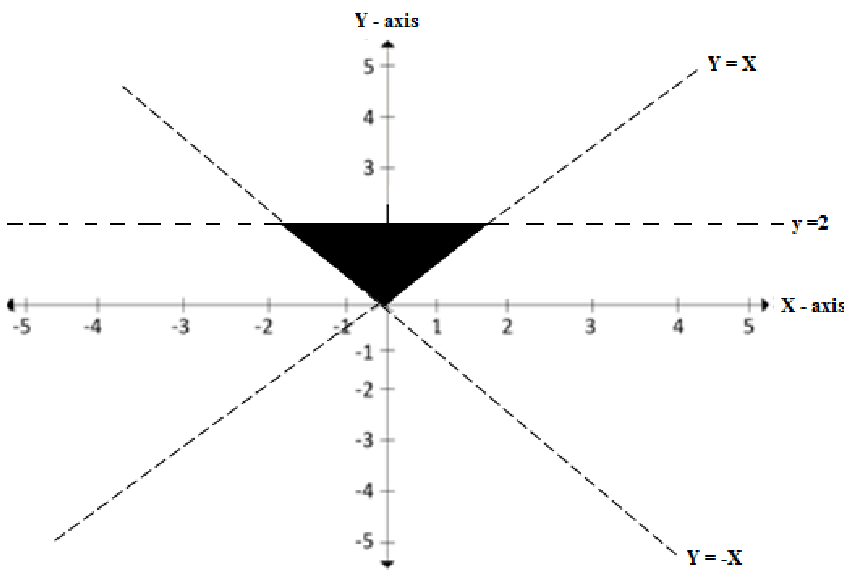
Table of values for x = y

x	0	1	2	3	-1	-2	-3
y	0	1	2	3	-1	-2	-3

Table of values x+y=0

y = -x

x	0	1	2	3	-1	-2	-3
y	0	-1	-2	-3	1	2	3



$Domain\ of\ R = \{x: -2 < x < 2\}$

$Range\ of\ R = \{y: 0 < y < 2\}$

Note

In sketching the graph of a relation of inequalities we use

1. Dotted line (------) for < and > Solid line (______) for =, ≤ and ≥

We always shade the required region for the inequalities graph

Example

Draw the graph for the relation

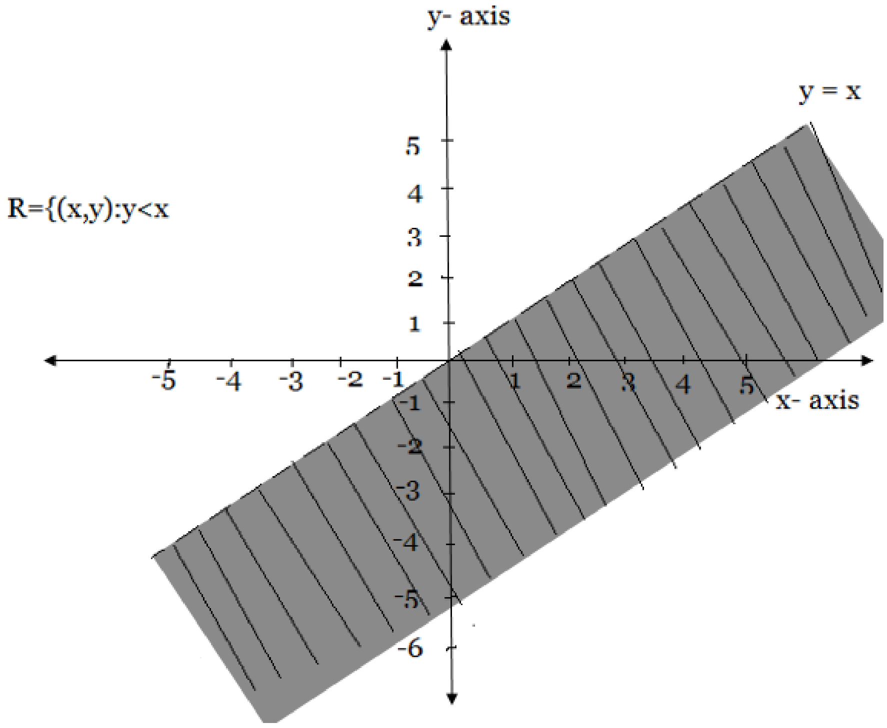
$R = \{(x, y): y < x\}$

Solution

The graph can be sketched as a graph of y=x

Some points belong to the relation R = {(x , y): y < x} are {(2,1), (4,3), (-2,-3), (-1,-4)}

The graph is



THE INVERSE OF THE RELATION

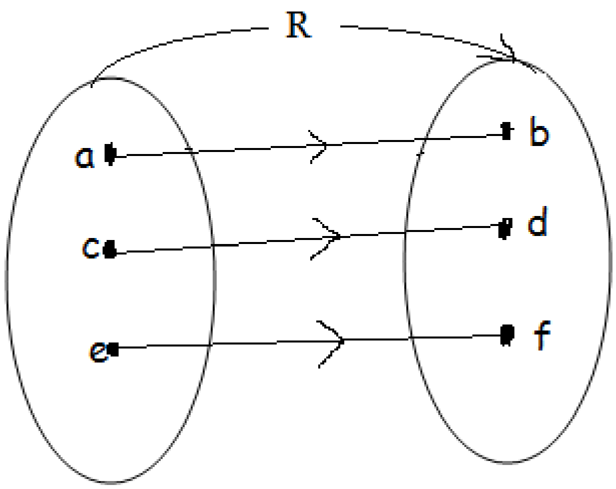
The inverse of the relation as R^{-1} can be obtained by reversing the order in all of the ordered pairs belonging to R. i.e If

$R = \{(a, b), (c, d), (e, f)\}$ *Then*

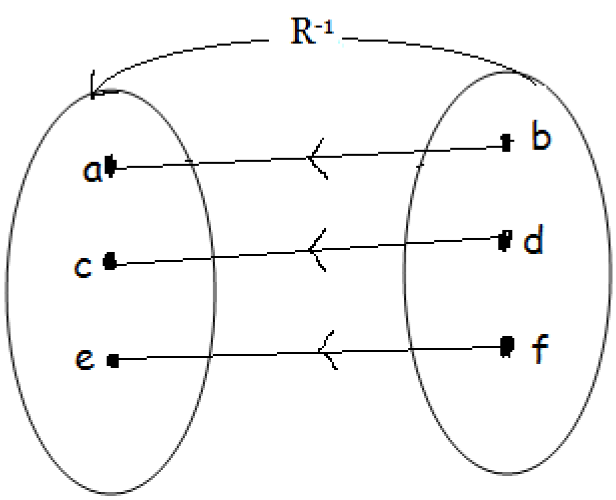
$R^{-1} = \{(b, a), (d, c), (f, e)\}$

The pictorial representation for R^{-1} can be obtained from the picture of R by reversing the direction of all the arrows

Pictorial representation of R



Pictorial representation of R^{-1}



The domain in R becomes the Ranges of R^{-1}

$\therefore \text{Domain of } R^{-1} = \text{Range of } R$

Range of $R^{-1} = domain\ of\ R$

The inverse of the above relation can also be found by first writing x in terms of y and then interchanging the variables. Therefore (x, y) becomes (y,x) in the inverse relation.

Example

1. 1. Given the relation $R = \{(x,y): y = 4x^2\}$

    (a) Find the inverse of R

    (b) Find the domain and range of R^{-1}

Solution

$R = \{(x,y): y = 4x^2\}$

$y = 4x^2$

Interchange the variables and make y the subject

$x = 4y^2$

$y^2 = \frac{x}{4}$

$\sqrt{y^2} = \sqrt{\frac{x}{4}}$

$y = \pm\frac{1}{2}\sqrt{x}$

$R^{-1} = \{\{x,y): y = \pm\frac{1}{2}\sqrt{x}\}$

(b). $Domain\ of\ R^{-1} = \{x: x \geq 0\}$

$Range\ of\ R^{-1} = \{y: y\ is\ all\ real\ numbers\}$

2.2. Given the relation $R = \{(x,y): 2x + 5 < y\}$ $Find\ the\ inverse$

$Then\ find\ the\ Domain\ and\ Range\ of\ the\ inverse$

Solution

The inverse of $R = \{(x,y): 2x + 5 < y\}$

Write x in terms of y

$2x + 5 < y$

$2x < y - 5$

$x < \frac{y-5}{2}$

$By\ interchanging\ the\ variables\ we\ get$

$y < \frac{x-5}{2}$

$R = \left\{(x,y): y < \frac{x-5}{2}\right\}$

$Domain\ of\ R^{-1} = \{All\ real\ numbers\}$

$Range\ of\ R^{-1} = \{All\ real\ numbers\}$

GRAPHS OF THE INVERSE OF THE RELATION

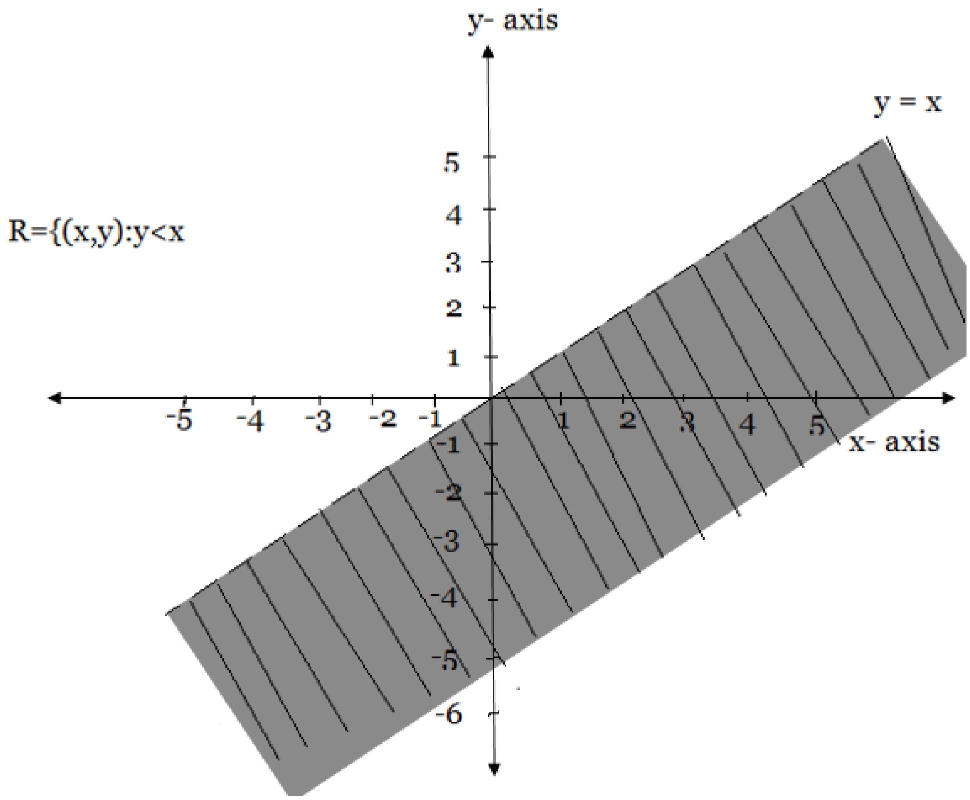
Consider the relation $R = \{(x,y): y < x\}$

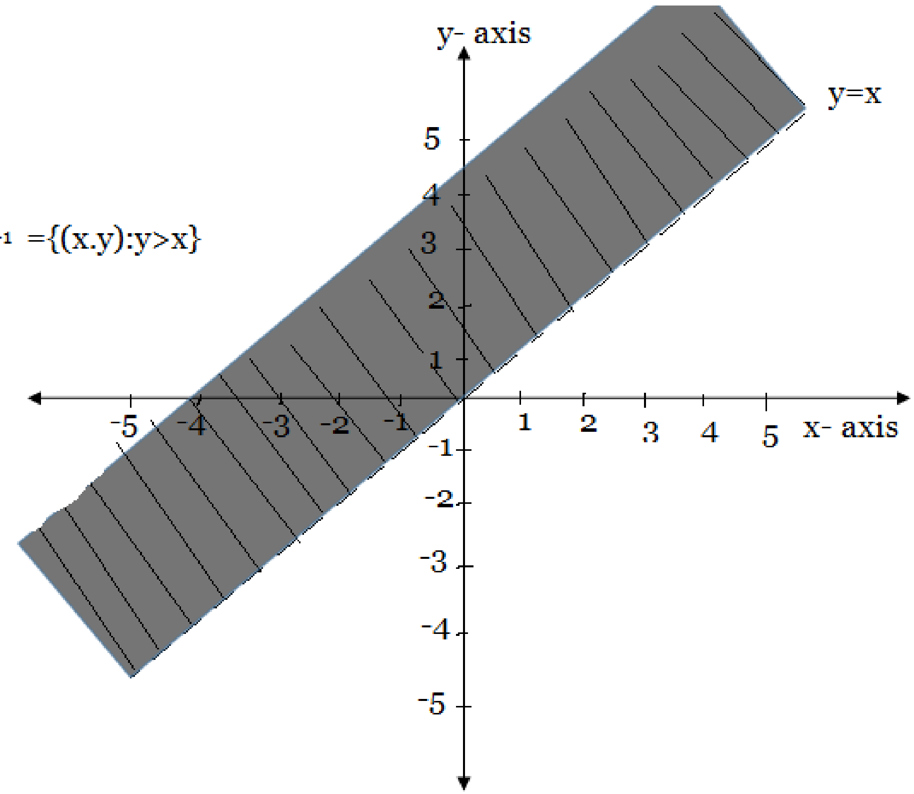
Its inverse is $R^{-1} = \{(x,y): y > x\}$

In this case R is the relation less than for all real numbers,

The graph of R $= \{(x,y): y < x\}$ and $R^{-1} = \{(x,y): y > x\}$

Are shown as shaded region below





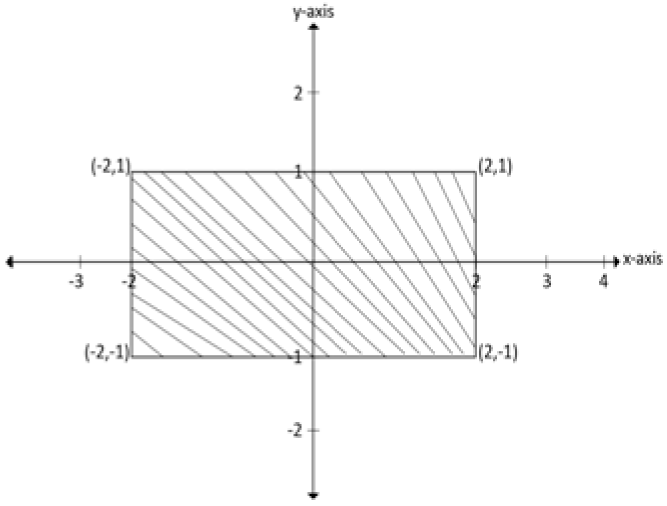
Note: The graph of R^{-1} for any relation can be obtained by reflecting the graph of R about the line y=x

Thus we can draw the graph of R^{-1} when R is given by first drawing R and then flecting it about the line y = x

Examples

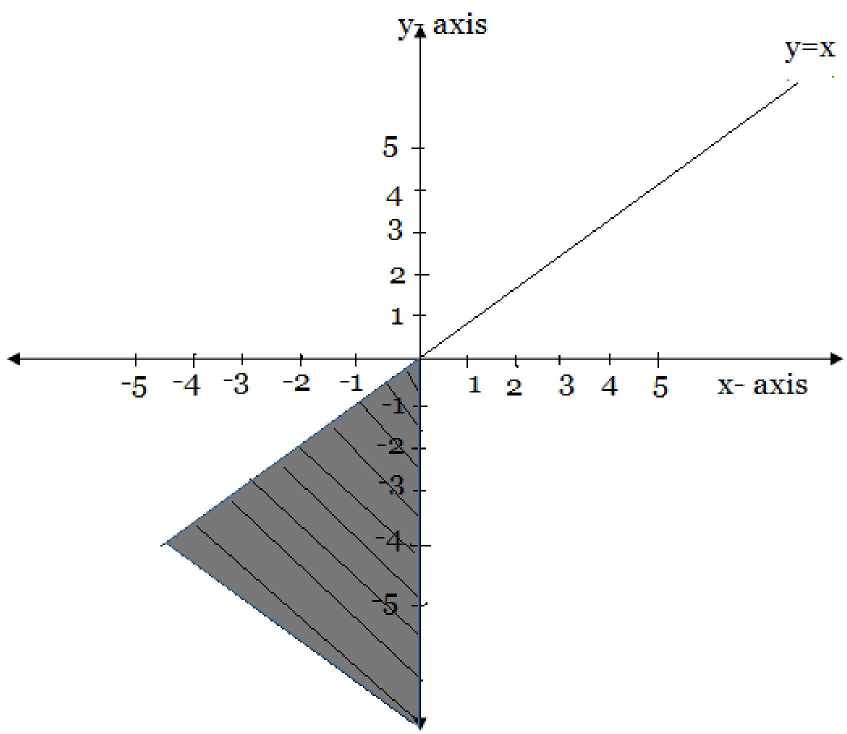
1. Draw the graph of the inverse of $R = \{(x,y): y \leq 0 \; and \; y \geq x\}$
Find its Domain and range
2. Draw the graph of the inverse of the relation shown in the figure

below. Find its domain and range



Solutions for question 1

$R^{-1} = \{(x, y): x \leq 0 \text{ and } y \leq x\}$



The domain and range of R^{-1} is the intersection of the domain of the two given relations

$\therefore \text{Domain of } R^{-1} = \{x: x \leq 0\}$

$\text{Range of } R^{-1} = \{y: y \leq 0\}$

Solution for question 2

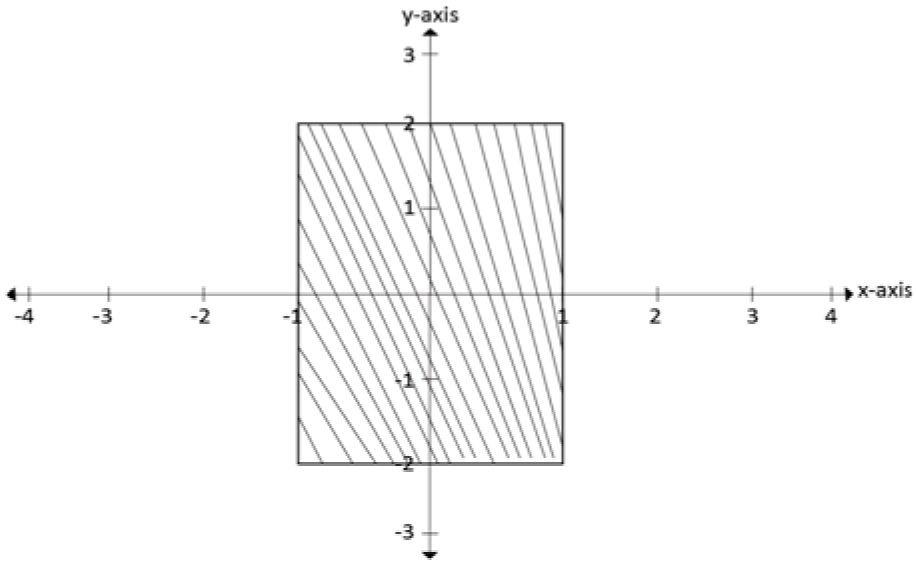
By using the coordinate on the boundary of R we have

$R = \{(2,1), (-2,1), (-2,-1), (2,-1), (o,1), (-2,0), (0,-1), (2,0)\}$

$R^{-1} = \{(1,2), (1,-2), (-1,-2), (-1,2), (1,0), (0,-2), (-1,0), (0,2)$

Use the ordered pair to plot the graph of R^{-1}

$\therefore \text{The graph of } R^{-1} \text{ is}$

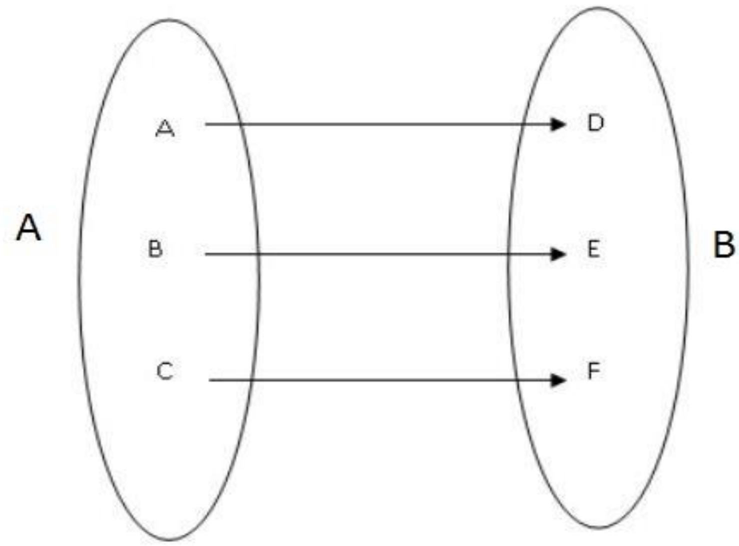


Domain of $R^{-1} = \{x: -1 \leq x \leq 1\}$

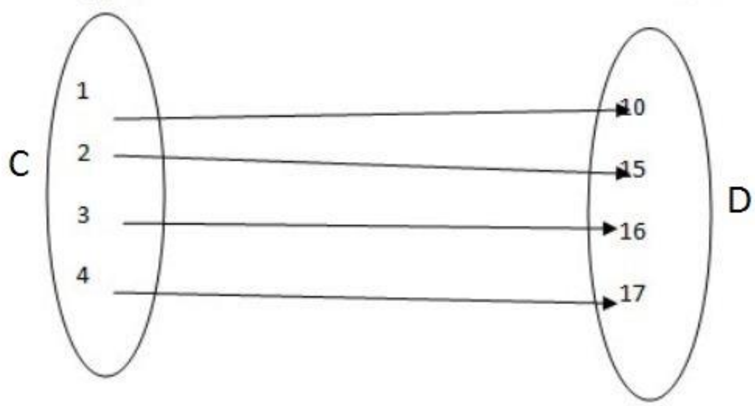
Range of $R^{-1} = \{y: -2 \leq y \leq 2\}$

FUNCTION

A function is a set of ordered pairs which relates two sets such that to each element of one set there is only one element of the second set



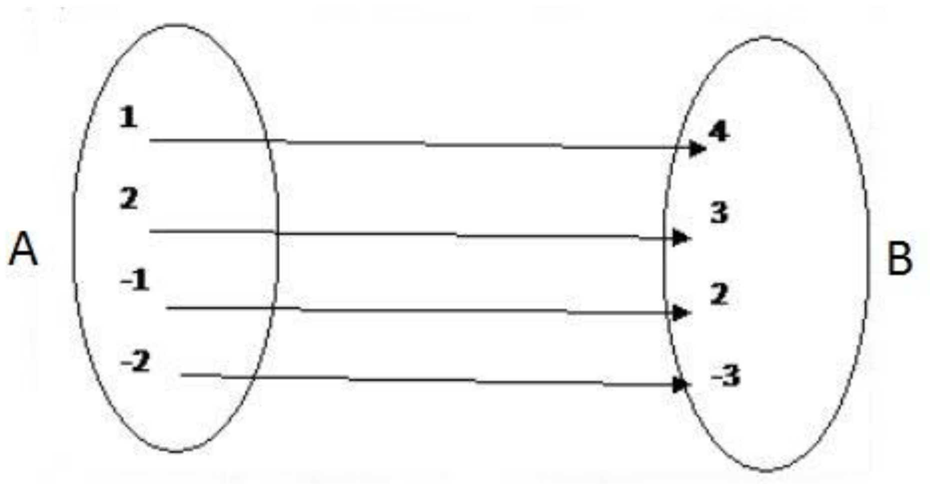
Or



Example

Represent the following set of ordered pairs in a pictorial diagram.
(1,4), (2,3), (-1,2), (-2,-3).

Solution



A function whose graph is such that any line drawn parallel to the x – axis at any point cuts it at only one point is **one-to-one function.**

Examples.

1. Write the expression of a function ‘ double plus one’

Solution

f:x ⟶ 2x +1

b)

2.Given the function G(x) = 4x-1. Find the value of G(-2).

Solution

G(x) = 4x -1

G(-2) = 4(-2) -1

G(-2) = -8 -1

G (-2) = -9

Exercise 2.1

1. Write each of the following function in the form f: x ⟶ f(x) use any functions symbol to represent the functions

(a)Divide by 5 and add 2.
(b)Subtract 7 and square
(b)Cube and then double

Solution

(a) F:x $\longrightarrow$ f(x)

F:x $\longrightarrow \frac{x}{5} + 2$

(b) F: x $\longrightarrow (x-7)^2$

(c) F:x $\longrightarrow x^3+2x$

2. Find the value of the function for each given value of x.

(a) f(x)= 2x+3; when
(i) x=1
(ii) x= -2
(iii) x =a

Solution

(a) (i) when x=1

f(x)= 2x+3

f(1)= 2(1)+3

f(1)= 2+3

∴ f(1)= 5.

(ii)when x= -2.

f(x) = 2(-2) +3

f(-2) = -4+3

∴f(-2) = -1

(iii) when x=a
f(x) = 2(a) +3

f(a) =2a+3

∴f(a) = 2a+3

(b) C(x) = x^3; when
(i) x=1
(ii) x = -1
(iii)x= 0
(iv)x=b

Solution

(i) C(x) =x^3

C (1) =1^3

C (1) =1

C (1) =1

(ii)C(x) =x^3

C (-1) = $(-1)^3$

C (-1) = -1

(iii)C(x) =x^3

C (0) =0^3

C (0) =0

(iv)C(x) =x^3

C(b) = b^3

(c) K(x) = 3-x; when
(i) x= -1
(ii)x=7

Solution

(i) K(x) = 3-x

K (-1) = 3- (-1)

K (-1) = 4

(ii) k(x)=3 - x
k(7) = 3-7
k(7) = -4

DOMAIN AND RANGE OF FUNCTIONS

Example 1.

1. Find the domain and range of f(x) =2x+1

let f(x) = y
y=2x+1

Solution

Domain = {x: x $\in$ *R*}

Range - make x the subject
y=2x+1
y-1=2x
(y-1)/2 =x
∴x=(y-1)/2
Range={y: y $\in$ *R*}

b **Example 2**
If y =4x+7 and its domain ={x : -10 $\leq$ x$\leq$ 10} find the range.

Solution

Domain = {x: -10 $\leq$ x$\leq$ 10}

Range ={y : -33 $\leq$ y $\leq$ 47}

y = 4x + 7

Table of values

X	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
y	-33	-29	-25	-21	-17	-13	-9	-5	-1	3	7	11	15	19	23	27	31	35	39	43	47

c **Example 3.**

Y= √x and domain is -5 ≤ x ≤ 5, Find its range.

Solution

Domain = {x: -5 ≤ x ≤ 5}

Range =y

Table of value of x = √(y)

X	0	1	2	3	4	5
y	0	1	√2	√3	2	√5

$(y)^2= (\surd x)^2$

$y^2= x$

x= y2

$\surd 5= \surd y^2$

y = √5

Range = {y: o ≤ y ≤ √5}

d **Example 4.**

Given F(x) = $\sqrt{1-x^2}$ find the domain and range.

Solution

Let f(x)=y

Domain of y= $\sqrt{1-x^2}$

For real values of y: $1-x^2 \geq 0$

$-x^2 \geq 0-1$

$-x^2 \geq -1$

$x^2 \leq 1$

$\sqrt{x^2} \leq \sqrt{1}$

$X \leq \sqrt{1}$

$X \leq 1$

∴Domain {x : X ≤1 }

let F(x) = y

$y = \sqrt{1 - x^2}$

To get the range, make x the subject

$y^2 = (\sqrt{1 - x^2})^2$

$y^2 = 1 - x^2$

$X^2 = 1 - y^2$

Therefore $\sqrt{x^2} = \sqrt{1 - y^2}$

$X = \sqrt{1 - y^2}$

For real value of x:

$1 - y2 \geq 0$

$y2 \leq 1$

$y \leq \sqrt{1}$

$y \leq 1$

∴Range ={y : y ≤1}

LINEAR FUNCTIONS

Is the function with form f(x) = mx + c.

Where:

f(x)= y

m and c are real numbers

m is called **gradient[slope].**

c is called **y – intercept.**

Example

1. Find the linear function f(x) given the slope of -2 and f(-1)=3

Solution

Given:
m(slope)=-2
x=-1
f(-1)=3

from;
f(x) =mx +c

f(-1) =(-2x-1)+c

3 =2+c

3-2 =c

c=1

∴f(x) = -2x+1

Example 2.
Find the linear function f(x) when m=3 and it passes through the points (2, 1)

Solution

f(x) =mx +c

f(2) = 3(2)+ c

1= 6+c

1-6 =c

-5 = c

c= -5

f(x) = 3x + -5

∴ f(x) =3x-5

Example 3
Find the linear function f (x) which passes through the points (-1, 1) and (0, 2)

Solution

Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{0+1}$

=1

m = 1

f(x) = mx +c

f(-1) = 1x(-1) +c

1 =-1+c
c=2

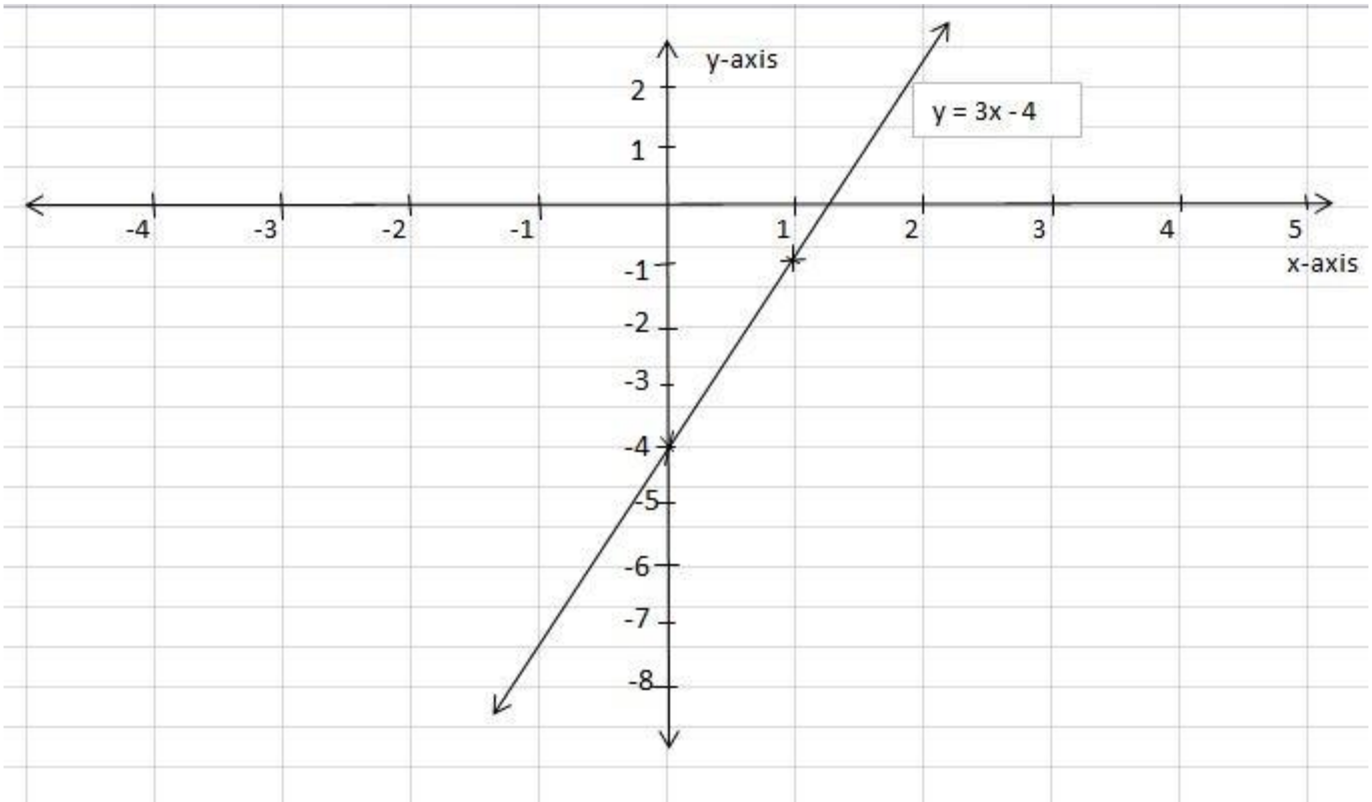
f(x) = 1x +2

∴f(x) = x +2.

4.Draw the graph of h(x) = 3x-4

Table of values of function

X	**-1**	**0**	**1**
h(x)	**-7**	**-4**	**-1**



Exercise

In problems 1 to 3 find the equation of a linear function f(x) which satisfies the given properties. In each case, m dissolves the gradient.

1). m =-3, f(1) = 3
2). m=2, f(0) =5
3). f(1) =2, f(-1) =3

4. Givenm= -4 , f(3) = -4 Find f(x)

In the problem 5 to 9 draw the graphs of each of the given functions without using the table of values

5) f(x)= $\frac{2}{5}x + \frac{1}{5}$

6) f(x) =4

Solution

1. f(x) = mx + c

f(x) = -3(1)+c

f(x) =-3 +c

3= -3 +c

3+3 =c

6=c

C=6

f(x) = -3x +6

2. f(x) =mx +c

f(0) = (2 x 0)+c

5= 0+c

c=5

f(x) =2x+5

3)

3.f(1) =2, f(-1) =3

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{3-2}{-1-1}$

$M = -{}^1/_2$

f(1)=2

f(x)= mx +c

$f(1) = -{}^1/_2 \times 1 + c$

$f(1) = -{}^1/_2 + c$

$2 = -{}^1/_2 + c$

$2 + {}^1/_2 = c$

Alternatively

f(-1) = m(-1) + c

3= -m + c......................(i)

f(1) = m(1) + c

2= m + c(ii)

Solve (i) and (ii) Simultaneously

-m + c = 3

+ m + c = 2

C=5/2

$C = 2\,{}^{1}/_{2}$ put c in (i)

$f(x) = -{}^{1}/_{2}x + 2\,{}^{1}/_{2}$ 3 = -m + 5/2

m = 5/2 - 3
m= -1/2
$\therefore f(x) = -{}^{1}/_{2}x + 2\,{}^{1}/_{2}$

4.

f(x)= mx + c

f(x) = -4(3)+c

-4 =-12+c

-4+12= c

8=c

C=8

f(x) = -4x+8

5. $f(x)= \frac{2}{5}x + \frac{1}{5}$

y- intercept, x=0

$f(0)= {}^{2}/5[0] + {}^{1}/5$

$f(0)= 0+{}^{1}/5$

$y={}^{1}/5$

{0,0.2}

x- intercept, y=0

$0={}^{2}/5[x] + {}^{1}/5$

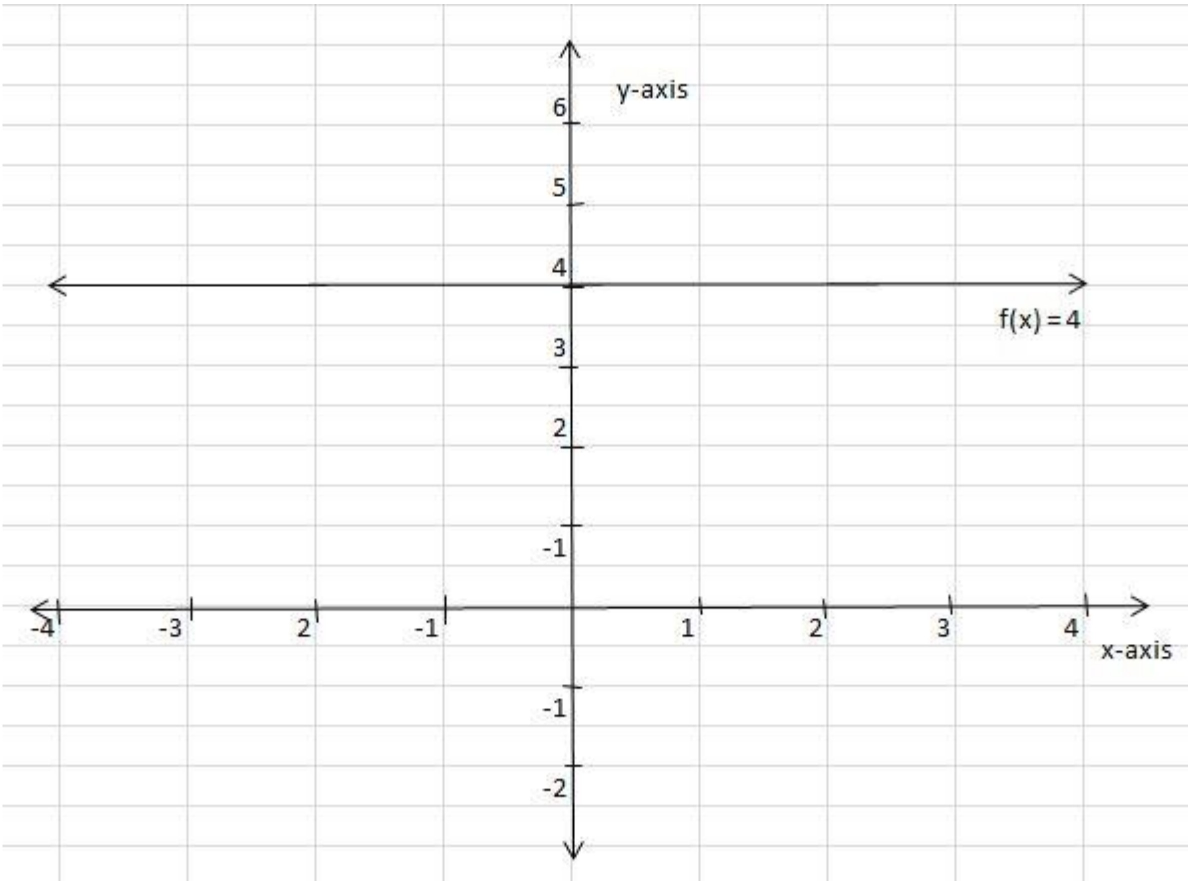
x intercept = - ½ (-1/2, 0)

y intercept = 1/5 (0,0.2)

6. f(x) =4

From f(x) = y

y = 4



QUADRATIC FUNCTIONS

A quadratic function is any function of the form;

$$f(x)=ax^2+bx+c$$

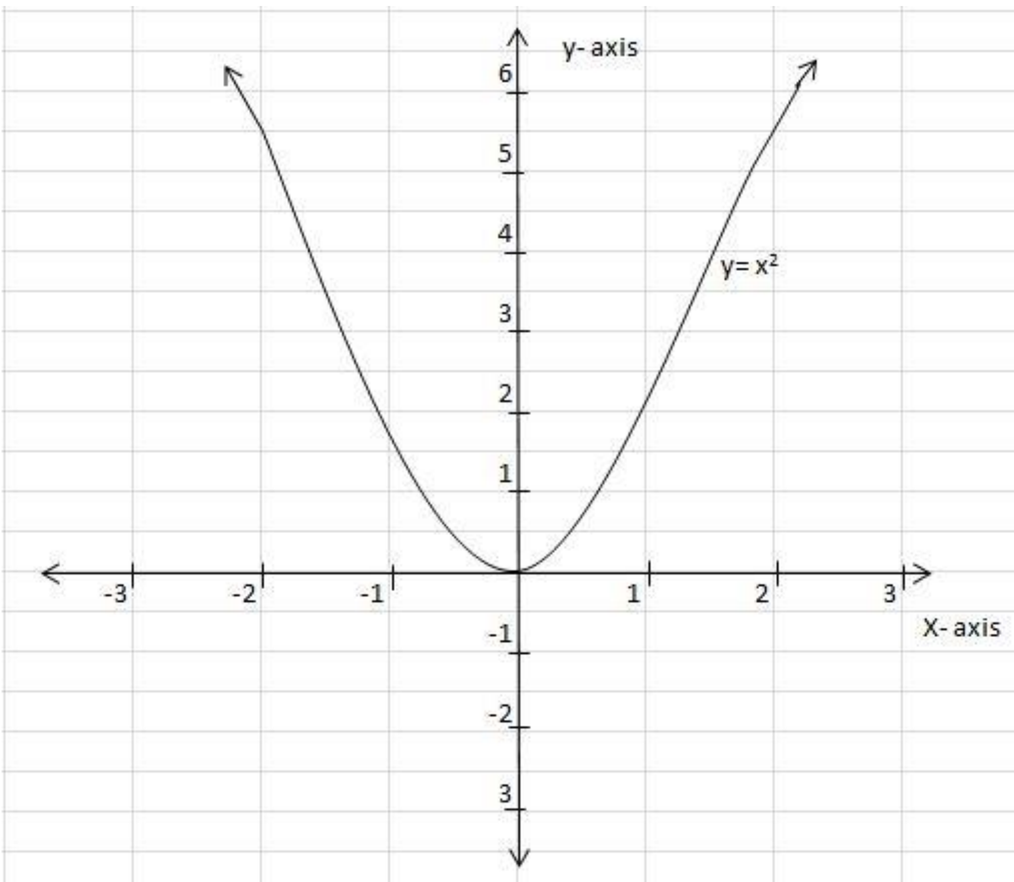
Where $a \neq 0$

a ,b and c are real numbers.

When a = 1, b=0, and c= 0

X	-3	-2	-1	0	1	2	3
f(x)	9	4	1	0	1	4	9

The shape of the graph of $f(x)=ax^2+bx+c$ is a parabola



•The line that divides the curve into two equal parts is called a **line of symmetry [axis of symmetry]**

•Point (0,0) in $f(x)=x^2$ called the **turning point (vertex).**

If “a” is positive the turning point is called **minimum point (least value).**

If "a" is negative the turning point is called **maximum point.**

PROPERTIES OF QUADRATIC FUNCTIONS

Think of $f(x)= ax^2+bx+c$

$y=a(x^2+bx/a)+c$

$y= a(x^2+bx/a+b^2/4a^2)+ c-b^2/4a^2$

$$=a\left(x + \frac{4b}{8a}\right)^2 + \frac{4ac - b^2}{4a}$$

x> 0 then

$a(x+\frac{b}{2a})^2 \geq 0$

$y = \frac{4ac - b^2}{4a}$

This is when $x = -\frac{b}{2a}$

The turning point of the quadratic function is $(-\frac{b}{2a}, \frac{4ac - b^2}{4a})$

Example

Find the minimum or maximum point and line of symmetry $f(x) = x^2 - 2x-3$. Draw the graph of f(x)

Solution:

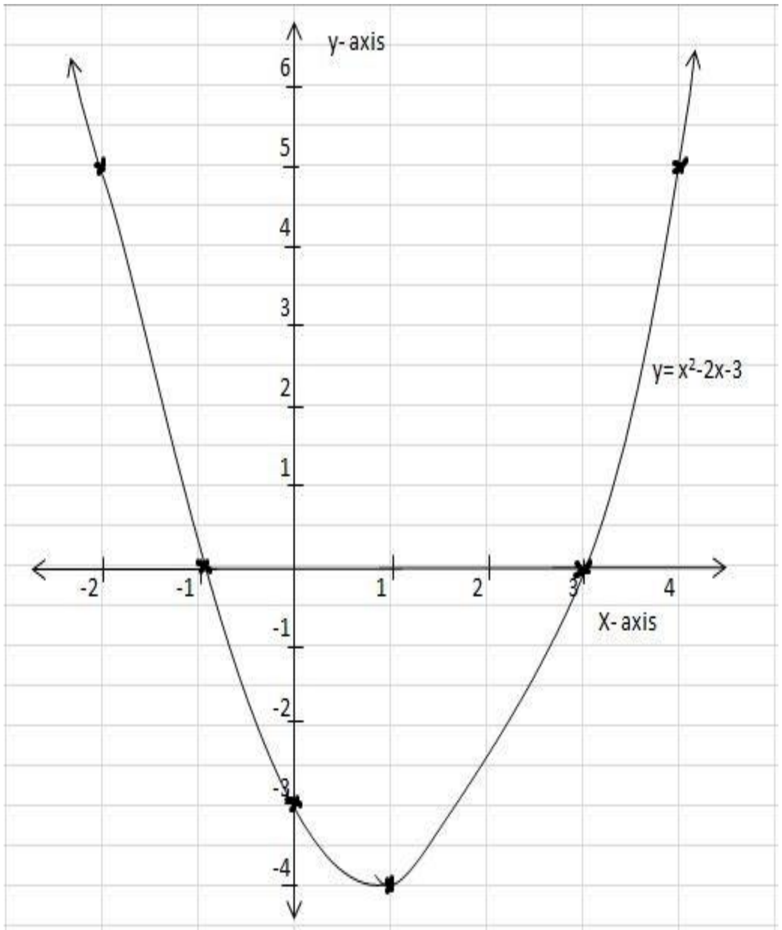
Turning point = $(-\frac{b}{2a}, \frac{4ac - b^2}{4a})$

$= (-\frac{2}{2 \times 1}, \frac{4(1)(-3) - 4}{2 \times 1})$

Maximum point = (1, -4)

Line of symmetry is x=1

x	**-5**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
f(x)	**15**	**5**	**0**	**-3**	**-4**	**-3**	**0**	**5**

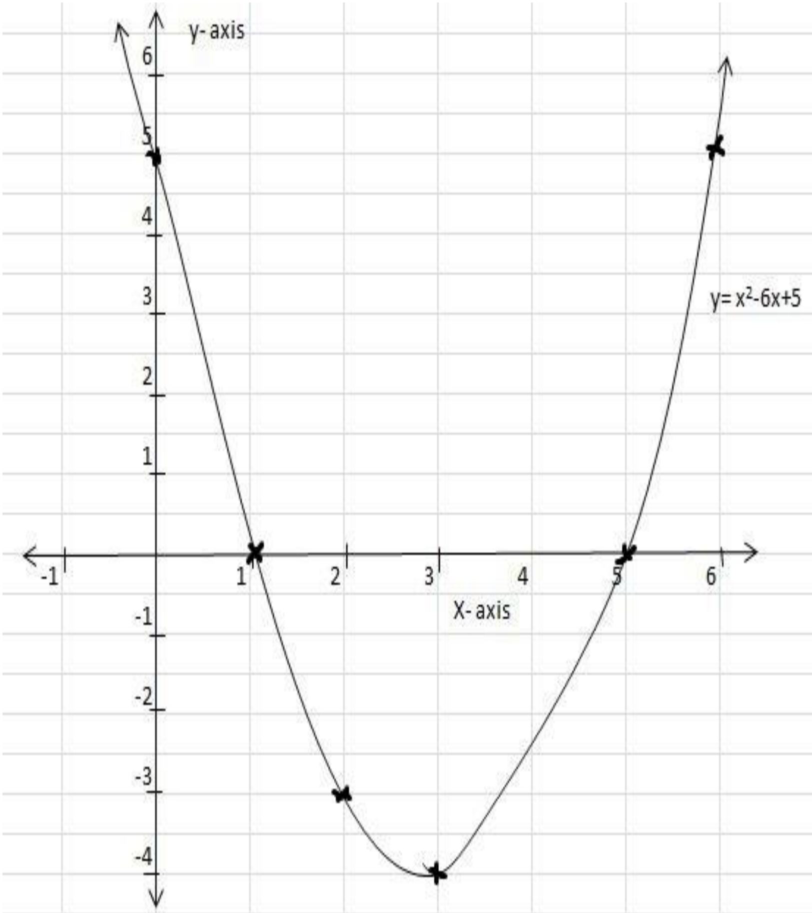


Exercise

1. Draw the graph of the function $y=x^2-6x+5$ find the least value of this function and the corresponding value of x

Solution

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**
y	**32**	**21**	**12**	**5**	**0**	**-3**	**-4**	**-3**	**0**



Least value.

y= - 4 where x=3

2. Draw the graph of the function y =x^2-4x+2 find the maximum function and the corresponding value of x use the curve to solve the following equations

a) X^2-4x-2 = 0

b) X^2-4x-2 = 3

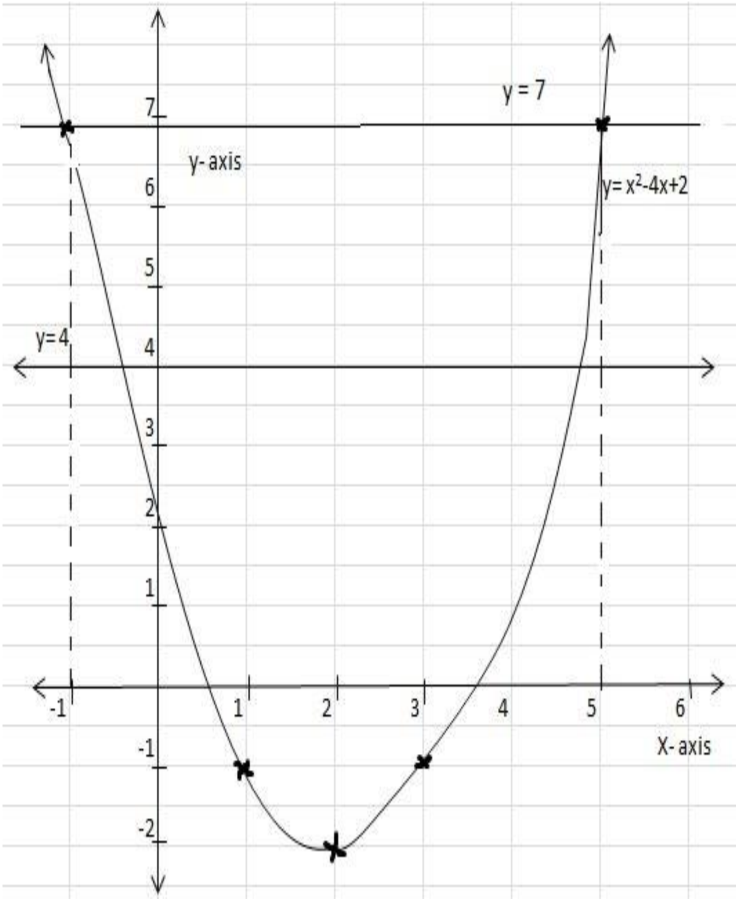
b)

Solution

Table of values of y= x^2-4x+2

x	-3	-2	-1	0	1	2	3	4	5	6
y	23	14	7	2	-1	2	-1	2	7	14

Find the maximum value of the function.;



Maximum value

$$= \frac{4ac - b^2}{4a}$$

$$Y = \left(\frac{4(1)(-2)+4}{2 \times 1}\right) = -2$$

Find the maximum value of the function;

Maximum value= $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

Maximum value= $\left(-\frac{4}{2 \times 1}\ ,\ \frac{4(1)(2)-(4)^2}{4 \times 1}\right)$

Maximum value

[2, -2]

The maximum value is = (-2,-2)

a) x^2-4x-2 = 0
add 4 both sides

x^2-4x + 2 = 4
but x^2-4x+2 = y
∴ y = 4
Draw a line y = 4 to the graph above. The solution from the graph is

x_1 = -1/2 , x_2 = 9/2
(x_1,x_2) = (-1/2,9/2)

(b) x^2-4x - 2 = 3

add 4 both sides
x^2-4x - 2 +4 = 3+ 4
x^2-4x + 2 = 7

but x^2-4x+2 = y
∴ y = 7
Draw a line y = 7 to the graph above. The solution from the graph is

x_1 = -1 , x_2 = 5
(x_1,x_2) = (-1,5)

3. In the problem 3 to 5 write the function in the form f(x)= a(x + b)^2 +c where a, b, c are constants

f(x) =5-x-9x^2

Solution

f(x) = -9x^2 – x + 5

= $-9\left(x^2 - \frac{x}{9}\right) + 5$

$= -9(\ x^2 - \frac{x}{9} + \frac{1}{324}) + 5 + \frac{1}{36}$

$= -9(x - \frac{1}{18})^2 + \frac{181}{36}$

4. In the following functions find:

a) The maximum value

b) The axis of the symmetry

$f(x)= x^2- 8x+18$

Solution

$(-\frac{b}{2a}, \frac{4ac-b^2}{4a}) = (-\frac{-8}{2}, \frac{4(1)(18)-(8)^2}{4})$

$(4, 2)$

Maximum value =2 where the axis of symmetry x=4.

5. $f(x) = 2x^2+3x+1$

Solution

Maximum value $(-\frac{b}{2a}, \frac{4ac-b^2}{4a})$ ___

$(-\frac{3}{2 \times 2}, \frac{4(2)(1)-3^2}{4 \times 2})$

The turning point of the graph is $(-\frac{3}{4}, \frac{-1}{8})$

The minimum value of the graph is $y = -\frac{1}{8}$

axis of symmetry

$= -\frac{3}{4}$

POLYNOMIAL FUNCTIONS

The polynomial functions are the functions of the form, " $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x^1 + a_0 x^0$ ". Where n is non negative integer and $a_n, a_{n-1}, a_{n-2} \ldots a_0$ are real numbers. The degree of a polynomial function is the highest power of that polynomial function.

Example

a) $f(x) = 5x^4 - 7x^3 + 8x^2 - 2x + 3$ is a degree of 4

b) $H(x) = 6x - 8x^2 + 9x^9 - 6$ is a degree of 9

c) $G(x) = 16x - 7$ is a degree of 1

d) $M(x) = 6$ degree is $0 = 6x^0$

GRAPHS OF POLYNOMIAL FUNCTIONS

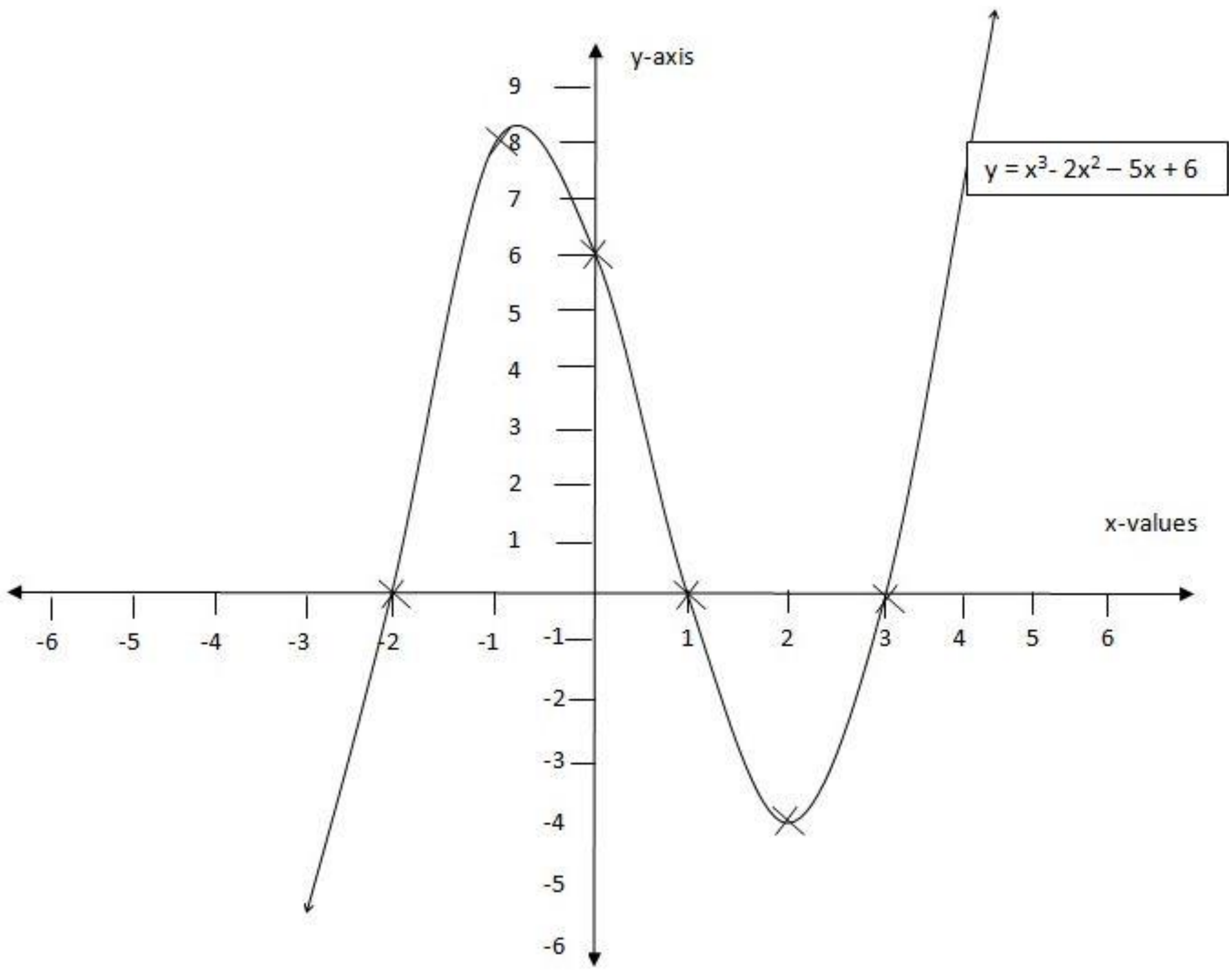
EXAMPLE

Draw the graph of $f(x) = x^3 - 2x^2 - 5x + 6$

Solution

Table of values

x	-3	-2	-1	0	1	2	3	4	
F[x]	-24	0	8	6	0	-4	0	18	



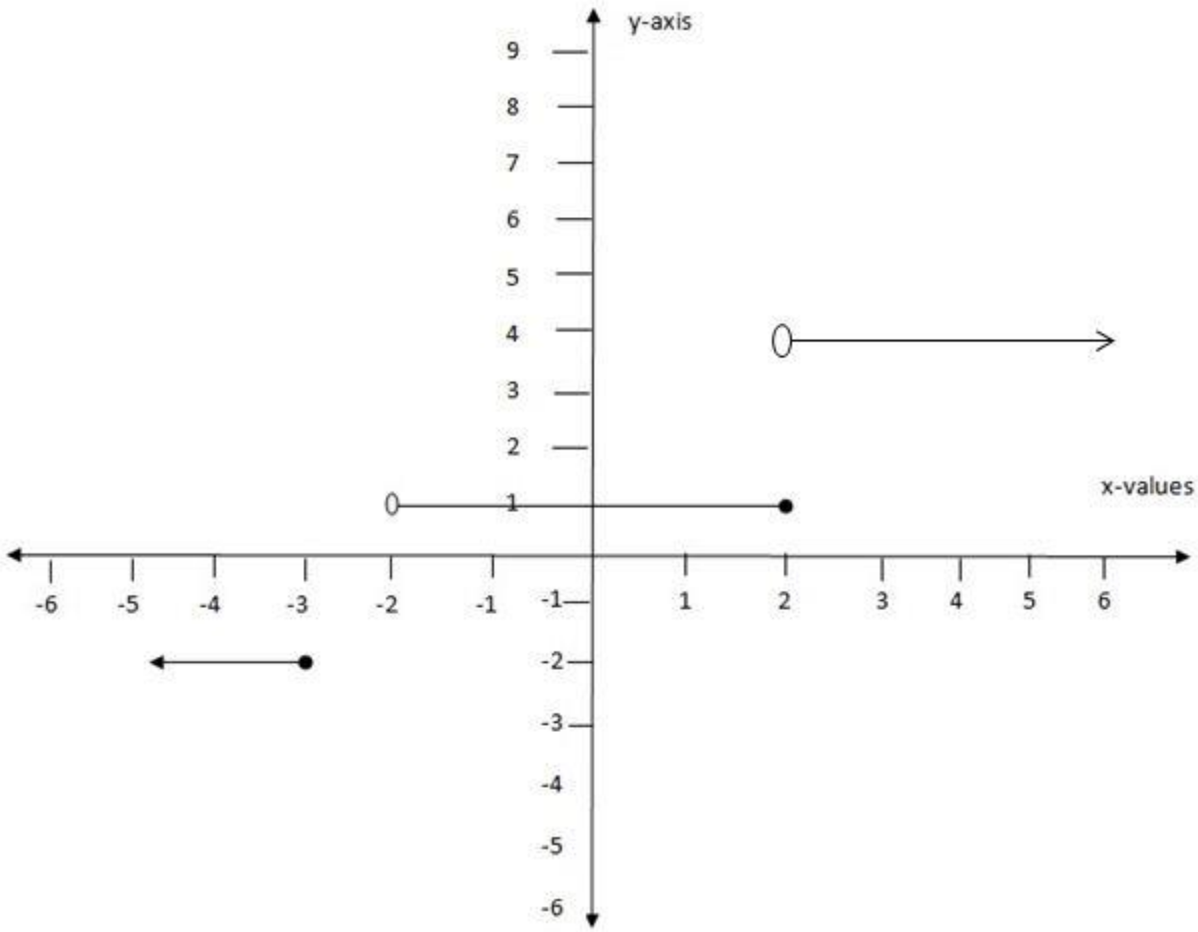
STEP FUNCTIONS

EXAMPLE

1. If f is a function such that;

$$f(x)=\begin{cases} -2 \text{ if } X \leq -3 \\ 1 \text{ if } -2 < x \leq 2 \\ 4 \text{ if } 2 < x \end{cases}$$

Draw the graph and find its domain and range.



Domain = {x: x $\in$ R, except -3$\leq$ x < -2}

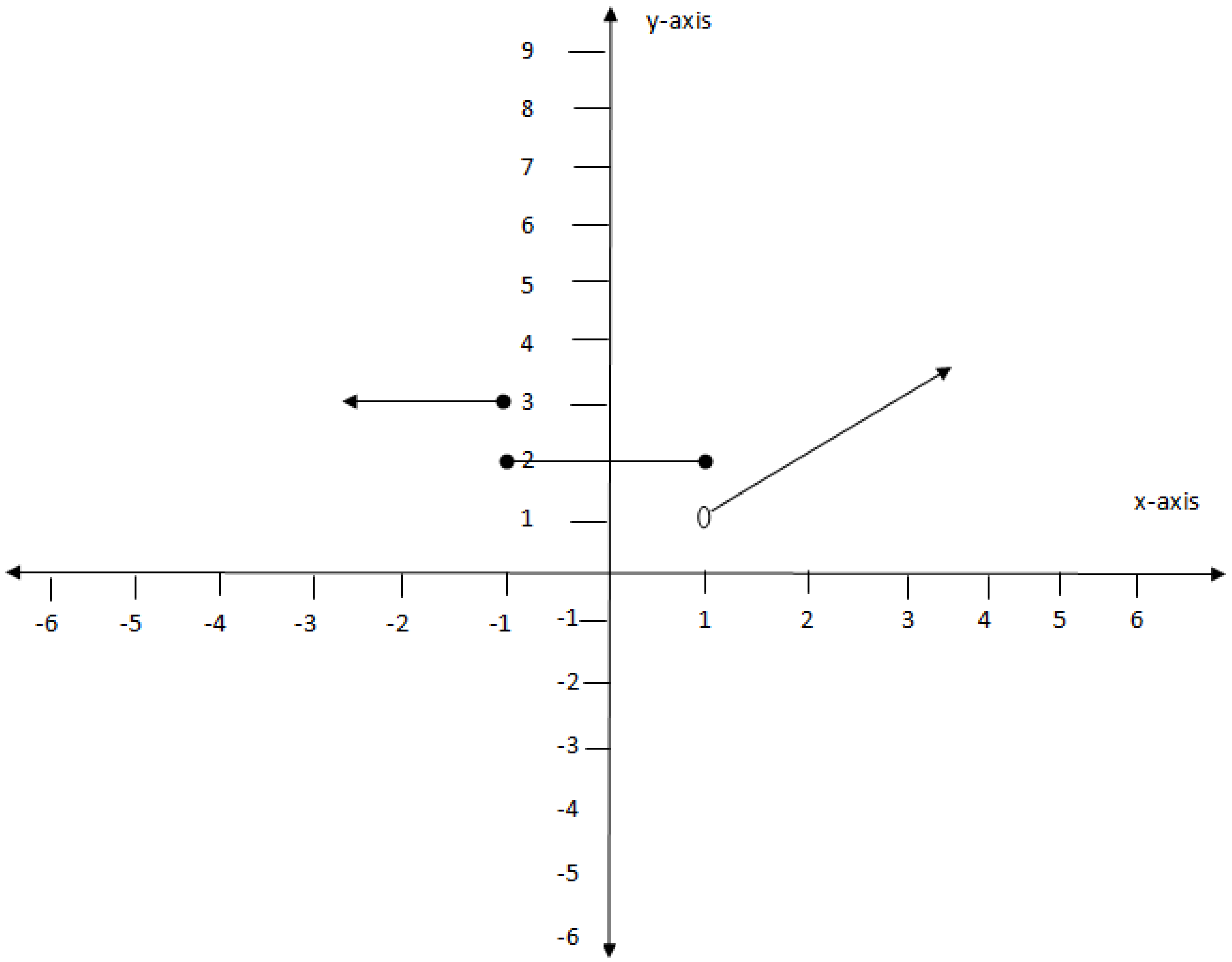
Range = {1,4, -2}

2. The function is defined by

$$f(x) = \begin{cases} 3 & \text{if } x < -1 \\ 2 \text{ if } -1 < x < 1 \\ X \text{ if } x > 1 \end{cases}$$

a)

Sketch the graph of f(x) use the graph to determine the range and the domain. Find the value of f(-6) , f(0). State if it is a one to one function



Domain

= {x : X $\in$ R}

Range

{ y:y >1}

f(-6) = -2

f(0)= 2

It is not a one to one function.

EXERCISE

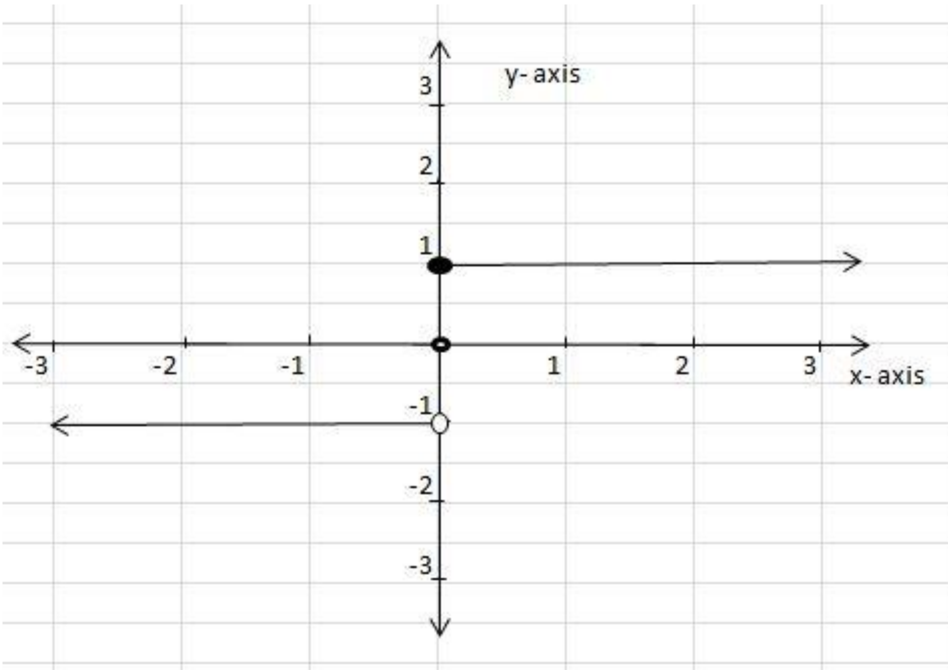
1. Draw the graph of the following defined as indicated

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

Solution:

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

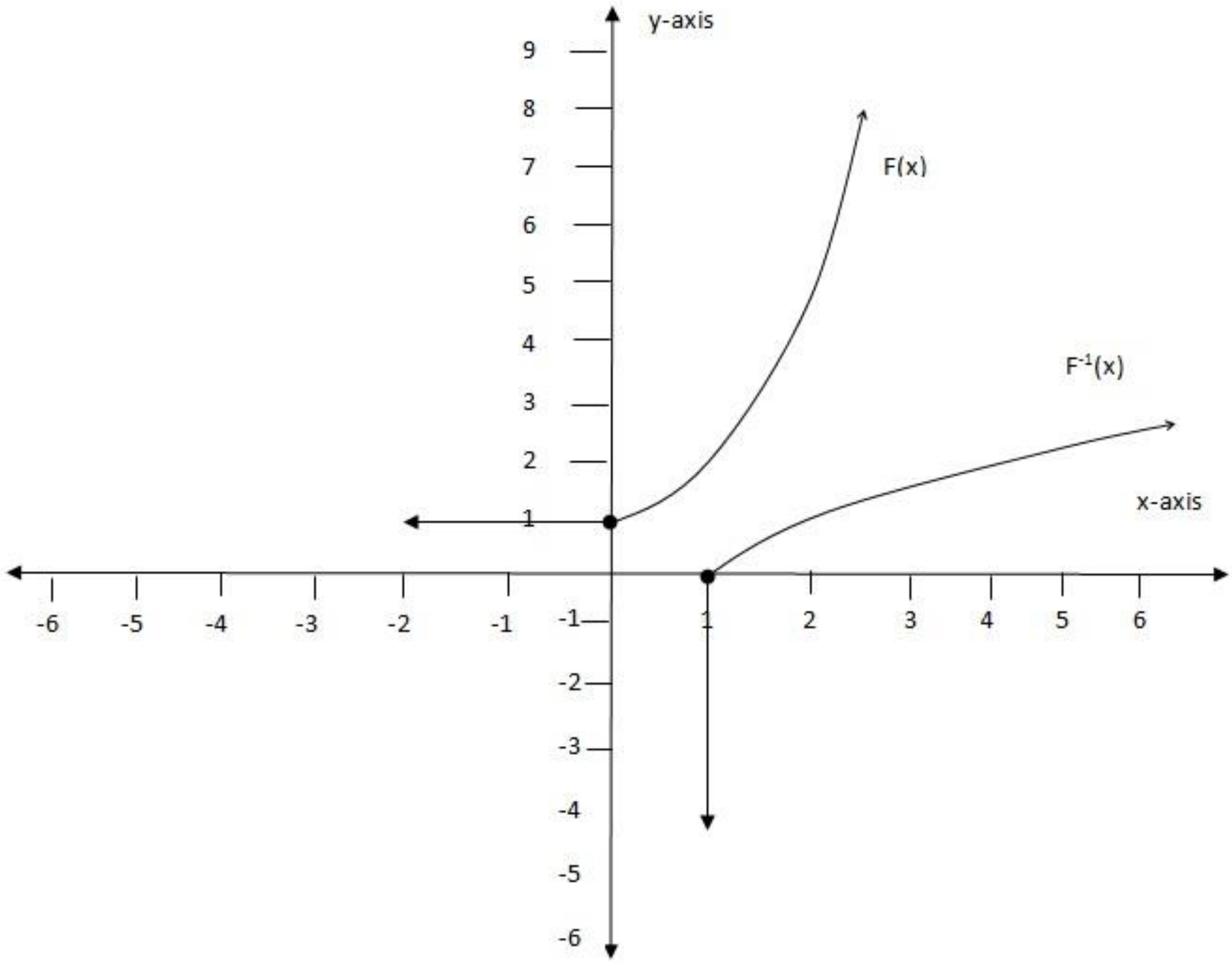
Solution



2. Given that f(x) = $\begin{cases} 1 & \text{when } x < 0 \\ x^2 + 1 & \text{when } x > 0 \end{cases}$

a) On the same set of axes sketch the graphs of f(x)and the inverse of f(x), From your graphs in [a] above determine;

- (a)The domain and range of f(x)
  (b)The domain and range if the inverse of f(x)
  (c)Find f(- 5)and f(5)
  (d)Is f (x)a one to one?
  (e)Is the inverse of f(x) a function?



(a) Domain of f(x) = { x: x $\in R$ }

(b) •Range of f(x) = { y : y ≥ 1}

•Domain of $f^{-1}(x)$ = { x : x ≥ 1}

(c) Range of $f^{-1}(x)$ = { y: x $\in R$ }

(d) f(-5) = 1 and f(5) = 6

(e)Yes the inverse of f(x) is a function and f(x) is one-to-one function

ABSOLUTE VALUE FUNCTIONS

The absolute value function is defined by f(x) $\begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$

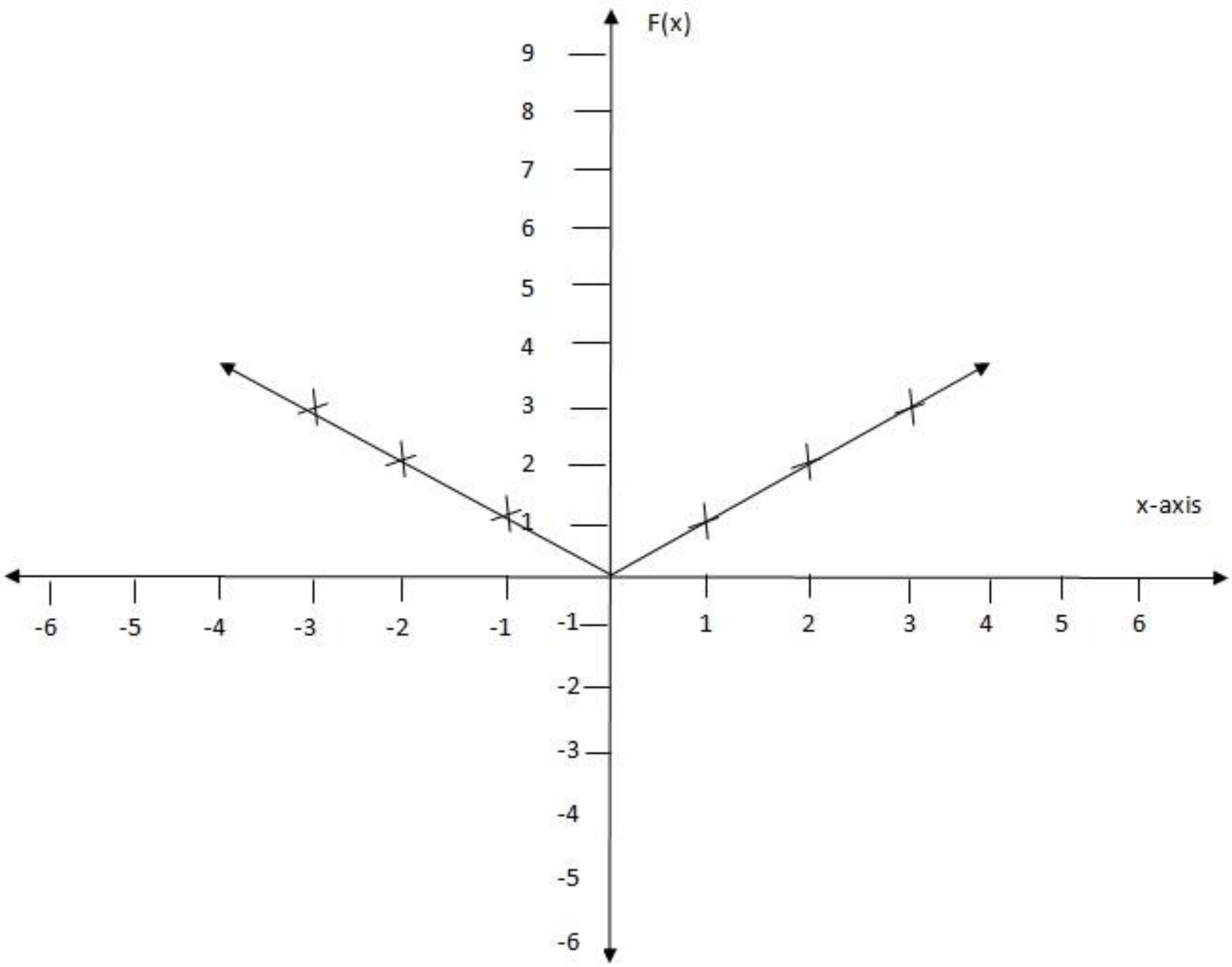
Example

1. Draw the graph of f(x) = | x |

Solution

Table of values f(x) = | x |

X	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
f(x)	**3**	**2**	**1**	**0**	**1**	**2**	**3**



THE INVERSE OF A FUNCTION

Given a functon y= f(x), the inverse of f(x) is denoted as $f^{-1}(x)$. The inverse of a function can be obtained by interchanging y with x (interchanging variables) and then make y the subject of the formula.

Example

1. Find the inverse of f(x) = 2x+3

Solution

Y= 2x+3

X=2y+3

2y=x-3

Y = $\frac{x}{2} - \frac{3}{2}$

∴ $f^{-1}(x) = \frac{x}{2} - \frac{3}{2}$

EXPONENTIAL FUNCTIONS

An exponential function is the function of the form $f(x) = n^x$ where n is called base and x is called **exponent.**

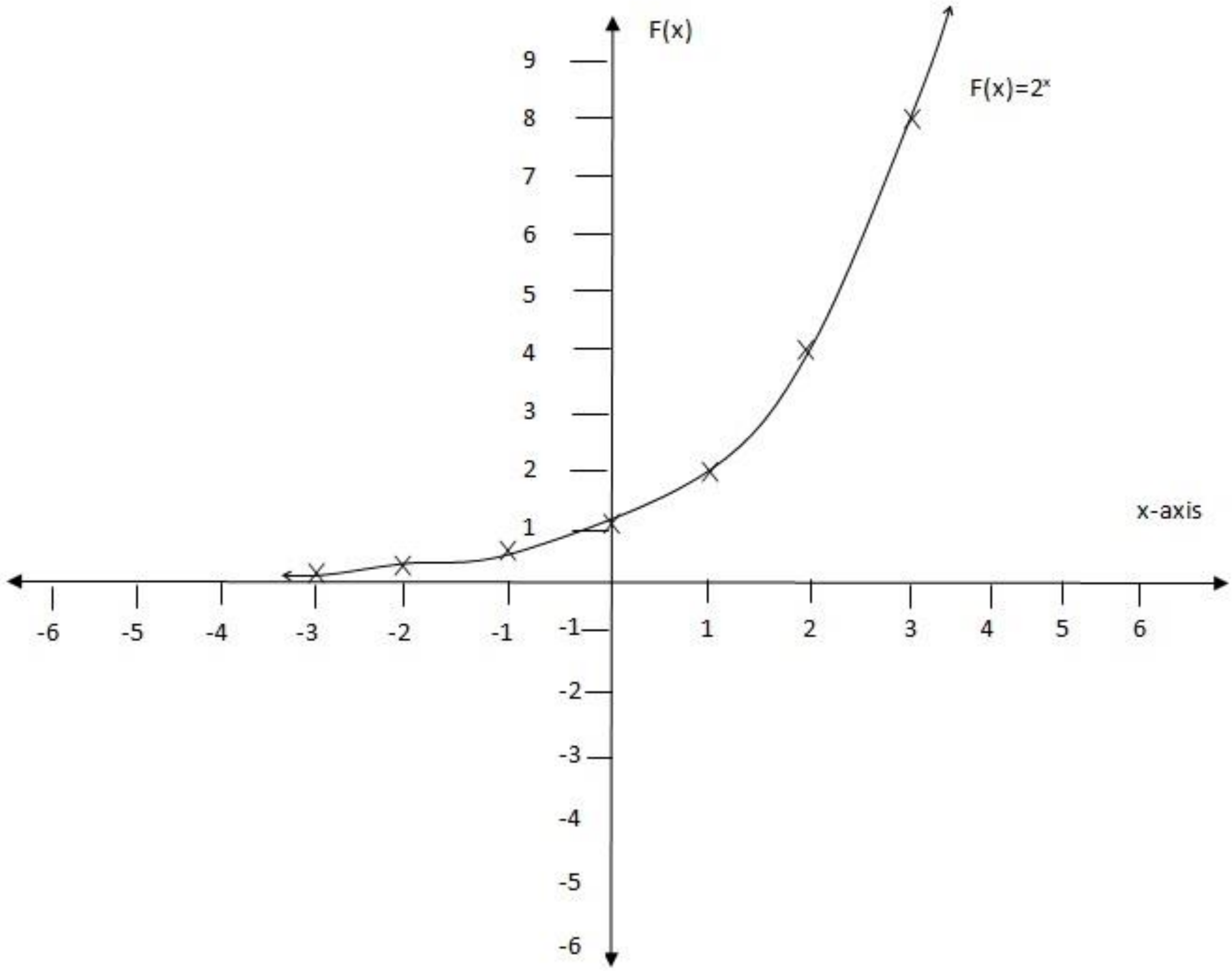
Example

1. Draw the graph of $f(x)=2^x$

Solution :

Table of values

X	-3	-2	-1	0	1	2	3
f(x)	1/8	1/4	½	1	2	4	8

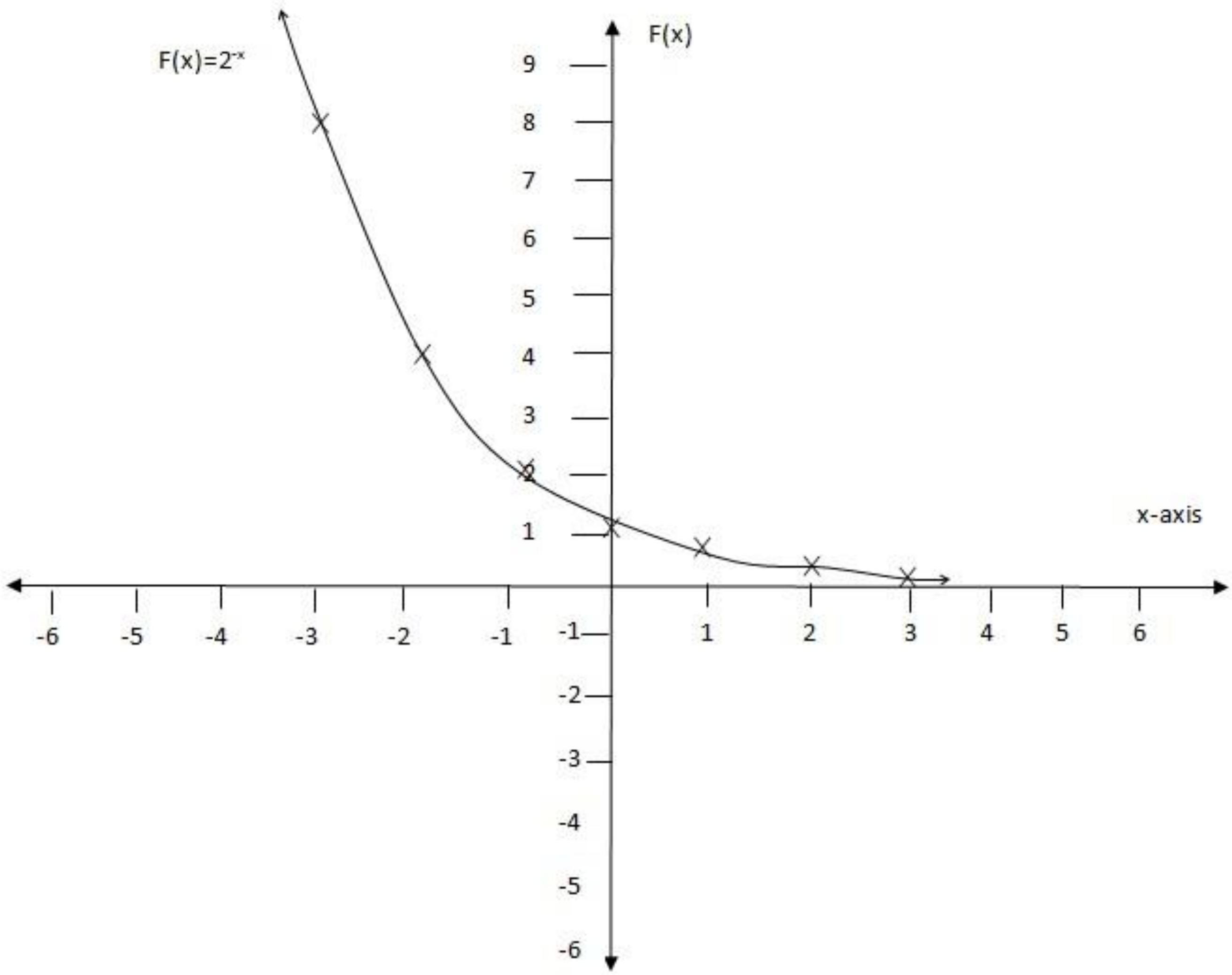


2. Draw the graph of f (x)=2^{-x}

Solution:

Table of values

X	-3	-2	-1	0	1	2	3
f(X)	**8**	**4**	**2**	**1**	**1/2**	**¼**	**1/8**



Properties of exponential function

- When x increases without bound, the function values increase without bound
- When x decreases, the function values decreases toward zero
- The graph of any exponential function passes through the point (0,1).
- The domain of the exponential function consists of all real numbers whereas the range consist of all positive values.

LOGARITHMIC FUNCTION

Logarithmic function is any function of the form f(x)= $\log_a x$ read as function of logarithm x under base a or f(x) is the logarithm x base a

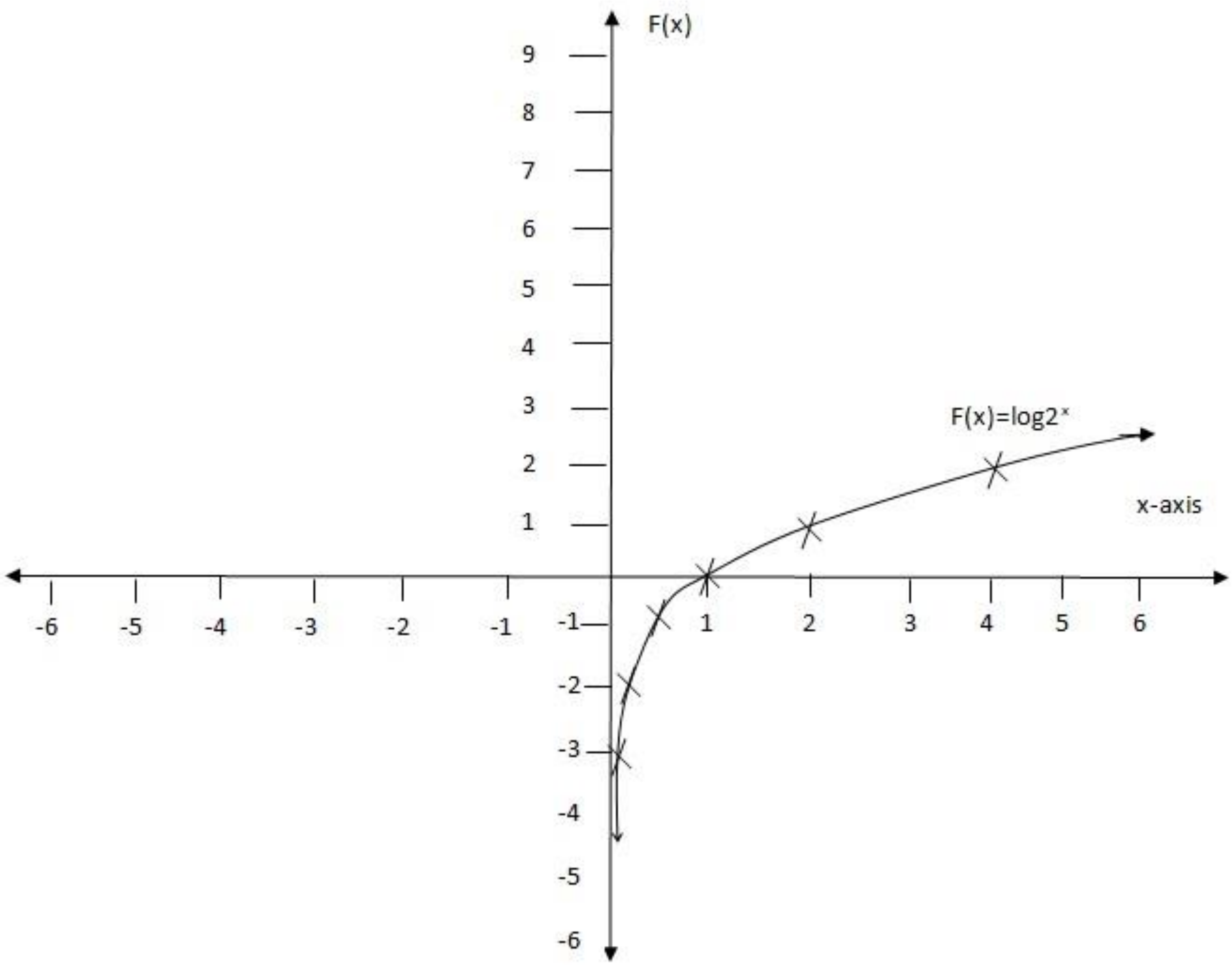
Example

Draw the graph of f(x)= $\log_2 x$

Solution

Table of values

x	1/8	¼	1/2	1	2	4	8
f(x)	-3	-2	-1	0	1	2	3



THE INVERSE OF EXPONENTIAL AND LOGARITHMIC FUNCTION

The inverse of the exponential function is the relation of logarithmic in the line y=x

EXAMPLE

1. Draw the graph of the inverse of f(x) = 2^x **and f(x)=** $\log_2 x$ **under the same graph.**

Solution

(i) y= 2^x

Apply log on both sides

log x=log 2^y

log x =y log 2

y = log (x- 2)

f-1(x) = log (x- 2)

(ii) f(x)= $\log_2 x$

y= $\log_2 x$

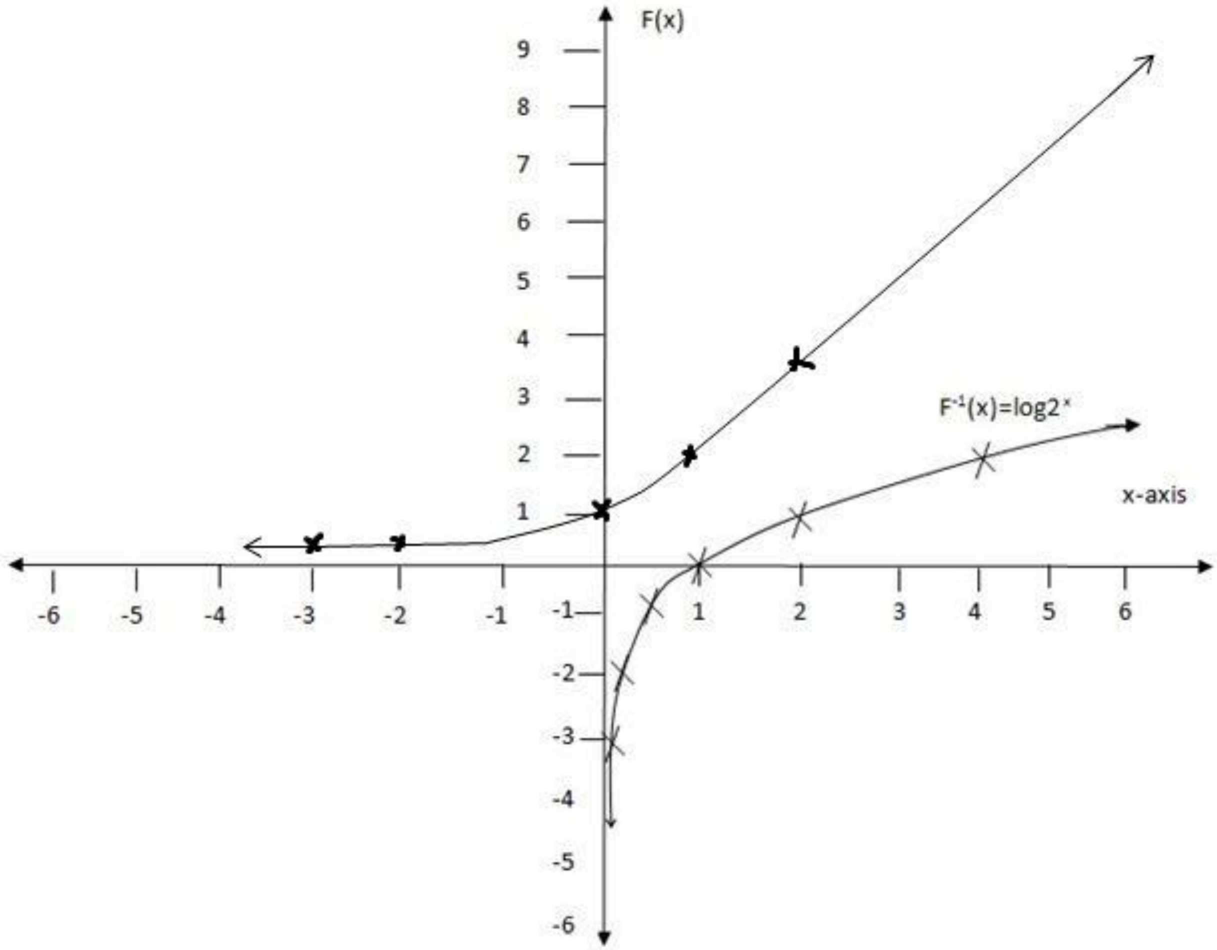
x= $\log_2 y$

v= 2^x

$f^{-1}(x)=2^x$

Table of values

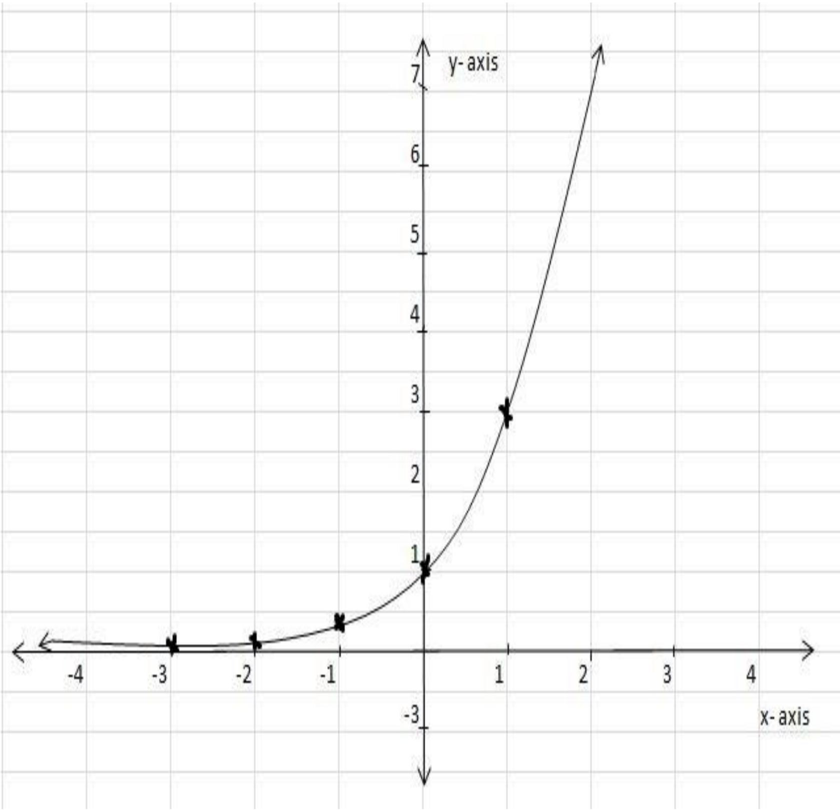
X	1/8	1/4	1/2	1	2	4	8
f(x)	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**



2. Draw the graph of the function $f(x)=3^x$

Table of values

x	-3	-2	-1	0	1	2	3
f(X)	1/27	1/9	1/3	1	3	9	27

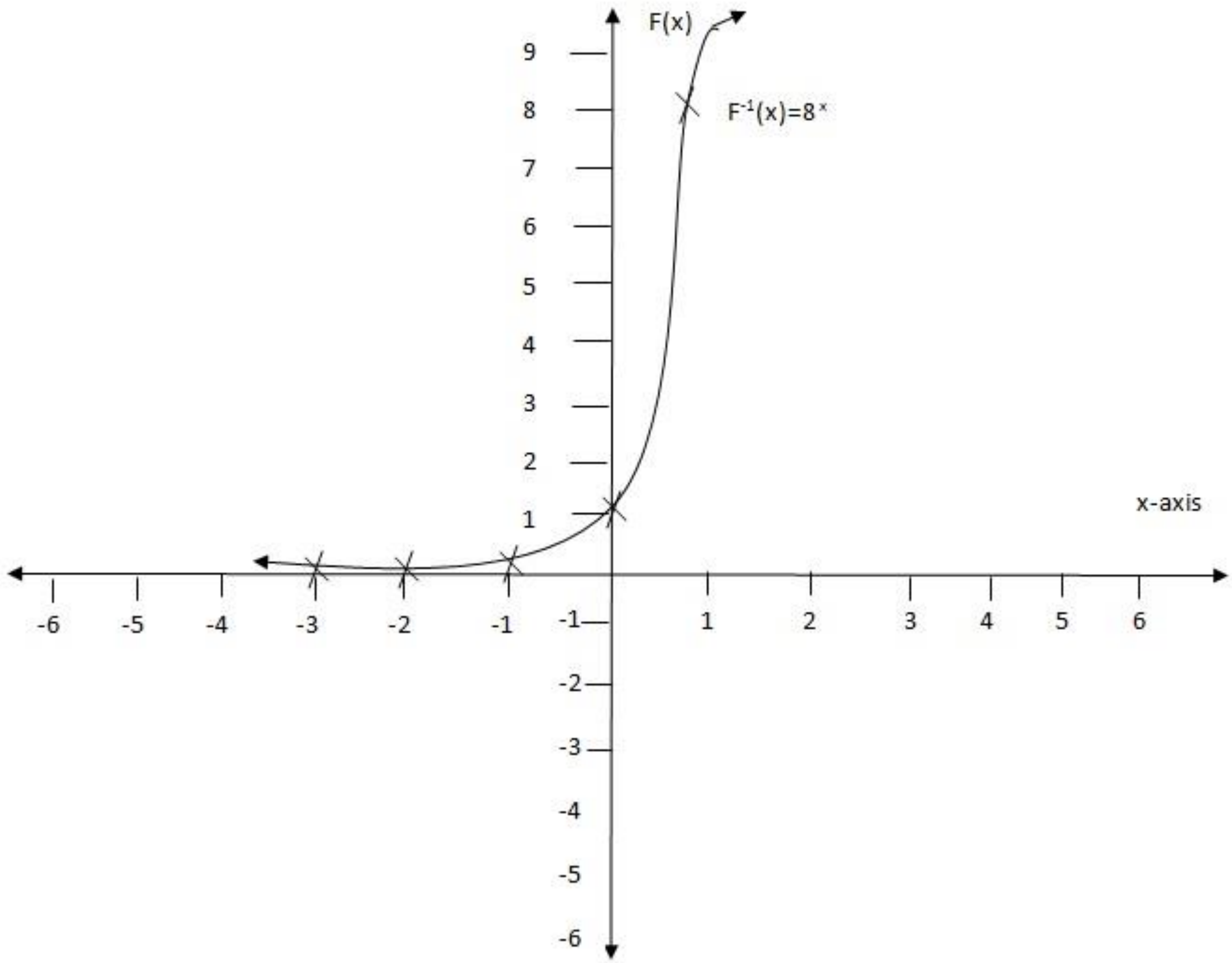


3. Draw the graph of the function $f(X)=8^x$

Solution

Table of values

X	-3	-2	-1	0	1	2	3
f(X)	1/512	1/64	1/8	1	8	64	512



Exercise

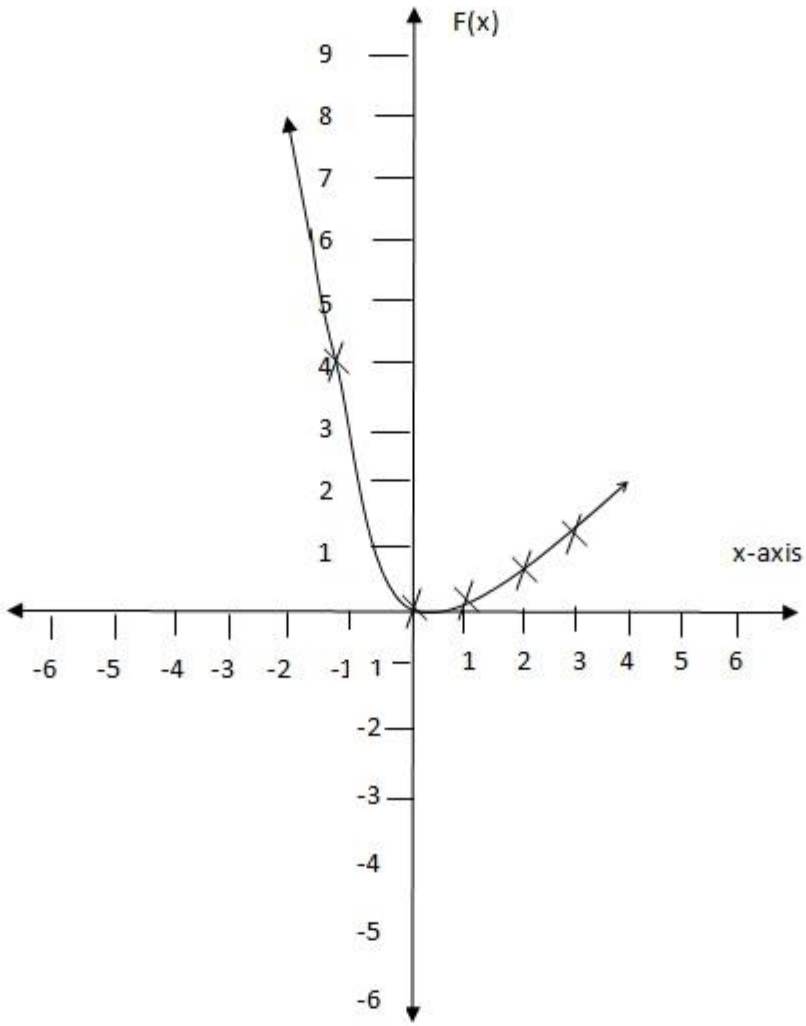
1. Find the graph of y =2^x and given that ¾ =$2^{-0.42}$ draw the graph of f(x)= $(3/4)^x$

Table of values if f(x) =$(3/4)^x$

X	-3	-2	-1	0	1	2	3
2^x	1/8	1/4	½	1	2	4	8
-0.42x	1.26	0.84	0.42	0	-0.42	-0.84	1.26
$2^{-0.42x}$	64/27	16/9	4/3	1	3/4	9/16	27/64

Copy and complete the following table and hence draw the graph of $f(x)=(1/4)^x$

X	-3	-2	-1	0	1	2	3
2x	**-6**	**-4**	**-2**	**0**	**2**	**4**	**6**
-2x	**6**	**4**	**2**	**0**	**-2**	**-4**	**-6**
2^{-2x}	**64**	**16**	**4**	**0**	**1/4**	**1/16**	**1/64**
$[1/4]^x$	**64**	**16**	**4**	**0**	**1/4**	**2/4**	**¾**



OPERATION OF POLYNOMIAL FUNCTION

ADDITIONAL AND SUBTRACTION

The sum of two polynomials is found by adding the coefficients of terms of the same degree or like terms, and subtraction can be found by subtracting the coefficient of like terms.

EXAMPLE

If p(x) = $4x^2$- 3x+7, Q(x) =3x+2 and r(x)= $5x^3-7x^2+9$

Solution

Sum =p(x)+q(x)+r(x)

$(4x^2-3x+7)+(3x+2)+(5x^3-7x^2+9)$
$4x^2-3x+7+3x+2+5x^3-7+9$

$4x^2+5x^2+7x^3+3x+7+2+9$

$5x^3-3x^2+18$

Alternatively the sum can be obtained by arranging them vertically

$4x^2-3x+7$

$0x^2+3x+2$

$\underline{5x^3-7x^2+0+9}$

$\underline{5x^3-3x^2+0+18}$

$P(x)+q(x)+r(x)=5x^3-3x^2+18$

Subtract $-5x^2+9+5$ from $3x^2+7x-2$

Solution

$(3x^2+7x-2)-(-5x^2+9x+5)$

$3x^2+7x-2+5x^2-9x-5$

$3x^2+5x^2+7x-9x-2-5$

$8x^2-2x-7$ answer

MULTIPLICATION

The polynomial R(x) and S(x) can be multiplied by forming all the product of terms from R(x) and terms from S(x) and then summing all the products by collecting the like terms.
The product can be denoted by RS(X)

example

If $p(x)=2x^2-x+3$ and $q(x)=3x^3-x$ find the product p(x)q(x)

Solution

$P(x)\ q(x)= (2x^2-x+3)\ (3x^3-x)$

$6x^5-2x^3-3x^4+x^2+9x^3-3x$

$6x^5-3x^4+7x^3+x^2-3x$

DIVISION

The method used in dividing one polynomial by another polynomial of equal or lower degree is the same to the one used for the long division of number

Example:

1. Given $p[x] =x^3-3x^2+4x+2$ and $q[x]=x-1$ find $\frac{p(x)}{q(x)}$

Solution

X^2-2x+2 remainder 4

$x-1 \overline{)\ x^3-3x^2+4x+2}$

$\underline{x^3-x^2}$

$0-2x^2+4x$

$\underline{-2x^2+2x}$

0+2x+2

2x-2

0+4

Therefore

$$\frac{x^3+3x^2+4x+2}{x+1} = x^2 - 2x + 2 + \frac{4}{x+1}$$

This means

X^3- $3x^2$+4x+2 is dividend x-1 is divisor

X^2-2x+2 is the quotient and 4 is the remainder

Then

$X^3-3x^2+4x+2 = (x^2-2x+2)(x-1) + 4$

Divided quotient x divisor + remainder

2. Divide p(x) = x^3-8 by q(x) = x-2

Solution:

X^2+2x+4

x- 2) X^3-8

$-x^3-2x^2$

$2x^2$-8

$-2x^2$-4x

4x-8

4x-8

– –

In the example [2] above there is a remainder i.e. the remainder is zero so dividing one function by another function is one of the way of finding the factors of the polynomial thus if we divide p(x) by q(x) is one of the factors of p(x) other factors can be obtained by fractionazing the quotient q(x).

3. Given p(x) = x^3-7x+6 and q(x) = x+3 determine whether or not d(x) is one of the factors of p(x) and hence find the factors if p(x)

Solution

X^2-3x+2

X+3 | X^3-7x+6

$\underline{X^3+3x^2}$

$-3x^2-7x$

$\underline{-3x^2-9x}$

2x+6

$\underline{-2x+6}$

– –

Since the remainder is zero d(x) =x+3 is one of the factors of p(x)

To factorize

X^2-3x+2

$(x^2-x) – (2x+2)$

x(x-1)-2(x-1)

(x-2) (x-1)

Therefore other factors are (x-2) and (x-1)

THE REMAINDER THEOREM

The remainder theorem is the method of finding the remainder without using long division

Example

1. If p(x) = (x-2) and q(x)+8. Dividend divisor quotient remainder, Then by taking x-2=0 we find

x-2= 0
x=2

subtracting x=2 in p(x)

p(2) =2-2 q(x) +r

p(2)= 0 x q(x)+r

p(2)= 0+r

p(2) =r

So the remainder r is the value of the polynomial p(x) when x=2

2. Give p(x) = x^3-3x^2+6x+5 is divided by d(x) = x-2 find the remainder using the remainder theorem

Solution

Let d(x)=0

x-2=0

X-2+2 =0+2

X=2

Subtracting x=2 in p(x)

P(2)= 23-3[2]2+6[2]+5

P(2)=8-12+12+5

P(2)=8+5

P(2)=13

The remainder is 13

3. The remainder theorem states that if the polynomial p[x] is divided by [x-a] then the remainder 'r' is given by p[a]

P(x) =(x-a) q(x)+r hence

P(a) =(a-a) q(x)+r

P(a)=o(q(x))+r

P(a)=0+r

P(a)=r

More Examples

1. By using the remainder theorem, Find the remainder when p[x] =4x^2-6x+5 is divided by d[x] =2x-1

Solution

d (x)=0

2x-1=0

2x-1+1=0+1

X = $\frac{1}{2}$

Substituting

X= $\frac{1}{2}$ in p(x)

P($\frac{1}{2}$) =4 x $\frac{1}{4}$ -6($\frac{1}{2}$)+5

P($\frac{1}{2}$)=1 - 6 x $\frac{1}{2}$ +5

P($\frac{1}{2}$)=1 -3+5

$P(\frac{1}{2})= 3$

The remainder is 3

2. P(x)= $3x^2$-5x+5 is divided by d(x) =x+4

Solution

Let d(x)=0

X+4=0

X+4=0-4

X=-4

Subtracting x=-4 in p(x)

P(-4)=3$(-4)^2$-=5(-4)+5

P(-4)=48+20+5

P(-4)=68+5

P(-4)= 73

The remainder is 73

3. P(x)= x^3+2x^2-x+4 is divided by d(x) = x+3

Solution

Let d(x)=0

X+3=0-3

X=-3

Substituting x=-3 in p(x)

P(-3)= -3^3+2(-3)2-(-3)+4

P(-3)=-27+18+3+4

P(-3)=-27+21+4

P(-3)=-27+25

P(-3)=-2

The remainder is -2

4. Find the value of 'a' if x^3-3x^2+ax+5 has the remainder of 17 when divided by x-3

Solution

By using remainder theorem

Let x-3 =0

x-3=0+3

x=3

Substituting x=3 we have

$x^3-3x^2+ax+5=17$

$(3)^3-3(3)^2+a(3)+5=17$

27-27+3a+5=17

3a+5=17-5

3a=12

a=4

5. If ax^2+3x-5 has a remainder -3 when divided by x-2. Find the value of a.

Solution

By using the remainder theorem

Let x-2=0

x-2=0+2

x=2

subtracting x=2 we have

a(2)2+3(2)-5 =-3

4a+6-5= -3

4a+1= -3

4a= -4

a = -1

Exercise

1. Divide p(x) by d(x) in the following

$P(x)=2x^2+3x+7$ $d(x)=x^2+4$

Solution

| 2

X^2-x+4) $2x^2$+3x+7

- $2x^2$-2x + 8

5x – 1

Then you get 2 and the remainder is 5x - 1

P[x]=x^5-1 d[x]=x-1

Solution

$x^4+x^3+x^2+x$

x - 1) x^5-1

x^5-x^4

X^4-1

X^4-x^3

X^3 - 1

X^3 - x^2

X^2-1

X^2-1

P[x]=x^3-4x+2, d[x]= x-2

Solution

X^2+2x remainder 2

$X - 2 \overline{) x^3 - 4x+2}$

$-x^3 - 2x^2$

$2x^2-4x$

$-2X^2-4X$

2

$P[x]= 28x^4- 14x^3$ and $d[x] = -21x^2$

Solution

$-\ ^4/_3 x^2 - ^2/_3$

$-21x^2 \overline{) 28x^4-14x^3}$

$28x^4$

$-14x^3$

$-14x^3$

Use remainder theorem to find the remainder when;

1. $P(x)=x^3-2x^2+5x-4$ is divide by $d(x)=x-2$

Solution

Let $d[x]=0$

$x-2 + 2=0+2$

$x=2$

substituting $x=2$ in $p(x)$

$p(2)=2^3-2[2]^2+5[2]-4$

p(2)=8-8+10-4

p(2)=10-4

p(2)=10-4

p(2)=10-4

Therefore the remainder is 6

2. $p(x)= 2x^4+ x^3+x-\ ^3/_4$ is divided by d(x)=x+2

solution

let d(x)=0

x+2 – 2 = 0-2

x = -2

Substituting x = - 2 in p(x)

$p(-2)=2(-2)^4+(-2)^3+(-2)(-\ ^3/_4)$

$p(-2)= 2(16)+(-8)-2(-\ ^3/_4)$

$p(-2)=32-8-2\ (-\ ^3/_4)$

$p(-2)=24-2(-\ ^3/_4)$

$p(-2)=21\ ^1/_4$

Therefore the remainder is $21\ ^1/_4$

STATISTICS

Is the study or the methods of collecting, summarizing and presenting data and interpreting the information.

MEASURES OF CENTRAL TENDENCY

1. Mean
2. Median
3. Mode

MEAN "X"

Is obtained by adding up all the data values then divide by the number of characters.

I.e.

$$\bar{x} = \frac{\text{sum of observations}}{\text{Number of Observation}}$$

$$\bar{x} = \frac{x_1 + x_2 + x_3}{N}$$

i.e. $\bar{X}$ =mean

$x_1+x_2+x_3$………. Sum of observations

N =number of observation

Example

1. Find mean score from the following scores of biology test 10, 25, 45, 15,63 42,7

$$\bar{X} = \frac{\text{sum of observations}}{\text{Number of Observations}}$$

$$\bar{X} = \frac{10+25+45+15+63+42+7}{7}$$

$$\bar{X} = 29.57$$

When the data is given with frequency or in grouped data;

$$\bar{X} = \frac{\sum fx}{N} \text{ or } \frac{\sum fx}{\sum x}$$

f= frequency

∑= summation

2. Find the mean number of children per family from the following table

No. of children [x]	0	1	2	3	4	5	6	7	8
No. of families [f]	3	6	7	8	10	12	8	4	2

Solution

Finding the mean of the numbers in the table below;

No. of children [x]	No. of families [f]	fx
0	3	0
1	6	6
2	7	14
3	8	24
4	10	40
5	12	60

6	8	48
7	4	28
8	2	16
Total	**60**	**236**

$$\bar{X} = \frac{\sum fx}{N} = \frac{236}{60}$$

$$\bar{X} = 3.93$$

Exercise

1. Football club has the following number of goals scored against them 0,1,0, 2, 9,0 , 1, 2,1. What is the mean number of goals scored against them?

Solution

$$\bar{X} = \frac{\text{sum of observations}}{\text{Number of Observations}}$$

$$\bar{X} = \frac{0+1+0+2+9+0+1+2+1}{9}$$

$$\bar{X} = \frac{16}{9}$$

$\overline{X}$ = 1.77

2. In a class of 30 girls the mean mass was 50kg calculate the total mass of the class.

Solution

$\overline{X}$ =50

N =30

$\overline{X} = \frac{\sum fx}{N}$

$\overline{X} = \frac{\sum fx}{30}$

$30 \times 50 = \frac{\sum fx}{30} \times 30$

∑fx = 1500kg

MEAN OF THE GROUPED DATA

1. The table below shows a distribution of 100 students find the mean mark.

Class interval	Class mark [x]	Frequency [f]	fx
91-95	93	0	0
86-90	88	1	88
81-85	83	6	498
76-80	78	10	780
71-75	73	15	1095
66-70	68	34	2312
61-65	63	22	1386

56-60	58	10	580
51-55	53	2	106
		N=100	∑fx=6845

Mean $\overline{X} = \frac{\sum fx}{N}$

$\overline{X} = \frac{6845}{100}$

The mean is 68.45

<u>MEAN BY ASSUMED MEAN METHOD</u>

$\overline{X} = A + \frac{\sum fd}{N}$

Where,

A = assumed mean

D = difference between the class marks and the assumed mean d= x-A

F= frequency

N= total frequency

From the above example use the data to find the mean by assumed mean method.Take the assumed mean as 58.

Class interval	**Class mark[x]**	**F**	**D=x-A**	**fd**
91-95	93	0	35	0
86-90	88	1	30	30
81-85	83	6	25	150

76-80	78	10	20	200
71-75	73	15	15	225
66-70	68	34	10	340
61-65	63	22	5	110
56-60	58	10	0	0
51-55	53	2	-5	-10
Total		100		1045

Total

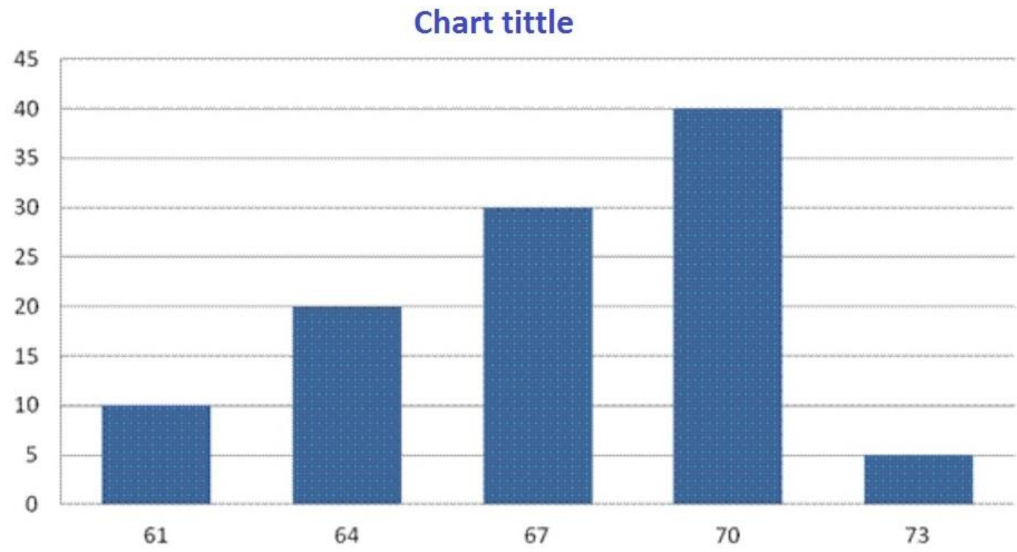
A=58

$\overline{X} = A + \frac{\sum fx}{N}$

$\overline{X} = 58 + \frac{1045}{100}$

$\overline{X} = 68.45$

The mass of students were recorded as shown below in the following figure.



Class mark[x]	f	fx
61	10	610
64	20	1280
67	30	2010
70	15	1050
73	5	365
Total	80	5315

$$\overline{X} = \frac{\sum fx}{N}$$

$$\overline{X} = \frac{5315}{80}$$

$\overline{X}$ = 66.4365

$\overline{X}$ = 66.44

Exercise

a)1. Show the distribution of the children's age in a month. Calculate the mean age in months using assumed mean that is the formula;

$$\overline{X} = A + \frac{\Sigma fx}{N}$$

Calculate the mean age in months using the formula;

$$\overline{X} = A + \frac{\Sigma fx}{N}$$

Class mark	frequency
41-46	3
35-40	4
29-34	9
23-28	12
17-22	18
11-16	28
5-10	26

Solution:

To calculate the mean age in months using an assumed mean that is the formula

$$\overline{X} = A + \frac{\Sigma fx}{N}$$

Class mark[x]	Frequency [f]	D= x-A	fd

41-46	3	30	90
35-40	4	24	96
29-34	9	18	162
23-28	12	12	144
17-22	18	6	108
11-16	28	0	0
5-10	26	-6	-156
Total	100		444

Total

Let A= 11-16

Let A = 13.5

$\overline{X} = A + \frac{\sum fx}{N}$

$\overline{X} = 13.5 + \frac{444}{100}$

$\overline{X} = 13.5 + 4.44$

$\overline{X} = 17.94$

2. Calculate the mean age in months using the formula for mean calculation.

Solution

Class interval	Frequency [f]	fx	Class mark (x)
41-46	3	130.5	43.5
35-40	4	150	37.5
29-34	9	283.5	31.5
23-28	12	306	25.5
17-22	18	351	19.5
11-16	28	378	13.5.
5-10	26	195	7.5
Total	100	1794.0	

$$\overline{X} = \frac{\sum fx}{N}$$

$$\overline{X} = \frac{1794}{100}$$

$$\overline{X} = 17.94$$

3. A survey was of 200 children under 10 years to see how many visits they made to the clinic during the courses of the year. The results were recorded as shown in the table below.

Number of visits	frequency
5	16
6	33
1	47
8	54
9	31
10	10
11	4

12	2
13	0
14	2
15	1

Solution

$\overline{X} = \frac{\sum fx}{N}$

Number of visits[x]	Frequency [f]	Fx
5	16	80
6	33	198
7	47	329
8	54	432
9	31	279
10	10	100
11	4	44
12	2	24
13	0	0
14	2	28
15	1	15
Total	200	1529

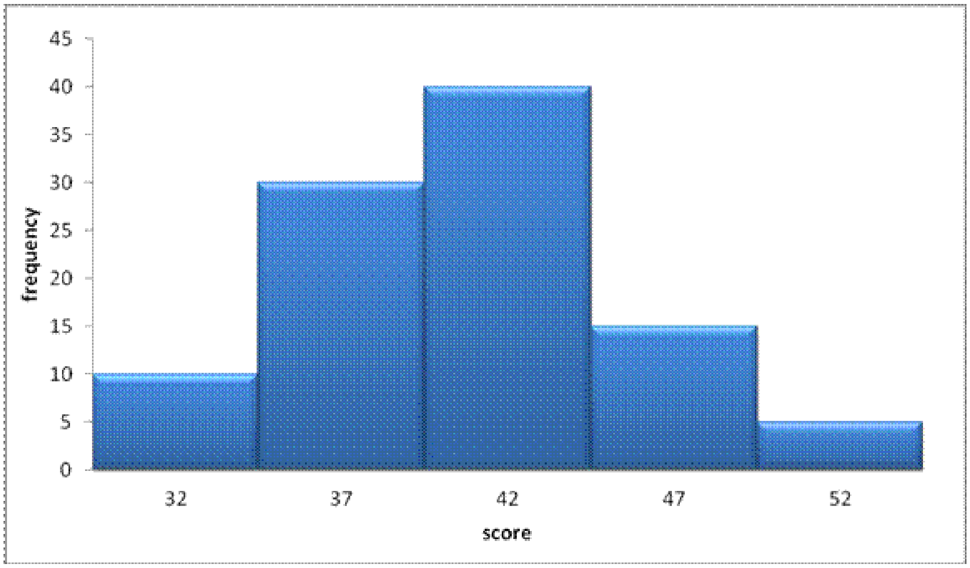
$\overline{X} = \frac{\sum fx}{N}$

$\bar{X} = \frac{1529}{200}$

$\bar{X} = 7.645$

Mean number of visits per child = 7.645

4. A histogram for 100 mathematics scores use the histogram to find the mean score



Solution

Score(x)	f	D= x-A	fd
32	10	-5	-50
37	30	0	0
42	40	5	200
47	15	10	150
52	5	15	75
Total	100		375

from

$\overline{X} = A + \frac{\sum fd}{N}$

Let A = 37

$\overline{X}$ =37+375/100

$\overline{X}$ = 40.75

MEDIAN

Median is a point that divides the data into two parts such that equal numbers of the data fall above and below that point.

Computation of the median depends on whether the data is ODD or EVEN or there is duplication of data [i.e. data with frequency]

MEDIAN OF ODD NUMBERS OF DATA

STEP 1

Arrange the numbers in ascending/descending order

1,1, 2,2,5, 5

STEP 2

Pick the number which is between those numbers. If it is even find the average of the two middle numbers
e.g:-
2+2= 4/2=2

Median of numbers 2,3,9,11, 2, 2,2, 2, 3, 9, 11

STEP 1

Arrange the numbers in ascending order
2,2,2,2,2,3,3,9,9,11,11

STEP
Pick the number which is between those numbers

Median =3

Example1.

1.Find the median of the following observations
1, 7, 4, 3, 8

Solution

1, 3, 4, 7, 8

Median =4

Exercise

1) 1, 2, 5, 3. Find the median of the given data

solution
Step 1

Arrange the data in ascending order

1, 2, 3, 5

Step 2

1, 2, 3, 5

$= \frac{2+3}{2}$

$= \frac{5}{2}$

Median is 2.5

2) 1, 1, 3, 2, find the median given even numbers of data

Step1

Arrange the data in ascending order

1, 1, 2, 3

Step 2

1, 1, 2, 3

$= \frac{2+1}{2}$

=1.5

Median is 1.5

3) 5, 3, 1, 6, 8 find the median given odd number of data

Step 1

Arrange the data in ascending order

1, 3, 5, 6, 8,

Step 2

1, 3, 5, 6, 8

Median is 5

4) Obtain the media of the following
1, 1, 6, 9, 8, 5

Step 1

Arrange the data in ascending order

1, 1, 5, 6, 8, 9

Step 2

1, 1, 5, 6, 8, 9

5+6

Median is 5.5

5) Obtain the media of the following 2, 3, 9, 7, 1.

Step 1

Arrange the data in ascending order

1, 2, 3, 7, 9

Step 2

1, 2, 3, 7, 9

Median is 3

MEDIAN FOR GROUPED DATA

$$\text{Median} = l + \frac{\left(\frac{N}{2} + n_b\right)}{n_w} i$$

Where by

L = the lower boundary of the median class

N = the total number of frequency

n_b = the number of items in classes below the median class.

n_w = the number of items within the median class

i = the class interval

Example

1. The following table shows the distribution of nails in [mm]. Calculate the median length.

Length [mm]	f	Cumulative frequency
88-96	3	3

97-105	5	8
106-114	9	17
115-123	12	29
124-132	5	34
133-141	4	38
142-150	2	40
Total	40	169

L=?

Median class=115-123

Median position= $\frac{N+1}{2} = \frac{40+1}{2} = \frac{41}{2}$ =20.5

L= lower limit - 0.5

From 115-123

L=114.5

N = 40

n_b=17

n_w =12

i= 9

$$Median = \frac{L + (\frac{N}{2} - n_b) \times i}{N_w}$$

$$Median = \frac{114.5 + (\frac{40}{2} - 17) \times 9}{12}$$

$$Median = 114.5 + \frac{9}{4}$$

$$Median = 116.75$$

Exercise

1. The following is the distribution of marks obtained in a test given to 50 candidates

Marks	frequency	Cumulative frequency
11-20	1	1
21-30	3	4
31-40	10	14
41-50	21	35
51-60	6	41
61-70	5	46
71-80	4	50
	50	

Find the median mark?

Solution

L=?

Median class= ?

Median position= $\frac{N+1}{2}$

Median position = $\frac{50+1}{2}$ = $\frac{51}{2}$ = 25.5

Median position =25.5

Median class= 41-50

L = lower limit- 0.5

From 41-50

L= 41-0.5

L= 40.5

N=50

n_b=14

n_w=21

i = 10

From the formula:

$$Median = L + \frac{\left(\frac{N}{2} - n_b\right)}{n_w} \times i$$

$$Median = 40.5 + \frac{\left(\frac{50}{2} - 14\right)}{21} \times 10$$

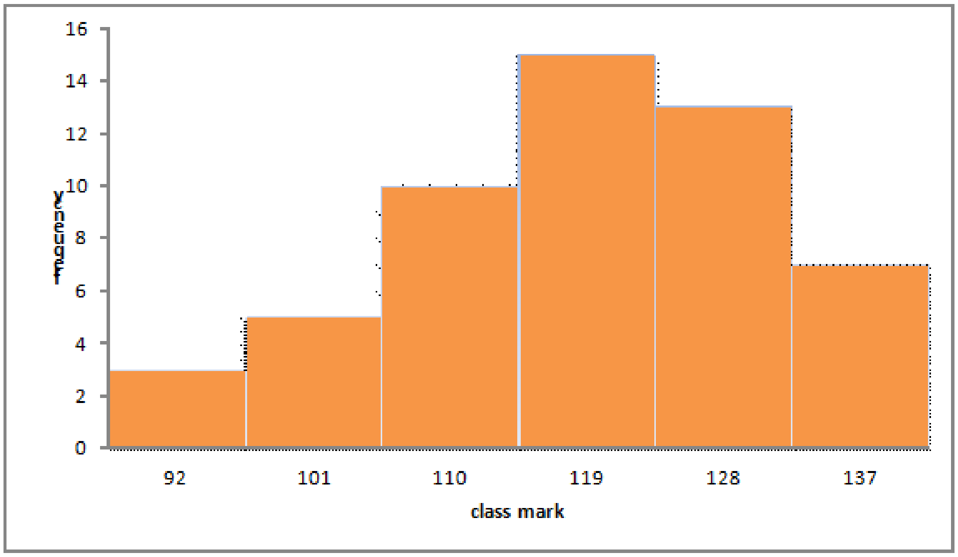
$$Median = 40.5 + \frac{110}{21}$$

$$Median = 45.74$$

The following figure represents the graph of frequency polygon of a certain data . To find the median distribution

Solution

a)



Class mark	frequency	Cumulative frequency
92	3	3
101	5	18
119	10	33
128	13	46
137	7	53
Total	53	

L =?

M.p = $\frac{N+1}{2} = \frac{53+1}{2}$ =27

L =114.5

N= 53

n_b=18

n_w=15

i= 9

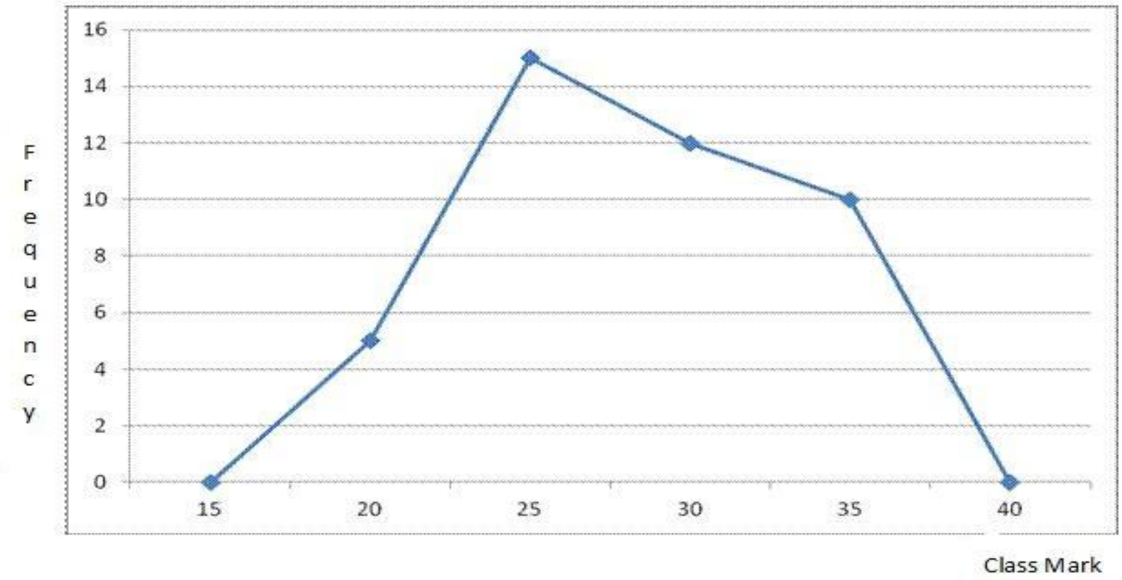
$\tilde{x}$ =114.5+ ($\frac{\frac{53}{2}-18}{15}$)x 9

$\tilde{x}$ =114.5+[8.5/15]9

$\tilde{x}$ = 114.5+5.1

$\tilde{x}$ = 119.6

b)



Solution

$$Median = L + \frac{\left(\frac{N}{2} - n_b\right)}{n_w} \times i$$

Class mark	frequency	Cumulative frequency
15	0	0
20	5	5
25	15	20
30	12	32
35	10	42
40	0	42

L =?

M.P $= \frac{N+1}{2} = \frac{42+1}{2} = 21.5$

L = 25+30 = 55/2= 27.5
L =27.5

N=42

n_b= 20

n_w=12

i =5

$$Median = 27.5 + \frac{\left(\frac{42}{2} - 20\right)}{12} \times 5$$

$$Median = 27.92$$

Exercise

1. The height in centimeters of 100 people was recorded as shown below.

Height [cm]	160	165	170	175	180	185
frequency	2	12	32	24	21	8

Find the median height?

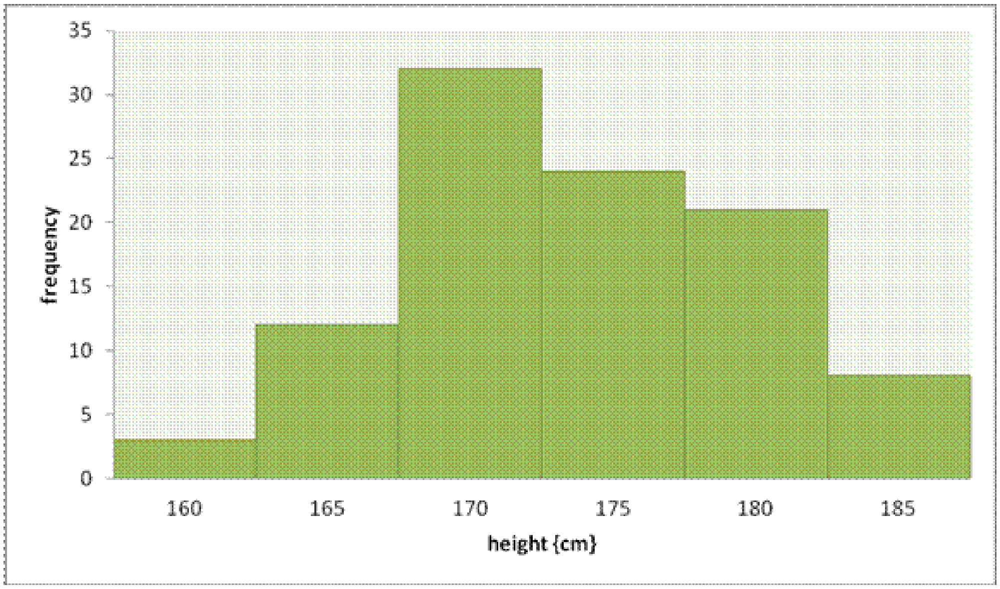
Solution

Height in [cm]	**frequency**	**Cumulative frequency**
160	3	3
165	12	15
170	32	47
175	24	71
180	21	92
185	8	100
total	100	

L =?

Median position = $\frac{N+1}{2} = \frac{100+1}{2}$

= 50.5



L= ?

L= 170 + 175 = 345/2= 172.5

L= 172.5

N =100

n_b=47

n_w=24

i= 5

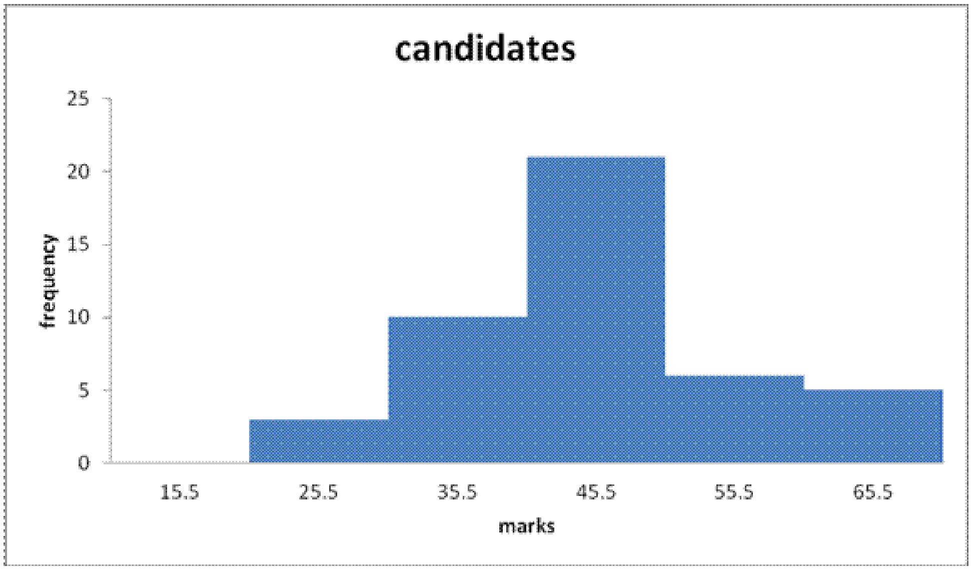
$$Median = L + \frac{\left(\frac{n}{2} - n_b\right)}{n_w} \times i$$

$$Median = 172 + \frac{\left(\frac{100}{2} - 47\right)}{24} \times 5$$

$$Median = 172.5 + \frac{50-47}{24} \times 5$$

$$Median = 173.125$$

Figure 5.13 is a histogram representing test marks of 50 candidates find the median mark.



Solution

L =?

Median Point = $\frac{N+1}{2} = \frac{50+1}{2}$

$= \frac{51}{2} = 25.5$

Class mark	frequency	Cumulative frequency
15.5	1	1
25.5	3	4
35.5	10	14
45.5	21	35
55.5	6	41
65.5	5	46

75.5	4	50
Total	50	

L = ?

L = 40.5

N =50

n_b =14

n_w=21

i=10

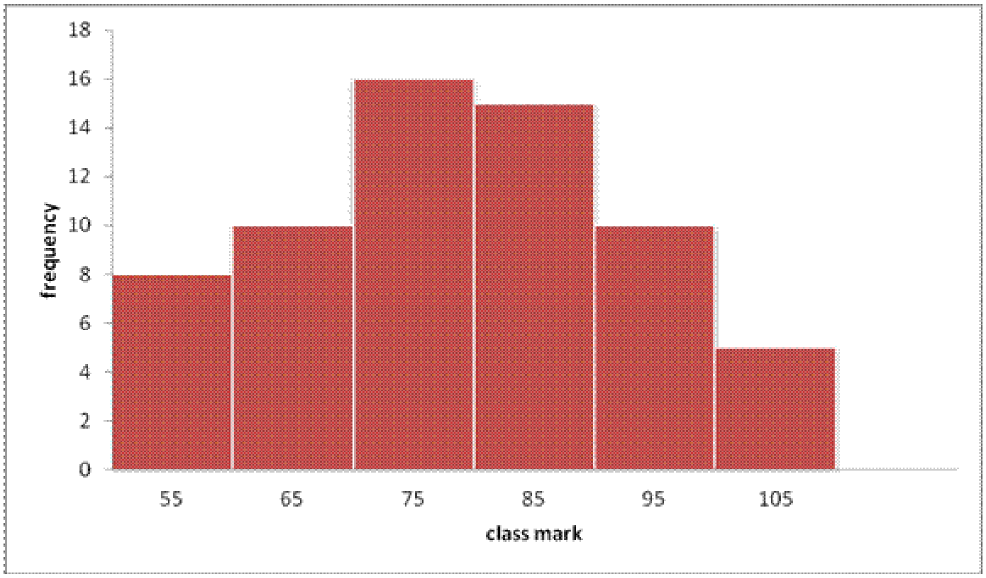
$$Median = 40.5 + \left(\frac{\frac{50}{2} - 14}{21} \right) \times 10$$

$$Median = 40.5+5.24$$

$$Median = 45.74$$

Figure 5.14 shows the frequency histogram for daily wages in TSHS of 70 people find the wages

Solution



Wages in [TSHS}	frequency	Cumulative frequency
55	8	8
65	10	18
75	16	34
85	15	49
95	10	59
105	5	64
Total	64	

Median position

$$\frac{N+1}{2} = \frac{64+1}{2} = \frac{65}{2}$$

= 32.5

L= 75+85 = 160/2
L= 80

N= 64

n_b = 34

n_w= 15

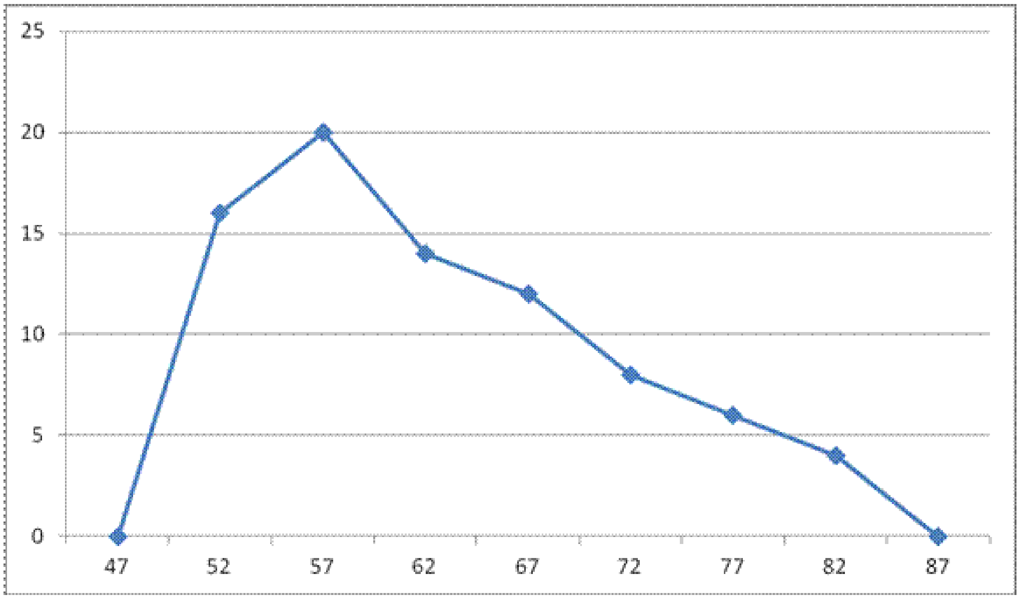
i=10

$$Median = L + \left(\frac{\frac{N}{2} - n_b}{n_w} \right) \times i$$

$$Median = 80 + \left(\frac{\frac{64}{2} - 34}{15} \right) \times 10$$

$$Median = 78.67$$

Figure 5.15 is a frequency Polygon for masses in kilogram's of 80 students find the median mass.



Solution

Mass in kg	frequency	Cumulative frequency
47	0	
47	0	0
52	16	16
51	20	36
62	14	50
61	12	62
72	8	70
77	6	76
82	4	80
87	0	80
Total	80	

L =?

Median position = $\frac{N+1}{2} = \frac{80+1}{2}$

$= \frac{81}{2} = 40.5$

L = 62+ 61 =123/2
L = 61.5

N =80

n_b=36

n_w=14

i = 5

$$\text{Median} = 61.5 + \left(\frac{\frac{80}{2} - 36}{14}\right) \times 5$$ \

Median = 62.93.

MODE

Mode is the value of data which occurs most frequently [data with the highest frequency].

Data may have only one mode, more than one mode or no mode at all.

Example

Find the mode from the following data

i) 3, 5, 7, 3, 2, 10, 8, 2, 7, 2

Mode is =2

ii) 2, 1, 2, 5, 3, 1, 1, 4, 2, 7.

Mode is=1 and 2

MODE FOR GROUPED DATA

$$\text{Mode} = L + \left(\frac{t_1}{t_1 + t_2}\right)i$$

Where by

L= Lower limit of the modal class

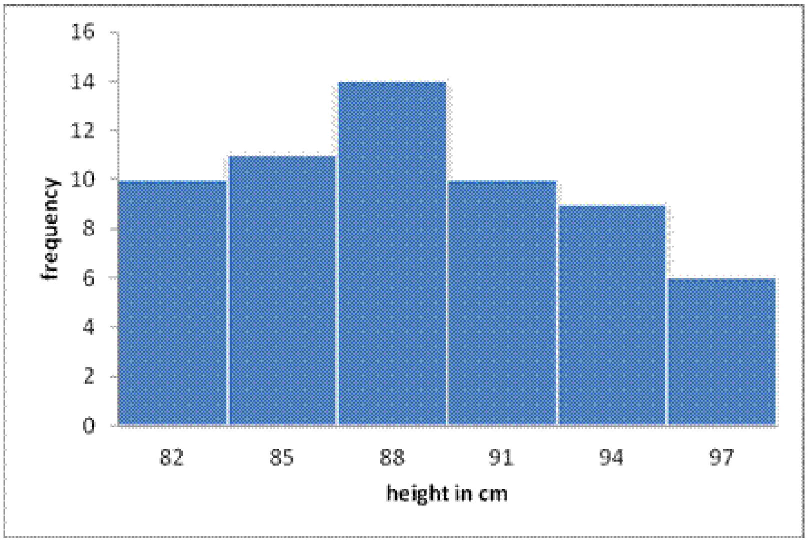
t_1 =The excess of modal frequency over the frequency of the next lower class.

t_2=The excess of modal frequency over the frequency of the next higher class

i = The Modal class interval

Figure 5.19 shows a histogram for heights of little children in centimeters calculate the mode of these heights

Solution



Height in cm	frequency
82	10
85	11
88	14

91	10
94	9
97	6
Total	60

Modal class =88

L = $\frac{88+85}{2}$ = 86.5

L= 86.5

t_1= 14-11 = 3

t_2= 14-10 = 4

i= 3

M = L + $\left(\frac{t_1}{t_1 + t_2}\right)$ x i

M = 86.5 + $\left(\frac{3}{3+4}\right)$ x 3

M = 87.79

The mode is 87.79

RATES AND VARIATIONS

RATES:-

When sets or quantities of different kinds are related, we use the word rate.

i.e 1. A rate of pay of 10,000/= Tsh per hour (money- time)

2. The price of juice is 700/= Tsh per litre (money -weight of juice)

3. The average speed of 80 kilometres per hour (distance- time)

Therefore the rate is the constant relation between two sizes of two quantities concerned.

NOTE:

Rates deals with the comparison of two quantities of different kinds.

Example

1. Hiring a car at a charged rate of Tsh 2,000/= per kilometer.

(a) A journey of 40 kilometers will cost 40 x Tsh 2,000= Tsh 80,000/=

(b) A journey of 100 kilometres, costs 100 x Tsh. 2,000= Tsh.200,000/=

If we state the rate we always give two quantities concerned and the unit measurement.

E.g: Average speed is written as 100 kilometres per 2 hours or 50 kilometres per one hour.

$$\therefore \text{Rate} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Rates can also written in a ratios form.

I.e. 100Km per 2 hours = $\frac{100\text{Km}}{2\text{ hours}}$ or $\frac{50\text{Km}}{1\text{hr}}$ = 50Km/hr.

Rate of Exchange

People in any country expect to pay and be paid in currency of their own country. It is necessary to exchange the currency of the first country for that of the second, when money is moved from one country to another.

i.e: The rate of exchange linked together various currencies of the world, which enable transfer of money and payment for goods to take place between countries.

Consider table below shows the exchange rates as supplied by the CRDB bank effective on May 17, 2007.

COUNTRY	CURRENCY	EQUIVALENT SHILLINGS
United states	1 Dollar	1272.50
Europe	1 Euro	1720.33
Japan	1 Yen	10.02
Britain	1 Pound stg	2513.68
Switzerland	1 Franc	1038.76
Canada	1 Dollar	1152.48
Australia	1 Dollar	1049.54
Kenya	1 Shilling	18.525
Uganda	1 Shilling	0.745
South Africa	1 Rand	181.60
Soud Arabia	1 Rial	338.695

India	1 Rupee	31.105
Sweden	1 Kronor	186.42
Zambia	1 Kwacha	0.317
Mozambique	1 Meticais	0.0535
Botswana	1 Pula	209.85

Examples

1. 1. A tourist from Sweden wishes to exchange 1,000 Kronors into Tanzanian shillings. How much does she receive?

Soln.

From the table above

1kron =Tsh. 186.42

1,000Kronor= ?

$$= \frac{1000\text{Kronor x T'sh. } 186.42}{1 \text{ Kronor}}$$

=T shs. 186420

$\therefore$ 1000 Kronor will by Tsh 186420

<u>The tourist will receive Tsh. 186420</u>

2. 2. How much 20,600 Tanzania shillings worth in Indian Rupees?

Soln.

1 Rupee = Tsh. 31.105

? = Tsh. 20,600

$$\frac{1 \text{ Rupee x Tsh. } 20600}{31.105}$$

= 662.273 Rupees

∴ Tsh 20600 worth 662.273 Indian rupees

Variations

Direct Variation

The two variables x and y are said to vary directly of the ratio is constant.

i.e. $\frac{y}{x} = K$, where K is a fixed real number.

The real number K is called the *constant of variation.*

Equation $\frac{y}{x} = k$ can be written in form $y = kx$.

And relationship may be written as $y \propto x$ which reads as *"y is proportional to x"*

If y varies directly as the square of x, then $\frac{y}{x^2}$=Constant.

And can be written as $y \propto x^2$ and the algebraic relation is $y=kx^2$

When having pairs of different corresponding values of x and y, this equation hold true.

$$\frac{y_1}{x_1} = k, \text{and } \frac{y_2}{x_2} = k \quad \text{or} \quad \frac{x_1}{x_2} = \frac{y_1}{y_2}$$

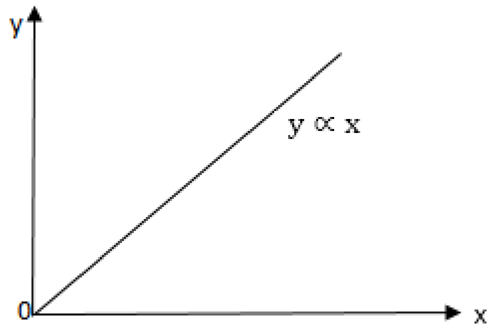
Therefore, we say that x and y vary directly if the ratios of the values of y to the values of x are proportional.

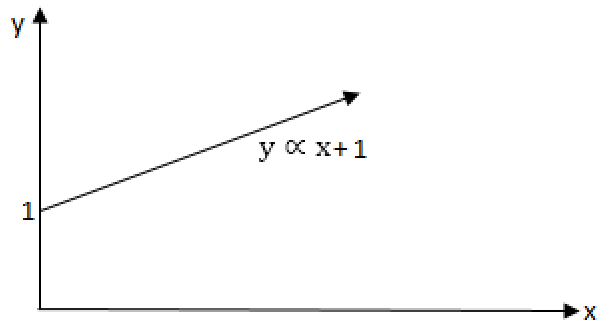
NOTE:

If x and y represent variables such that $y \propto x$, then $y=kx$, The form of this equation y=kx is similar to y=mx. The graph of y=mx is a straight line passing through the origin, M being the gradient same to the equation y=kx,

The graph is a straight line passing through the origin and gradient is k.

A sketch is like





Examples

If x varies directly as the square of y, and x=4 where y=2, find the value of x when y=8.

Solution

Let $x_1 = 4$, $y_1 = 2$, $y_2 = 8$, x_2 is required

But

$$\frac{x_1}{x_2} = \frac{{y_1}^2}{{y_2}^2}$$

$$\frac{4}{x_2} = \frac{2^2}{8^2}$$

$$\frac{4}{x_2} = \frac{4}{64}$$

$$4x_2 = 4 \times 64$$

$$x_2 = \frac{4 \times 64}{4}$$

$$\therefore x_{2=64}$$

Inverse variation

The graph of y against $\frac{1}{x}$ is the straight line through the origin.

i.e I f y varies inversely as x, and then y varies directly as $\frac{1}{x}$.

Other cases of inverse variation

One quality may vary inversely some power of another quantity.

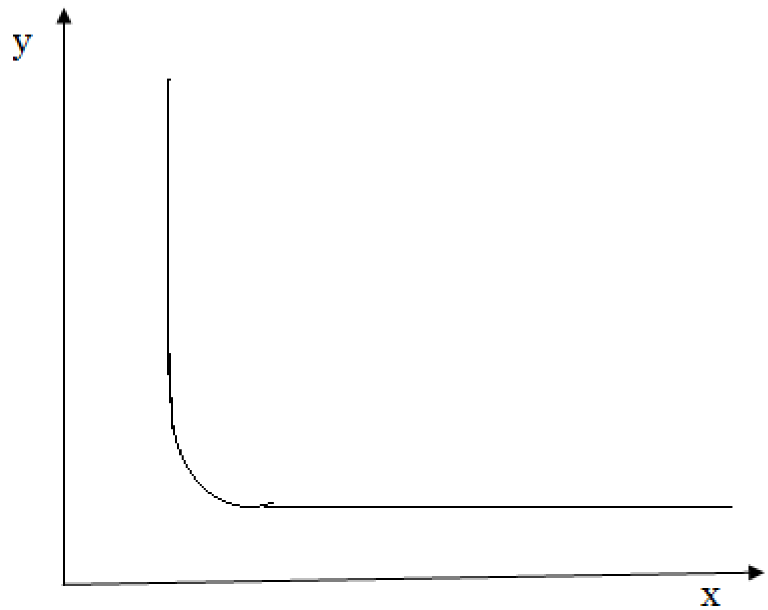
i. If $y \, \alpha \, \frac{1}{x^2}$, y varies inversely as the square of x.

Equation connecting x and y is $y = \frac{k}{x^2}$ or $yx^2 = k$.

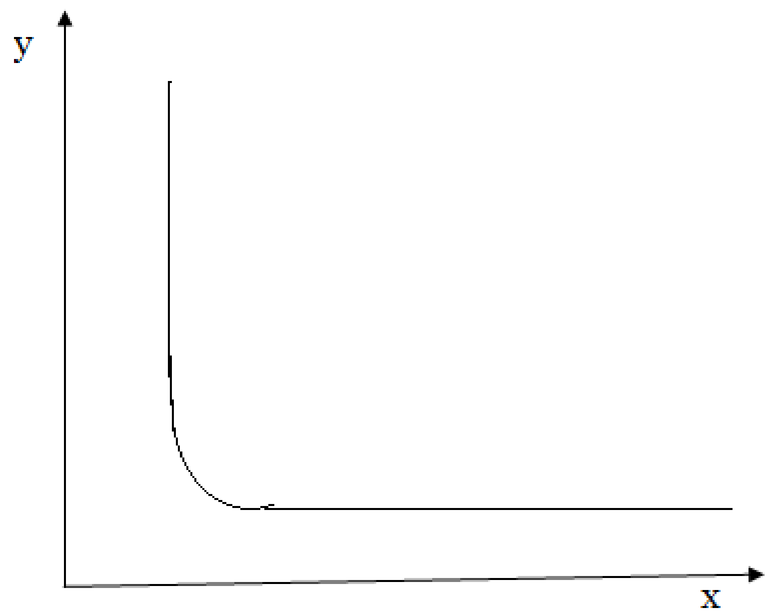
ii. If $y \, \alpha \, \frac{1}{\sqrt{x}}$, y varies inversely as the square root of x.

The equation connecting x and y is $y = \frac{k}{\sqrt{x}}$ or $y\sqrt{x} = k$.

The sketch of $y \, \alpha \, \frac{1}{x}$ is



The sketch of $y \, \alpha \, \frac{1}{\sqrt{x}}$ is



NOTE: The graph does not touch the axis because division by 0 (zero) is impossible.

Example 1

If x varies inversely as y, and x=2, when y=3

Find the value of y when x=18.

Solution.

$Y_1=\frac{k}{x1}$, $y_2=\frac{k}{x2}$

$X_1y_1=k$ $x_2\,y_2=k$

$\frac{x1}{x2}=\frac{y2}{y1}$, where $x_1 = 2$, $y_1 = 3$

$x_2 = 18$, y_2 = uknown

Then $\frac{2}{18} = \frac{y2}{3}$

$18y_2 = 6$

$y_2=\frac{6}{18}$

$y_2=\frac{1}{3}$

When x = 18, $y_2=\frac{1}{3}$

Example 2

3 tailors are sewing 15 clothes in 5 days. How long would it take for 5 tailors to sew 20 clothes?

Solution

- Let t = tailors, d = days c= clothes.

A number of tailors is inversely proportional to the number of days.

$t\ \alpha \frac{1}{d}$……………….(i)

- The number of tailors in directly proportional to the number of clothes.

$t \; \alpha \; c$..................(ii)

$\therefore t\alpha \frac{c}{d}$ in algebra can be written as $t = \frac{kc}{d}$

Given t = 3, d = 5 and c = 15

$k = \frac{td}{c} = \frac{3 \times 5}{15} = 1$

Then, $t = \frac{c}{d}$

When t = 5, c= 20, d can be found as

$t = \frac{kc}{d}$, $t = \frac{c}{d}$

$5 = \frac{20}{d}$

$d = \frac{20}{5}$

d = 4

It takes 4days for to tailors to sew 20 clothes

JOINT VARIATION

If a quantity is equal to a constant times the product of the two other quantities, then we say that the first quantify varies jointly as the other two quantities.

If x = k $\times$ y $\times$ z where k is a fixed real number then x varies jointly as y and z. Similarly if $x_1 \times y_1 \times z_1$ and $x_2 \times y_2 \times z_2$ are corresponding values of the variables x, y and z, then $x_1 = k \times (y_1 \times z_1)$ and $x_2 = k \times (y_2 \times z_2)$

$\frac{x_1}{y_1 \times z_1} = k$ and $\frac{x_2}{y_2 \times z_2} = k$

From these we get

$$\frac{x1}{y1 \times z1} = \frac{x2}{y2 \times z2}$$

OR

$$\frac{x1}{x2} = \frac{y1}{y2} \times \frac{z1}{z2}$$

Examples 1

1. If x varies directly as y and inversely proportional as z and x = 8, when y= 12 and z = 6. Find the value of x when y = 16 and z =4

Solution

Given x varies directly as y and inversely proportional as z.

Then $x \propto \frac{y}{z}$

$\rightarrow x = \frac{ky}{z}$.

From which $k = \frac{xz}{y}$

Given x = 8, y = 12 and z = 6

$k = \frac{8 \times 6}{12} = 4$

Now when y = 16, z = 4, x can be found as

$x = \frac{ky}{z}$,

$x = \frac{4 \times 16}{4}$,

$\therefore x = 16$

Example 2

9 workers working 8 hours a day to complete a piece of work in 52 days. How long will it takes 13 workers to complete the same job by working 6 hours a day.

Solution

Let w= workers

h=hours

d=days

It is a joint variation problem and can be written as

$\frac{w_1}{w_2} = \frac{h_2}{h_1} \times \frac{d_2}{d_1}$ where $d_1 = 52$ days, $h_1 = 8$ hours, $w_1 = 9$ workers

d_2= required, $h_2 = 6$ hours, $w_2 = 13$ works

Thus $\frac{9}{13} = \frac{6}{8} \times \frac{d2}{52}$

$\frac{9}{13} = \frac{6d2}{8 \times 52}$

$2d_2 = 96$

d_2= 48days.

<u>It will take 48 days for 13 workers to complete the job by working 6hours daily</u>

SEQUENCE AND SERIES

<u>SEQUENCE</u>

Is a set of numbers written in a definite order such that there is a rule by which the terms are obtained.

Or

Is a set of number with a simple pattern.

Example

1. A set of even numbers

- 2, 4, 6, 8, 10 ……

2. A set of odd numbers

- 1, 3, 5, 7, 9, 11….

Knowing the pattern the next number from the previous can be obtained.

Example

1. Find the next term from the sequence

- 2, 7, 12, 17, 22, 27, 32

The next term is 37.

2. Given the sequence

- 2, 4, 6, 8, 10, 12………

What is

i)The first term =2

ii) The 3^{rd} term =6

iii)The 5^{th} term =10

iv)The n^{th} term [the general formula]

2=2x1

4=2x2

6=2x3

8=2x4

10=2x5

12=2x6

N^{th} =2xn

Therefore n^{th} term =2n

Find the 100^{th} term, general formula =2n

100^{th} term means n=100

100^{th} term =2x100

100^{th} term =200

3. Find the n^{th} term

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5} \ldots$

Solution

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5} \ldots \frac{n}{n+1}$

4. Given the general term $3[2^n]$

a) Find the first 5terms

b) Find the sum of the first 3 terms

Solution

$3[2^n]$ when;

n=1, $3 [2^1] =6$

n=2, $3 [2^2] =12$

n=3, $3 [2^3] =24$

n=4, $3 [2^4] =48$

n=5, $3 [2^5] =96$

First 5 terms = 6, 12, 24, 45, 96

Sum of the three terms

Sum of the first three = 6+12+24

Sum of the first three = 42

Exercise 1.

1. Find the nth term of the sequence 1, 3, 5, 7…..
2. Find the nth term of the sequence 3, 6, 9, 12……
3. The n^{th} term of a sequence is given by 2n+1 write down the 10^{th} term.
4. The n^{th} term of a certain sequence is 2^{n-1} find the sum of the first three terms.
5. If the general term of a certain sequence is $\frac{1}{2+(-1^{n})}$
   Find the first four terms increasing or
   decreasing in magnitudes

Solution

1. 1, 3, 5, 7…n^{th}

   From the sequence the difference between the consecutive terms is 2 thus

   n^{th} =2+n

2. 3, 6, 9, 12……..n^{th}

   The difference between the consecutive terms is 3 thus

   n^{th} =3n

3. 10^{th} = 2[10+1]

   10^{th} =20+1

   10^{th} =21

4. When

n=1, 2^{1-1}

n=2, 2^{2-1}

n=3, 2^{3-1}

Sum of the first three terms =1, 2, 4

Sum = 1+2+4

Sum = 7

5. **When**;

n = 1, $\frac{1}{2+(-1^{1})} = 1$

n = 2, $\frac{1}{2+(-1^{2})} = \frac{1}{3}$

n = 3, $\frac{1}{2+(-1^{3})} = 1$

n = 4, $\frac{1}{2+(-1^{4})} = \frac{1}{3}$

The first four terms are $1, \frac{1}{3}, 1, \frac{1}{3}$

SERIES

Defination: When the terms of a sequence are considered as the sum, the expression obtained is called a serier or a propagation

Example

(a). 1+2+3+4+5+..........
(b). 2+4+6+8+10+.........+100.
(c). -3-6-9-.......

The above expression represent a series. There are two types of series

1.Finite series
Finite series is the series which ends after a finite number of terms
e.g. 2+4+6+8+............+100.
-3-6-9-12-............-27.
2. infinite Series
Is a series which does not have an end.
e.g. 1+2+3+4+5+6+7+8.............
1-1+1-1+1-1+1-1+1.............

Exercise 5.2.1
1. Find the series of a certain sequence having $2(-1)^n$ as the general term
2. Find the sum of the first ten terms of the series -4-1+2+.......
3. The first term of a certain series is k. The second term is 2k and the third is 3k. Find
   (a). The n^{th} term
   (b). The sum of the first five terms

Exercise 5.2.1 Solution

1. n=1, $2(-1)^1 = -2$
   n=2, $2(-1)^2 = 2$
   n=3, $2(-1)^3 = -2$
   n=4, $2(-1)^4 = 2$
   n=5, $2(-1)^5 = -2$

The series is -2+2-2+2-2+2-2+2-2+2+...................$+2(-1)^n$

2. The sum of the first n terms of the series -4-1+2+5+8+11+14+17+20+23
   = 95

3. (a) k + 2k + 3k + 4k +.......... + nk
   The n^{th} term of the series is nk
   (b) k + 2k + 3k + 4k + 5k = 15k

∴ The sum of the first 5 terms = 15k

ARITHMETIC PROGRESSION [A.P]

An arithmetic progression is a series in which each term differ from the preceding by a constant quantity known as the **common difference** which is denoted by "**d**".

For instance 3, 6, 9, 12….. is an arithmetic progression with common difference 3.

The n^{th} term of an arithmetic progression

If n is the number of terms of an arithmetic progression, then the n^{th} term is denoted by A_n

Therefore $A_{n+1} = A_n + d$

e.g. First term = A_1

Second term = $A_1 + d = A_2$

Third term = $A_2 + d = A_3$

Consider a series 3+6+9+12+.....

$A_1 = 3$, $d = 3$

$A_2 = A_1 + d$

$A_3 = A_2 + d = A_1 + d + d = A_1 + 2d$

$A_4 = A_3 + d = A_1 + 2d + d = A_1 + 3d$

$A_5 = A_4 + d = A_1 + 3d + d = A_1 + 4d$

$A_6 = A_5 + d = A_1 + 4d + d = A_1 + 5d$

$A_n = A_{[n-1]} + d = A_1 + (n-1)d$

∴ The general formula for obtaining the nth term of the series is

$$A_n = A_1 + (n-1)d$$

The general formula for obtaining the n^{th} term in the sequence is also given by $A_n = A_1 + [n-1]d$

Question

1. $A_7 = \frac{5}{2}A_2$ and $A_4 = 16$. Find A_1 and d.

Solution

$A_7 = A_1 + 6d$

$= A_1 + 6d = \frac{5}{2}(A_1 + d)$

$2A_1 + 12d = 5A_1 + 5d$

$3A_1 = 7d$

$A_1 = \frac{7}{3}d$

$A_4 = 16$

$A_4 = A_1 + 3d = 16$

$\frac{7}{3}d + 3d = 16$

$\frac{16}{3}d = 16$

$d = 3$

But $A_1 + 3d = 16$

$A_1 + 9 = 16$

$A_1 = 7$

Exercise 2

1. The p^{th} term of an A.P is x and the q^{th} term of this is y, find the r^{th} term of the same A.P
2. The fifth term of an A.P is 17 and the third term is 11. Find the 13^{th} term of this A.P.
3. The second term of an A.P is 2 and the 16^{th} term is -4 find the first term.
4. The sixth term of an A.P is 14 and the 9^{th} of the same A.P is 20 find 10^{th} term.
5. The second term of an A.P is 3 times the 6^{th} term. If the common difference is -4 find the 1^{st} term and the n^{th} term
6. The third term of an A.P is 0 and the common difference is -2 find;
   (a) The first term
   (b) The general term

7. Find the 54^{th} term of an A.P 100, 97, 94
8. If 4, x, y and 20 are in A.P find x and y
9. Find the 40^{th} term of an A.P 4, 7, 10……..
10. What is the n^{th} term of an A.P 4, 9, 14

11. The 5^{th} term of an A.P is 40 and the seventh term of the same A.P. is 20 find the

    a) The common difference
    b) The n^{th} term

12. The 2^{nd} term of an A.P is 7 and the 7^{th} term is 10 find the first term and the common difference

Exercise 2 Solution

1. $d = y-x$

$r^{th} = A_1+[n-1]d$

$= A_1+nd-d$

$= A_1+n[y-x]-[y-x]$

$= A_1+[ny-nx-[y-x]$

$=A_1+[ny-y]-[nx-x]$

$A_1+g[n-1]-x[n-1]$

$r^{th}=A_1+[y-x][n-1]$

2.

$A_5=17$

$A_3=11$

$A_{13}=?$

$A_5 =A_1+4d = 17$

$A_3=A_1+2d =17$

$A_1+2d=11$

Solve the simultaneous equation by using equilibrium method.

$A_1+4d=17$
$A_1+2d=11$

$$\frac{2d}{2} = \frac{6}{2}$$

d = 3

$A_1+4[3]=17$
$A_1+12=17-12$
$A_1=5$

$A_{13}=A_1+12d$
$A_{13}=5+12[3]$
$A_{13}=5+36$
$A_{13}=41$

3.

$A_2=2$
$A_{16}=-4$
$A_1=?$

$A_2=A_1+d$
$A_1+d=2$

$A_{16}=A_1+15d$
$A_1+15d=-4$

Solve the two simultaneously equations by using the elimination method

$A_1 + d=2$

$A_1 +15d=-4$

$$\frac{-14d}{-14} = \frac{-6}{14}$$

d= $-\frac{7}{3}$

From the 1st equation

$A_1 + -\frac{7}{3} = 2$

$A_1 = 2 + \frac{7}{3}$

$A_1 = \frac{17}{7}$

4.

$A_6 = 14$

$A_9 = 20$

$A_6 = A_1 + 5d = 14$

$A_9 = A_1 + 8d = 20$

Solve the simultaneous equation by using the elimination method

$A_1 + 5d = 14$

$A_1 + 8d = 20$ (difference between two equations)

$$-\frac{3}{3}\,\mathrm{d} = \frac{-6}{3}$$

Then d = 2

From 1st equation

$A_1 + 5[2] = 14$
$A_1 + 10 = 14 - 10$
$A_1 = 4$

$A_{10} = A_1 + 9d$
$A_{10} = 4 + 9[2]$
$A_{10} = 4 + 18$
$A_{10} = 22$

$A_n = A_1 + [n-1]d$
$A_n = 4[n-1]2$
$A_n = 4+2n-2$
$A_n = 2n+4-2$
$A_n = 2n+2$

5.

$A_2 = 3 \times A_b$
D= -4
$A_1 = ?$
$A_n = ?$

$A_z = 3 \times A_6$
$A_1 + d = 3 \times A1+5d$
$A_1 + d = 3A1+15d$
$2A_1 + 14d = 0$
d= -4

$2A_1 + 14[-4] = 0$
$2A_1 + -56 = 0$
$2A_1 = 56$
$A_1 = 28$

$A_n = A_1 + [n-1]d$
$A_n = 28+[n-1]-4$
$A_n = 32-4n$
$A_n = 32-4n$

6.

(a) $A_3 = 0$
d= -2

From the formula

$A_1 + 2d = 0$
$A_1 + 2[-2] = 0$
$A_1 - 4 = 0$
$A_1 = 4$

The first term is 4.

b) The general term

$A_n = A_1 + [n-1]\ d$
$A_n = 4 + [n-1] - 2$
$A_n = 4 + -2n + 2$
$A_n = 6 - 2n$

The general term is 6-2n.

7.

$A_{54} = ?$
100, 97, 94 = A_1, A_2, A_3

$A_{54} = A_1 + 53d$
$d = A_2 - A_1 = A_3 - A_2$
d= 97-100= 94-97
d= -3

$A_{54} = 100 + 53[-3]$
$A_{54} = 100 + -159$
$A_{54} = -59$

8.

4, x, y, 20
$A_4 = 20$
$A_1 + 3d = 20$, but $A_1 = 4$

4+3d= 20
3d=16
$d = {}^{16}/3$

A_1, A_2, A_3, A_4

$A_2 = A_1 + d$

$4 + \frac{16}{3}$

$X = \frac{28}{3}$

$A_3 = A_1 + 2d$

$4 + 2 \times \frac{16}{3}$

$A_3 = \frac{44}{3}$

Hence $x = \frac{28}{3}$ and $y = \frac{44}{3}$

9.

$A_{40} = ?$
$A_1 = 4$
$A_2 = 7$
$A_3 = 10$
$A_1 + 39d = ?$

$d = 7-4 = 10-7$
$d = 3$

$A_1 + 39[3]$
$A_1 + 117$
$4 + 117$
$A_{40} = 121$

10.

$A_1 = 4$
$A_2 = 9$
$A_3 = 14$
$d = 9-4 = 14-9$
$d = 5$

$A_n = A_1 + [n-1]d$
$A_n = 4 + [n-1]5$
$A_n = 4 + 5n - 5$
$A_n = 5n - 1$

The nth term is 5n-1.

11.

a) the common difference

$A_5 = 40$
$A_7 = 20$

$A_1 + 4d = 40$ ………… (1)
$A_1 + 6d = 20$ ………….(2)

Subtracting equation (2) from equation (1) we obtain

$-2d = 20$
$d = -10$

b) the tenth term

$A_{10} = A_1 + 9d$,
But $A_1 + 4d = 40$
$A_1 = 80$

$\therefore A_{10} = A_1 + 9d$
$= 80 - 90$
$A_{10} = -10$

12.

$A_2 = A_1 + d$
$A_7 = A_1 + 6d$
$A_1 + 6d = 10$
$A_1 + d = 7$

Solving the simultaneous equations by using the elimination method;

$-5d = -3$
$d = {}^{3}/5$
$A_1 + \frac{3}{5} = 7$
$A_1 = \frac{32}{5}$

SUM OF THE FIRST n TERMS OF AN ARITHMETIC PROGRESSION

Consider a series with first term A_1, common difference d. If the sum n terms is denoted by S_n, then

$S_n = A_1 + (A_1 + d) + (A_1 + 2d) + ... + (A_1 - d) + A_n$
$+ \ S_n = A_n + (A_{n-} d) + (A_n - 2d) + ... + (A_1 + d) + A_1$

$2S_n = (A_1 + A_n) + (A_1 + A_n) + (A_1 + A_n) + \ldots + (A_1 + A_n) + (A_1 + A_n)$

There are n terms of $(A_1 + A_n)$ then

$$2S_n = n(A_1 + A_n)$$

$$S_n = \frac{n(A_1 + A_n)}{2}$$

∴ The sum of the first n terms of an A.P with first term A_1 and the last termA_n is given by

$$S_n = \frac{n}{2}(A_1 + A_n)$$

But $A_n = A_1 + (n-1)\, d$

Thus, from

$$S_n = \frac{n}{2}(A_1 + A_n)$$

$$S_n = \frac{n}{2}[A_1 + A_1 + (n-1)\, d]$$

$$S_n = \frac{n}{2}[2A_1 + (n-1)\, d]$$

∴ therefore, the sum of the first n term of an A.P with the first A_1 and the common difference d in given by

$$S_n = \frac{n}{2}[2A_1 + (n-1)\, d]$$

Where

n =number of terms
A_1= first term
A_n =last term
d= common difference

Example

i) Find sum of the first 5^{th} term where series is 2, 5, 8, 11, 14 first formula

Solution

$S_5 = \frac{5}{2}[2 + 14]$

$S_5 = 40$

(ii) $S_n = \frac{n}{2}[2A_1 + [n-1]\ d]$

$S_5 = \frac{5}{2}[2 \times 2 + [5-1]\ (3)]$

$S_5 = 40$

Arithmetic Mean

If a, m and b are three consecutive terms of an arithmetic. The common difference
d= M - a
Therefore M- a = b - M
$2M = a + b$
$M = \frac{a + b}{2}$
M is called the arithmetic mean of and b
E.g. Find the arithmetic mean of 3 and 27
$M = \frac{3 + 27}{2}$
$= \frac{30}{2} = 15$

GEOMETRIC PROGRESSION (G.P)

Defination:
Geometric progression is a series in which each term after the first is obtained by multiplying the preceding term by the fixed number.
The fixed number is called the common ratio denoted by r.
E.g. 1+2+4+8+16+32+...
3+6+12+24+48

The n^{th} term of Geometric Progression

If n is the number of terms of G.P, the n^{th} term is denoted by G_n and common ratio by r. Then $G_{n+1} = rG_n$ for all natural numbers.

$G_1 = G_1$
$G_2 = G_1r$
$G_3 = G_2r = G_1r.r = G_1r^2$
$G_4 = G_3r = G_1r^2.r = G_1r^3$
$G_5 = G_4r = G_1r^3.r = G_1r^4$
$G_6 = G_5r = G_1r^4.r = G_1r^5$
$G_7 = G_6r = G_1r^5.r = G_1r^6$
$G_8 = G_7r = G_1r^6.r = G_1r^7$

The n^{th} term is given by
$G_n = G_1r^{n-1}$

Example1: Write down the eighth term of each of the following.

(a). 2+ 4+ 8+...

(b). 12+ 6+ 3+ ...

Solution

(a) The first term G_1 =2, the common ratio r=2 and n=8, Then from

$G_n = G_1r^{n-1}$

$G_8 =(2)(2)^{8-1}$

$G_8 = (2).\ 2^7$

$G_8 = 256$

(b) The first term G_1 =12, the common ratio r=1/2 and n=8, Then from

$G_n = G_1r^{n-1}$

$G_8 =(12)(1/2)^{8-1}$

$G_8 = (12).\ (1/2)^7$

$G_8 = 12/128 = 3/32$

Example2: Find the numbers of terms in the following 1+2+4+8+16+...+512.

Solution

The first term G_1 =1, the common ratio r=2 and G_n= 512, Then from

$G_n = G_1r^{n-1}$

$512 =(1)(2)^{n-1}$

$512 = 2^{n-1}$

$512 = 2^n.2^{-1}$

$512 \times 2= 2^n$

$1024= 2^n$

$2^{10} = 2^n$

n = 10

The sum of the first n terms of a geometrical progression.

Let the sum of first n terms of a G.P be denote by S_n

$S_n = G_1+ G_2+ G_3+ G_4+...+ G_n$

Since the common ratio is r

From

$G2 = G1r$

$G3 = G1r^2$

$G4 = G1r^3$

$...G_n = G_1rn-1$

$S_n = G_1 + G_1r+ G_1r2+ G_1r3+...+ G_nrn-1$

Multiplying by common ratio r both sides we have

$rS_n = rG_1+ + G_1r3+ G_1r4+...+ G_nrn$

Substract S_n from rS_n

$rS_n - S_n = - G_1$

$S_n(r-1) = (r^n-1)$

$S_n = G_1\left(\frac{r^n-1}{r-1}\right)$ where r ≥ 1 for r≠1

G_1= first term of G.P

r= common ratio

Sn = sum of the first n^{th} term

n= number of terms

for r< 1 the formular is given by

$s_n - rs_n = G_1 - G_1r^n$

$S_n(1-r) = G_1(1-r^n)$

$S_n = \frac{G1(1-r^n)}{1-r}$ for r<1

When r= 1, the sum is simply given by

$S_n = G_1+G_1+G_1+G_1+G_1+G_1+...+G_1$

$s_n = nG_1$ for r=1

Example

1. Sum of the first 5^{th} term of G.P where series is 2+4+8+16+32

$S_5 = G_1\left(\frac{2^5-1}{2-1}\right)$

$S_5 = 2\left(\frac{32-1}{1}\right)$

$S_5 = 62$

$S_n = G_1\left(\frac{1-r^n}{1-r}\right)$

2. The sum of the first n terms of a certain series is given by $Sn = 3^n-1$ show that this series is a G.P

Solution

When

n= 1;
$S_1 = 3^1 - 1 = 3 - 1$
$= 2$

n= 2;
$S_2 = 3^2 - 1 = 9 - 1$
$= 8$

n= 3;
$S_3 = 3^3 - 1 = 27 - 1$
$= 26$

n= 4;
$S_4 = 3^4 - 1 = 81 - 1$
$= 80$

2 + 6 + 18 + 54...

r= $\frac{6}{2} = \frac{18}{6} = \frac{54}{18}$

r= 3

Exercise

1. An arithmetic progression has 41 terms. The sum of the first five terms of this A.P is 35 and the sum of the last five terms of the same A.P is 395 find the common difference and the first term.

Solution

$S_5 = 35$
$A_5 = 395$
d= ?
$A_1 = ?$

$S_5 = \frac{5}{2}$ [$2A_1$ + [5-1] d]

$S_5 = \frac{5}{2}$[$2A_1$+4d]

S_5= $5A_1$+10d

35= $5A_1$+10d ... (1)
395=5 A_1+190d… (2)

Solve the simultaneous equations

Then the value of d = 2

From the 1st equation

$5A_1$+10d=35
$5A_1$+10[2]=35
$5A_1$+ 20 =35
A_1 = 3

Therefore the first term is = 3

2. An arithmetic progression has the first term of 4 and n th term of 256 given that the sum of the nth term is 1280. Find the value of the nth term and common difference

Solution

A_1 =4
A_n =256
S_n =1820
n=?
d=?

$S_n = \frac{n}{2}$ [A_1 + A_n]

1820 = $\frac{n}{2}$ [4 + 256]

1820 = n [130]

n= $\frac{1820}{130}$

n= 14

Therefore the n term = 14

$A_n = A_1 + [n-1]d$
$A_n = 4+[14-1]d$
$256= 4+13d$
$256-4=13d$
$252 = 13d$

$d= \frac{252}{13}$

$\therefore$ Common difference = $\frac{252}{13}$

3. The 4th, 5th and6th terms of an A.P are (2x +10), (40x-4) and (8x+40) respectively. Find the first term and the sum of the first 10

$A_4= 2x+10$
$A_5= 40x-4$
$A_6=8x+40$

Solution

$A_4= A_1+3d = (2x + 10)$…i
$A_5 = A_1+4d = (40x - 4)$…ii

Solve the equations by using elimination method

$A_1+3d= 2x+10$
$A_1+4d= 40x-4$

$- d = 2x + 10 - 40x + 4$
$d= 38x - 14$

From 1st equation

$A_1+3[38x-14] =2x+10$
$A_1+114x - 42=2x +10$
$A_1 = 2x+10 - 114x + 42$
$A_1= -112x +52$

Therefore the first terms = -112x+52

$S_{10}= \frac{10}{2}$ [2(-112x+52) +[10-1] 38x-14]

S_{10}=5[-224x+104] +9 [38x-14]

S_{10}=5[-224x+104+342x-126]

S_{10}=5[118x-22]

S_{10}=590x-110

4. The sum of the first n terms of an A.P is given by s_n=n[n+3] for all integral values of n. write the first four terms of the series

Solution

When;

n = 1 then s_n=n[n+3] =1[1+3]

= 4

n = 2 then =2[2+3]

= 10

n = 3 then =3[3+3]

= 18

n = 4 then =4[4+3]

= 28

The first four terms of the series is 4+6+8+10

5. The sum of the first and fourth terms of an A.P is 18 and the fifth terms is 3 more than the third term. Find the sum of the first 10 terms of this A.P

Solution

$A_1+A_4=18$
$A_5 = (A_1 + 4d) = 3(A_1 + 2d)$
$S_{10} = ?$
$A_1+A_1+3d = 18$

$2A_1+3d=18$……….i and $(A_1 + 4d) = 3(A_1 + 2d)$

$A_1 + 4d = 3A_1 + 6d$
$2A_1 = -2d$
$-A_1 = d$

Substitute the value of d into equation … (i)

$2A_1+3d=18$
$2A_1+3(-A_1)=18$
$A_1 = -18$

But $-A_1 = d$ this give us d = 18

The sum of the first ten terms $S_n = \frac{n}{2}[2A_1 + [n-1] d$

$S_{10} = \frac{10}{2}[2 \times -18 + [10-1](18)]$

$= 630$

6. How many terms of the G.P 2+4+8+16…… must be taken to give the sum greater than 10,430?

Solution

$G_1+G_2+G_3+G_4$………… 2+4+8+16

$S_n = G_1\left(\frac{r^n - 1}{r - 1}\right)$

$S_n = 2\left(\frac{2^n - 1}{2 - 1}\right)$

$10430 = 2\left(\frac{2^n - 1}{2 - 1}\right)$

$5215 = 2^n - 1$

$5216 = 2^n$ then

n= $\log_2 5216$

Then more than $\log_2 5216$ term should be taken to provide the sum greater than 10430

7. In a certain geometrical progression, the third term is 18 and the six term is 486, find the first term and the sum of the first 10 terms of this G.P

Solution

$G_3 = G_1 r^2 = 18$ ………… (1)
$G_6 = G_1 r^5 = 486$……….. (2)

Take equation (2) divide by equation (1)

$$\frac{G_1 r^5}{G_1 r^2} = \frac{486}{18}$$

$r^3 = 27$

r= 3

But $G_1 r^2 = 18$

$$G_1 = \frac{18}{9}$$

$G_1 = 2$

$S_n = G_1\left(\frac{r^n - 1}{r - 1}\right)$ then

$S_{10} = 2\left(\frac{2^{10} - 1}{2 - 1}\right)$

= 2046

The first term is 2

The sum of the first 10 terms is 2046

8. Given that p-2, p-1 and 3p-5 are three consecutive terms of geometric progression find the possible value of p

Solution

R=$\frac{p-1}{p-2}$ = $\frac{3p-5}{p-1}$

r=[p-1][p-1]=[p-2][3p-5]

r=p^2-2p+1=3p^2-5p-6p+10

p^2-2p+1=3p^2-11p+10

p^2-2p+1=3p^2-11p+10

0=3p^2-p^2-11p+2p+10-1

2p^2-9p+9=0

From the general quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$p = \frac{9 \pm \sqrt{(-9)^2 - 4(2)(9)}}{(2)(2)}$$

p= $\frac{9 \pm \sqrt{9}}{4}$

p= $\frac{9 \pm 3}{4}$

P=3 or p= $\frac{3}{2}$

<u>**Geometric mean**</u>

If a, m and b are consecutive terms of a geometric progression then the common ratio
r= M/a = b/M
M^2 = ab
M = $\sqrt{ab}$

Example: Find the geometric mean of 4 and 16.

from G. M = $\sqrt{ab}$

G.M= $\sqrt{4 X 36}$

G.M = $\sqrt{144}$

G.M = 12.

5.7. APPLICATION OF A.P AND G.P.

Simple interest is an application of arithmetic progression which is given by;

I = $\frac{PRT}{100}$

Where

I=simple interest

P=principal

R= rate of interest

T= period of interest

Compound interest is an application of geometric progression it is given by;

A_n=p+1 or A_n =p$\left(1+\frac{RT}{100}\right)^n$

Where

A_n =an amount at the end of the New Year
R= rate of interest
n= number of years
T=period of interest
p= principal

Example

1. Find the simple interest on Tshs 10,000/= deposited in a bank at the rate of 10% annually for 4 years

Solution

$I = \frac{PRT}{100}$

$I = \frac{(10000)(10)(4)}{100}$

I=1000x4

I=4000

The interest for 4 years will be 4000/=

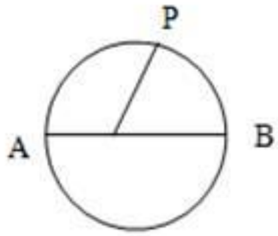
CIRCLE

DEFINITION AND PROPERTIES OF A CIRCLE

A circle can be defined in two ways.

A circle: Is a closed path curve all points of which are equal-distance from a fixed point called centre OR

- Is a locus at a point which moves in a plane so that it is always of constant distance from a fixed point known as a centre.



O - Is called the centre of the circle.

OP - Is the line segment is the radius.

AB - Is the line segment of diameters of the circle.

O – (Centre) - Is a fixed point of circle.

OP (Radius) – Is the constant distance from the centre to any point on a circumstance of a circle.

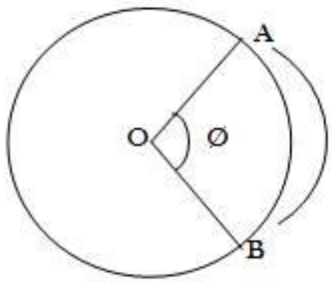
AB (Diameter) – Is a line segment which passes through the centre of a circle.

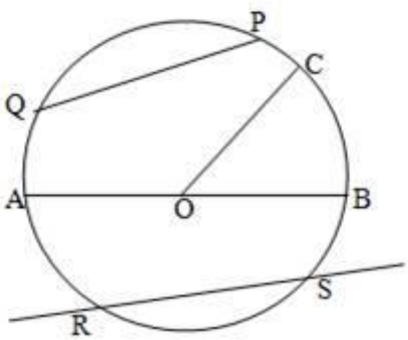
A circumference – Is a length of a locus which moves around the centre.

Diameter =2 x Radius

$$D = 2r$$

Hence the diameter of a circle is equal to two times radius.





OC - Are called the Radii at circle
PO (Chord) - Is a live segment

RS (Secant) - Is a line segment whose points are on the circle.

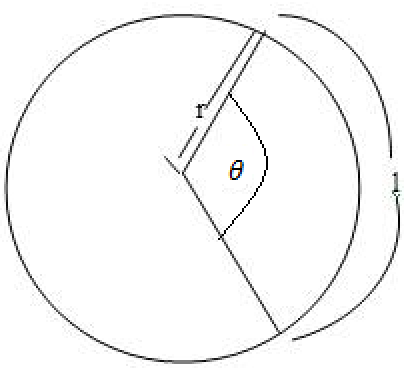
BOC – Is called a central angle

(PTQ) – A Segment – Is the part of a circular region included within the chord and its arc.

(COB) Sector - Is the part of a circular region bounded by two radii and an arc.

CENTRAL ANGLE

Consider a circle of radius r, length of arc l, subtending a central angle.



- The length of the circumference C of the circle is C = 2πr. This means that the length of the arc intercept by a central angle 360⁰ is 2πr.

- The length of an arc is proportional to the measure of the central angle of the central angle. Thus if the central angle is 1

$$\frac{2\pi\theta}{360°}$$

Example 1.

An arc subtends an angle of 20⁰ at the centre of a circle of radius 25m.

Find the length of the arc.

Length of an arc is given by

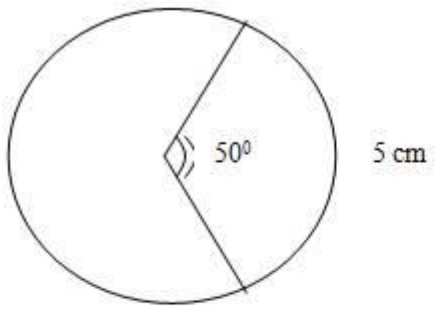
$$\frac{\pi r\theta}{180°}$$

$$L = \frac{3.14 \text{ x } 25\text{m x } 20°}{180°}$$

$$= \frac{3.14 \text{ x } 25\text{m}}{9}$$

= Length of an arc = 8.72m

An arc of length 5cm subtends 50⁰ at the centre of a circle. What is the radius of the circle.



Data

$\theta = 50^{o}$

Length = 5 cm

Radius = required

Length of an arc is equal to

$$\frac{\pi r\theta}{180°}$$

$$L = \frac{3.14 \text{ x } r \text{ x } 50°}{180°}$$

$$5 = \frac{3.14 \text{ x } 5}{18}$$

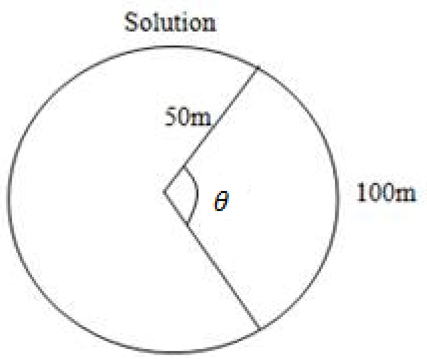
$$= \frac{1800}{314}$$

=5.73cm

Example 3

A circular running track has radius 50m. A sprinter runs 100m along the track.

Through what angle has she turned?



Radius = 50m

Length of an arc = 100m

θ = Required

Length at an arc is equal to

$$\frac{\pi r \theta}{180°}$$

$$100\text{m} = \frac{3.14 \text{ x } 50\text{m x } \theta}{180°}$$

$$100 \text{ x } 18 = 3.14 \text{ x}5 \text{ x } \theta$$

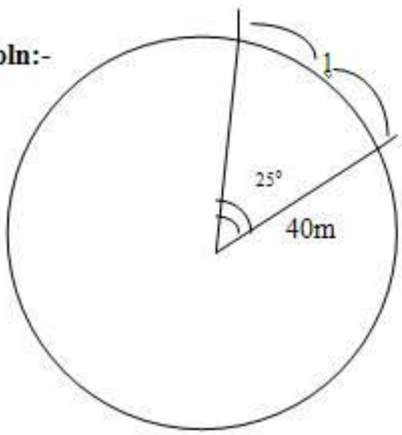
$$\frac{1800}{157} = \frac{157\theta}{157}$$

The central angle is 114 .650

Questions:

1. An arc subs tends 25 at the centre of a circle of radius 40m. What is the length of the arc?

Soln:-



Data

Length of an arc = required

Radius = 40m

θ =- 25

Length at an arc is equal to

$$\frac{\pi r\theta}{180°}$$

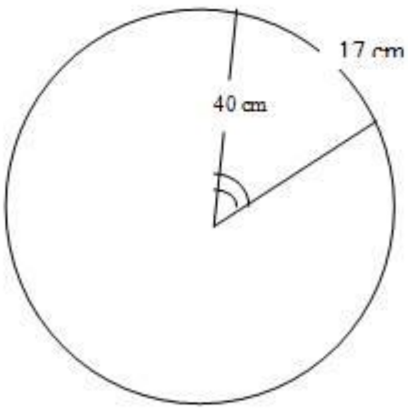
$$= \frac{3.14 \text{ x } 40\text{m x } 25°}{180°}$$

$$= \frac{3.14 \text{ x } 2 \text{ x } 25}{9}$$

$$= \frac{157}{9}$$

The length at an arc = 17.44m

2. An arc of length 17cm forms a circle of radius 40cm what angle does the arc subtend?



Data :-

Length of an arc = 17cm

Radius = 40cm

θ= Required

Length of arc is equal to

$$\frac{\pi r\theta}{180°}$$

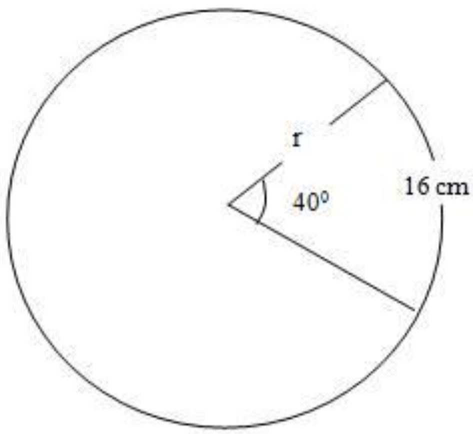
$$17 = \frac{3.14 \text{ x } 40\text{cm x } \theta°}{180°}$$

$$18 \; x \; 17 = 3.14 \; x \; 4 \; x\theta$$

$$\theta = \frac{30600}{1256}$$

The arc subtends 24.330

3. An arc of length 16m subtends 400 at the centre of the circle. What is the radius at the circle?



Data

Length an arc = 16m

Radius = Required

$\theta = 40^o$

Length of arc is equal to

$$\frac{\pi r\theta}{180^\circ}$$

$$16 = \frac{3.14 \text{ x r x } 40°}{180°}$$

$$16 \; x \; 9 = 3.14 \; x \; r \; x \; 2$$

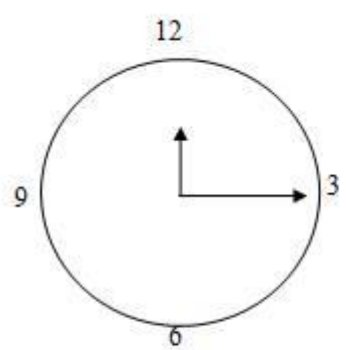
$$r = \frac{14400}{628}$$

The radius = 22.85m

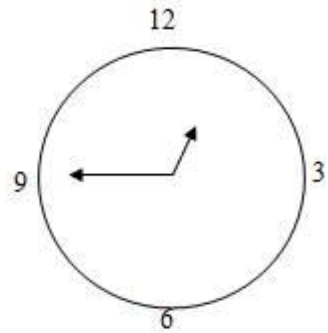
RADIAN MEASURE

Angles can also be used to measure the amount of turning. Turns of a minute hand of a clock and a wheel can be measured in both angles and radians. Example a minute hand of clock turns through an angle of 90⁰ of ½ π radians between noon and 12:15pm as shown in the figure.

- The tip of the hand has covered a distance of ½ π radians.



From noon to 12:45 pm, the hrs turned through an angle of 270⁰or 3/2πr radians. The angle 270⁰ is reflex angle.



- One complete turn at hand clock represents an angle of 360⁰ or 2π radians.

- Measures of angles more than 360⁰ or 2 π radius can be obtained if hand of a clock measures more than one complete turn. Example from noon to 1:15 pm, the hand has turned through 1 ¼ turns. Now one turn is 360⁰ or 20 radians.

- 5/4 turns or 3600 x 5/4 or 2π x 5/4 radians which reduce to 450⁰ or 5/2π radians.

- There fore from noon to 1:15 pm the hand turns through 450⁰ or 5/2π radians.

Questions

1. Give the size in degree at an angle through which a minute hand of a clock has returned between noon and the following times.

(a) 12:40

Solution:

1min = 6

40min =?

X= 40min x 6

X = 240

(b) 3:00

Solution

1hour = 360

3 hour =?

X = 3 hour x 360^0

X = 1080^0

(c) 9:00

Solution

1hour = 360^0

9 hour =? X

X = 9 x 360^0

X = 3240^0

2. Give the size in radians at angles through which the minute hand of a clock has turned between noon and the following times.

(a) 12:20pm

Solution

1min = 6^0

20 min =? X

X= 20min x 6

X = 120^0

πrad = 180^0

? = 120^0

x = π rad x 120

X = 2/3 π rad

(b) 2:15

Solution

1hour = 60 min

2 hour =? X

x =60 x 15min

X =120

120 x 6^0=720^0

$= 720^o + 90^o$

$= 810^o$

$$x = \frac{810 \text{ x } \pi\text{rad}}{180^o}$$

$$\therefore x = \frac{9}{2}\text{rad}$$

(c) 24:00 noon

Solution

1 hour = 360^o

24hrs = ?

X = 360^o x 20

= 8640^o

πrad = 180^o

?x = 8640^o

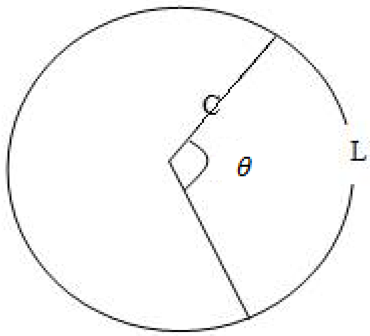
x = 8640^o x πrad

$$x = \frac{8640}{180}\pi\text{rad}$$

X = 48πrad

RADIAN MEASURE

- The relation between the arc length l, the central angle θ, and the radius r, can be used to compare the measurement of an angle in radius with the measurements in degree.



Circumference of the circle for the given radian C = 2πr. Circumference sector

$$C = \frac{\theta}{360°}2\pi r$$

$$C = \frac{\pi\theta}{180°}$$

But C = length at an arc

$$\frac{l}{r} = \frac{\pi\theta}{180°}$$

$$\frac{l}{r} = \textit{is called radian}$$

In abbreviation is written as:-

$$S = \frac{\pi\theta}{180°}\textit{ where by}$$

S = is radian measured of an angle

θ = is the degree measure

$$\theta = \frac{18 \text{ x } 3}{3.14}$$

$$S = \frac{54}{3.14} x \frac{100}{100}$$

$$= \frac{5400°}{314}$$

θ= 17 .19°

The angle in degree is 17. 19°

Class Activity:-

1.Find the degree of each of the following :-

(i) 3/2π

Solution:-

$$S = \frac{\pi\theta}{180°}$$

$$\theta = \frac{180° S}{\pi}$$

$$= \frac{180° \times \frac{3}{2}\pi}{\pi}$$

= 90° x 3

= 3/2π = 270°

(ii) ¾π

Solution

$$S = \frac{\pi\theta}{180^\circ}$$

$$\theta = \frac{180^\circ S}{\pi}$$

$$= \frac{180^\circ \times \frac{3}{4}\pi}{\pi}$$

¾ π = 135^{o}

Example 1:

Find in radian as multiple of π for each of the following degrees.

(a) 315^{o} (b) 240^{o}

Solution:

$$S = \frac{\pi\theta}{180^\circ}$$

$$\theta = \frac{\pi \times 315^\circ S}{180^\circ}$$

$$= \frac{63}{36}$$

$$S = \frac{7}{4}\pi radian$$

$$\therefore 315 = \frac{7}{4}\pi radian$$

(b) 240°

Solution:-

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{240° \times \pi}{180°}$$

$$= \frac{12}{9}\pi$$

$$S = \frac{4}{3}\pi radian$$

$$\therefore 240° = \frac{4}{3}\pi radian$$

2. Change the following radians into degree

(a) 0.3 (b) 5

solution

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{180° S}{\pi}$$

$$\theta = \frac{180° S}{\pi}$$

$$\theta = \frac{5400}{314}$$

$$\therefore \theta = 17.19°$$

Class activity.

1. Find the degree of each of the following

i) 3/2π

Solution.

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{180°S}{\pi}$$

$$\theta = \frac{180°S}{\pi}$$

$$\theta = \frac{180 \times \frac{3}{2}}{\pi}$$

$$= 90° \times 3$$

$$\therefore \frac{3}{2}\pi = 270°$$

ii 3/4 π

Solution

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{180°S}{\pi}$$

$$\theta = \frac{180°S}{\pi}$$

$$\theta = \frac{180 \times \frac{3}{4}\pi}{\pi}$$

$$\therefore \frac{3}{4}\pi = 135°$$

iii) 2 π

Solution

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{180°S}{\pi}$$

$$\theta = \frac{180°S}{\pi}$$

$$\theta = \frac{180 \times 2\pi}{\pi}$$

$$\therefore 2\pi = 360°$$

2.Find the radians multiple of the π following

(i)80°

Solution

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{\pi \times 80}{180}$$

$$S = \frac{4}{9}\pi radian$$

$$\therefore 80° = \frac{4}{9}\pi$$

ii) 215^0

Solution

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{\pi \times 215°}{180°}$$

$$S = \frac{43}{36}\pi radian$$

$$\therefore 215° = \frac{43}{36}\pi$$

iii) 60°

Solution

$$S = \frac{\pi\theta}{180°}$$

$$= \frac{\pi \times 60°}{180°}$$

$$S = \frac{1}{3}\pi$$

$$\therefore 60° = \frac{1}{3}\pi$$

3. (i) Change the radians into the degree of 0.3

$$S = \frac{\pi\theta}{180°}$$

$$\theta = \frac{180° S}{\pi}$$

$$\theta = 180° \times 0.3$$

∴The angle the degree = 54°

(ii) 5

Solution

$$S = \frac{\pi\theta}{180°}$$

$$\theta = \frac{180°S}{\pi}$$

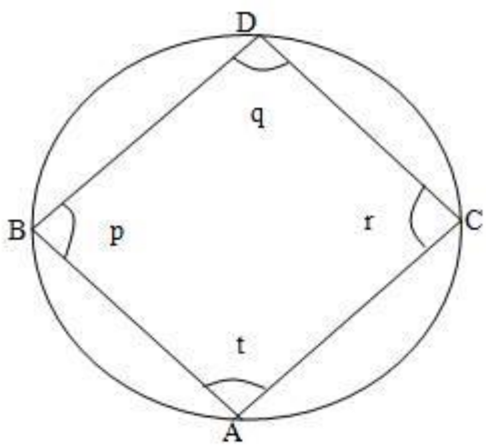
$$\theta = 180° \times 0.3$$

$$\theta = 180° \times 5$$

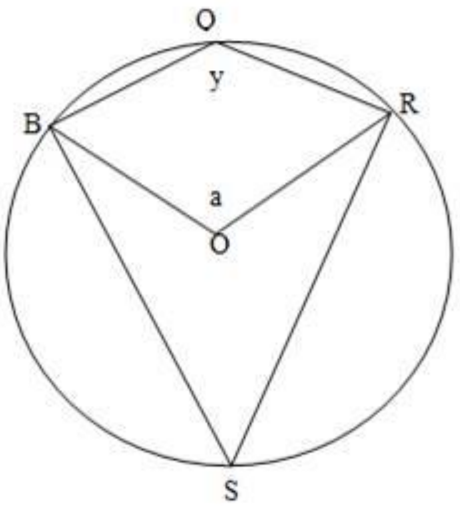
The angle degree = 900°

ANGLES IN CYCLIC QUADRILATERAL

These are four angles whose vertices are lying on the circumference of a circle.



The angles p, q, r, and t are called cyclic angles in a quadrilateral ABCD, q and t, p and r are opposite angles.



1. **THEOREM:**

The opposite angles of a cyclic quadrilateral are supplementary (add up to 180⁰).

Given; A quadrilateral SPQR inscribed in a circle centered at o

Required to prove: x+ y = 180⁰

Constuction; join OR and OP

Proof: in the above figure

a = Zx (angle s on a circle) PQR)

b = Zy (angle on arc PSR)

a+b = Zx + 2y but a+ b = 360⁰

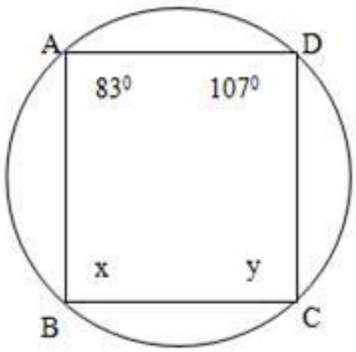
360⁰ = 2 (x+y) divide by 2 both sides

$x+y = 180^o$

∴ $x+ y = 180^o$ hence proved.

Example:

Find the size of each lettered angle.



The opposite angles A and C, B and D

:. $x +83^o = 180^o$

$y = 180^o – 83$

$y = 97^o$

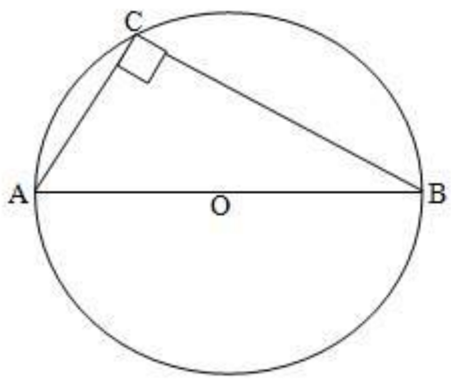
$x+107^o = 180^o$

$x= 180^o – 19$

$x = 63^o$

2. THEOREM:

Any inscribed angle in a semicircle is a right angle.



Given

AB – Is a diameter at a circle.

O - Is the center.

C - Is any point on the Circumference

Required to prove that ACB = 90^o

2 ACB= A$\hat{O}$B (Angle at the centre)

But,

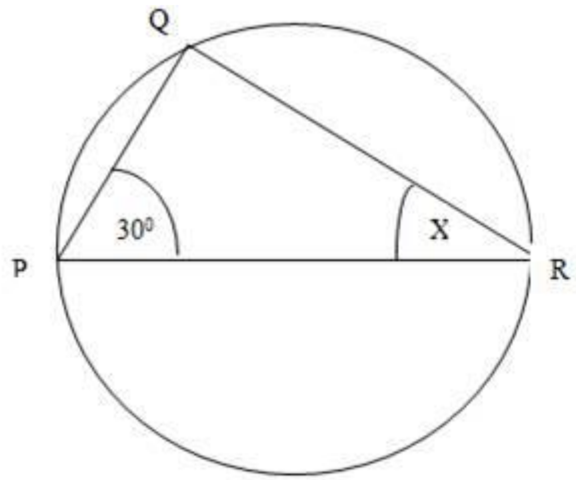
A$\hat{O}$B = 180^o (Straight angle)

$$\frac{2A\hat{C}B}{2} = \frac{180}{2}$$

ACB = 90º Hence proved.

Example:

Calculate the Value of x



Soln:

PQR = 90º (Angle in a semi Circle)

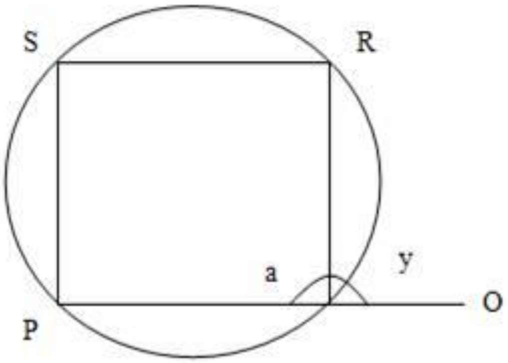
Then 90º + 30º + x = 180º

(Angles sum in A)

x = 180º – 120º

x = 60º

An exterior angle is an angle formed outside a cyclic quadrilateral. An internal angle is formed inside the cyclic quadrilateral when you provide line from this angle you will form an angle which is outside called exterior angle.



THEOREM: 3

Exterior angle of a cyclic quadrilateral is equal to the inside opposite angle.

Aim: to prove that PSR = PQT

Proof: let angle PQR = a

Angle PRS = b

Angle PQT = y

(i) PQR + PSR = 180^o (Because it is opposite angle of cyclic quadrilateral).
Thus a+b = 180^o

(ii) Therefore when you equate them since both are 180^o

(iii) PQR + RQT = 180^o (Because they are adjacent angles on straight line)

Thus a + y = 180^o

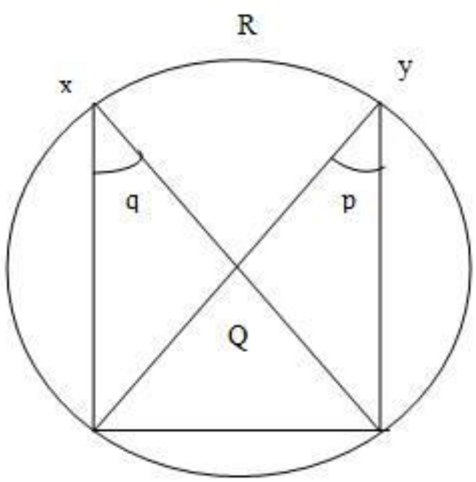
Therefore when you equate them since both are 180^o you will have:

$$a + b = a + y$$

B = y
But b = PSR
And y = PSR
PSR = PQT
Proved.

THEOREM 4:Angles in the same segment are equal.

THEOREM 5: Angles in the semi circle are right angled triangle.



Aim: To prove that PxQ = PyQ

Construction: join OP and OQ

Let PÔQ = P

PVQ = q

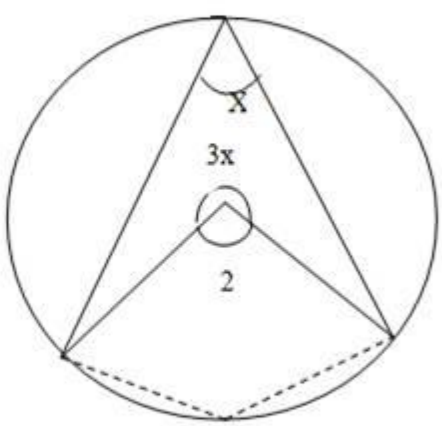
PYR = r

(i) PÔQ = 2p x Q and
P = 2q (Because angle at the centre is twice the angle at the circumference)

(ii) PÔQ = 2PyQ (Because angle at the centre is twice the angle at the circumference)
P = 2r
x = P = 2q, P = 2r
= 2q = 2r
q = r

TO ANSWER THE QUESTIONS:

1. Find the value of X. if O is the centre of Circle



Soln:

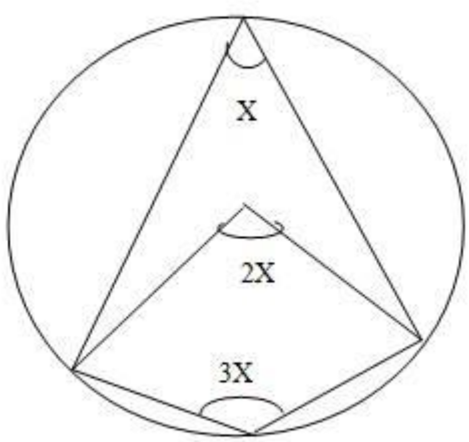
Angle at the centre is twice the angle at the circumference.

Since the circle at the centre is 360^o

$3x + 2x = 360^o$

$5x = 360^o$

∴= 72^{o}



(b) Angle at the centre is twice the angle of the circumference.

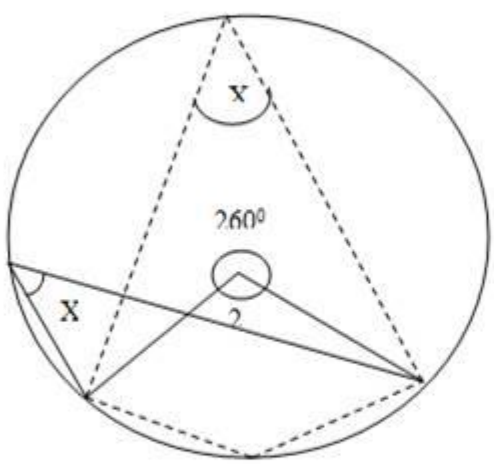
Opposite angle at a cyclic quadrilateral are supplementary (add up to 180^{o})

x + 3x = 180^{o}

4x = 180

x=45

(c)



Soln:

Angle at the same segment are equal.

Angle at the centre is twice the angle at the circumference.

$2\text{ x} + 260^o = 360^o$

Since the circle at the centre is 360^o

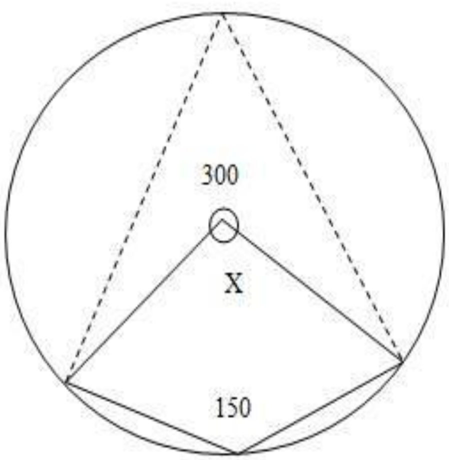
$2x + 260^o = 360^o$

$2x = 360^o - 260^o$

$2x = 100^o$

$\therefore x = 50^o$

(d)



Soln:

Angle at the centre is twice the angle at the circumference.

$150^o \text{ x } 2 = 300^o$

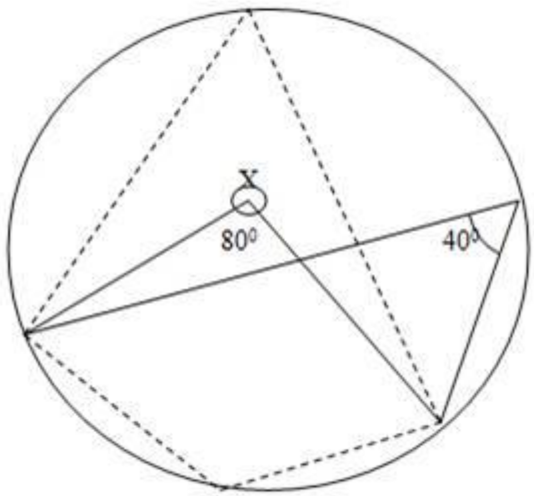
Since the circle at the centre is 360^o

x + 300° = 360°

x = 360° – 300

∴x = 60°

(e)



Soln:

Angle at the centre is twice the angle at the circumference.

40° x 2 = 80°

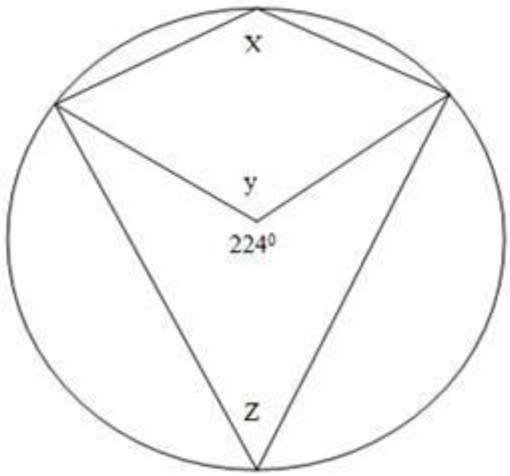
Since the circle at the centre is 360°

x + 80° = 360°

x = 360° – 80°

x = 280°

(f) Find the value of angles marked x, y and z



Soln:

Angle at the centre is twice the angle at the circumference.

$X \times 2 = 224^o$

$X = 112^o$

Since the circle at the centre is 360^o

$y + 224^o = 360^o$

$y = 360^o - 224^o$

$\therefore y = 136^o$

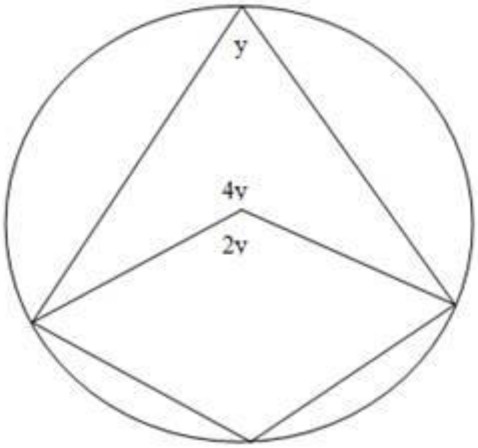
Opposite angle of a cyclic quadrilateral are supplementary (add up to 180^o)

$Z + 112^o = 180^o$

$Z = 180^o - 112$

$\therefore z = 68^o$

(g) Find ‘y’



Soln:

Angle at the centre is twice the angle at the circumference.

y x 2 = 2y

Since the circle at the centre is 360^o

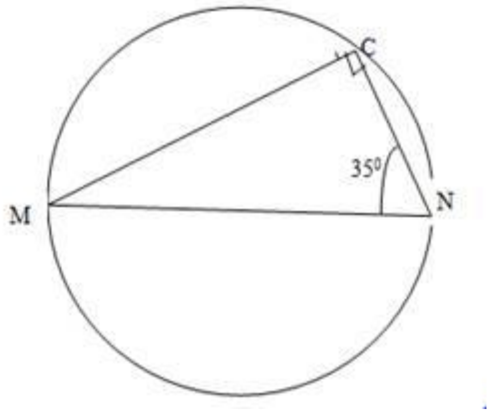
$4y + 2y = 360^o$

$\therefore y = 60^o$

Class activity

1. MN is a diameter of a circle and L is a point on the circle. If MNL = 135^o,

Find NML



Soln:

Any inscribed angle in a semicircle is a right angled triangle.
let NML=X

$X + 35^o + 90^o = 180^o$

$X + 125^o = 180^o$

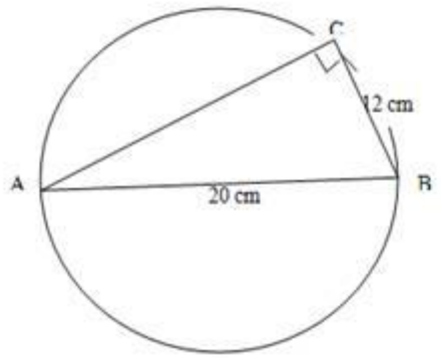
$X = 180^o - 125^o$

$\therefore NML = 55^o$

2. AB is a diameter of a circle radius 10 cm and e is a point on the circumference.

CB = 12, find CA (Remember Pythagoras theorem)

Soln:



By using Pythagoras theorem

$20^2 = CA^2 + 12^2$

$400 = CA^2 + 144$

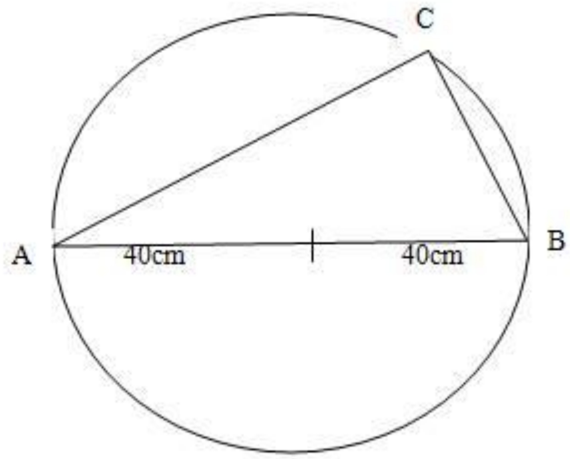
$CA^2 = 400 - 144$

$CA^2 = 256$

$\therefore CA = 16cm$

3. AB is a diameter of a circle radius 40cm and C is point on the circumference.

If CBA = 62^0, then find $C\hat{A}B$



Soln:

Let X = CÂB

$X + 62^o + 90^o = 180^o$

$X + 152^o = 180^o$

$X = 180^o - 152^o$

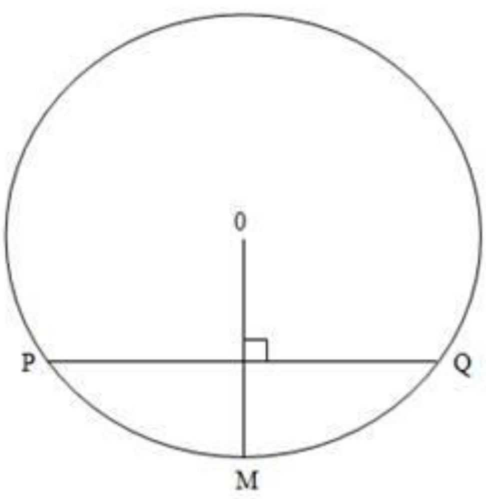
∴CÂB = 28^o

THE CHORD PROPERTIES OF A CIRCLE

The chord of a circle is the line segment whose end point are on the circle. A chord which passes through the centre of a circle is called a diameter. It is very important for you to know what a chord is and how to identify the chord properties of a circle because it will summarize you with this unit.

Therefore in this section you are going to study about the chord itself and the **chord properties** of a circle.

You are also going to study how to develop theorem which relate to these properties at chord. At the end of the section you will be able to identify the chord, prove the theorem of the chord. Properties in a circle and then apply these theorems on solving related problems in order to identify the properties of the chord properties it easier if you draw a circle with centre O.



You can see that O is a centre of the OM is the radius of the circle and PQ is chord of the circle.

Therefore you will discover that.

(a) The centre of the circle lies on the perpendicular bisector of the chord.

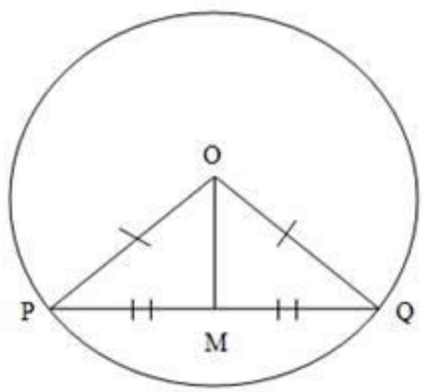
(b) The perpendicular from the centre of the circle to the chord

(c) The line joining the centre of the circle to the midpoint of the chord.

Then from the information above you can develop the theorem which can be written as;

THEOREM

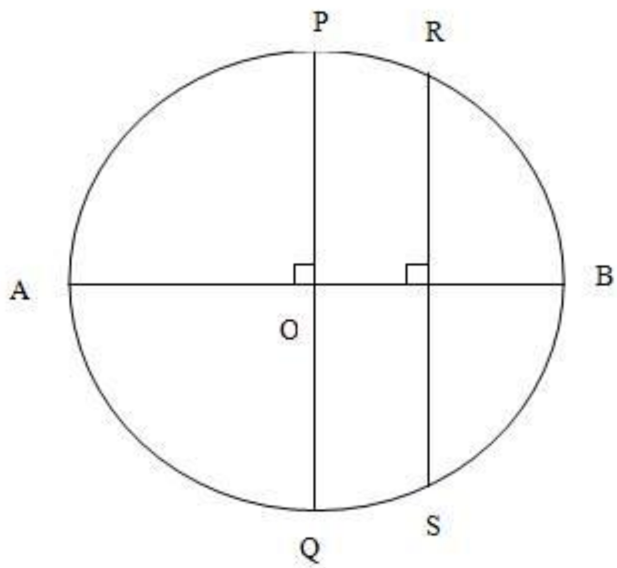
The perpendicular bisector of a chord passes through the centre of the circle.



AIM: To prove that OMP = OMQ = 90°
Construction: Join OP, OQ and OM
Proof:
(i) OP = OQ (Radii)
(ii) PN = QM (M is midpoint given)
(iii) ON = OM (common)
(iv) OPM = OQM (Bisected angles)

∴The corresponding angle are congruent and hence ΔOMP = OMQ = 90° proved.

THEOREM: Parallel chords intercept congruence arc.



Aim: To prove that arc PR≡Are

Proof:

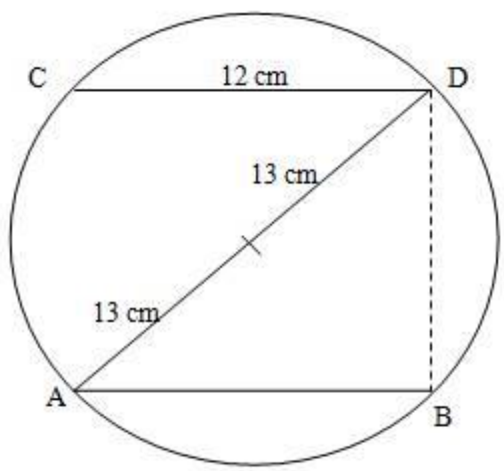
Arc AQ≡ Arc AP (AOB is diameter)
Arc AS≡ Arc AR (AOB is a diameter)

Arc PR ≡Arc AR – Arc AP and also

Arc QS≡ Arc AS – Arc AQ

By step (i) up to step (iii) above you can conclude that Arc PR≡ Arc QS proved

Class Activity:
Two chords, AB and CD of the circle whose radius is 13cm are equal and parallel.
If each is 12cm long, find the distance between them.



Soln: - By using Pythagoras theorem

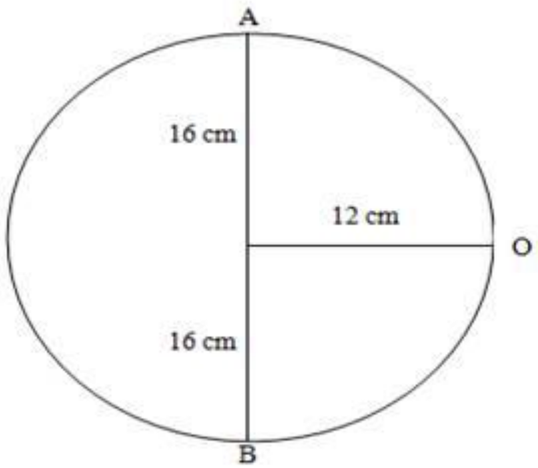
$AD^2= AB^2 +DB^2$
$26^2 =12^2 + DB^2$
$676 = 144 + DB^2$
$DB^2= 676 – 144$
$DB^2 = 532$
Square root both sides

$$DB = \sqrt{532}$$

$\therefore$ The distance between them is $2\sqrt{133}$ cm

2.A chord of length 32cm is at a distance of 12cm from the centre of a

circle.
Find the radius of a circle.



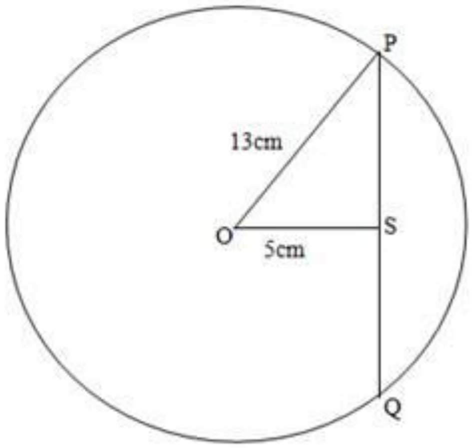
Soln:

By using Pythagoras theorem

$AO^2 = 12^2 + 16^2$
$= 144 + 256$
$= 400$
Apply square root both sides
$AO = \sqrt{400}$

$\therefore$ The radius is 20 cm

3. A distance of a chord PQ from the centre of a circle is 5cm. If the radius of the circle is 13cm. Find the length of PQ



Soln:

By using Pythagoras theorem.

$13^2 = 5^2 + PQ^2$

$169 = 25 + PQ^2$

$PQ^2 = 169 - 25$

$PQ^2 = 144$

Apply square root both sides

PQ = 12cm

Since OS is perpendicular to PQ

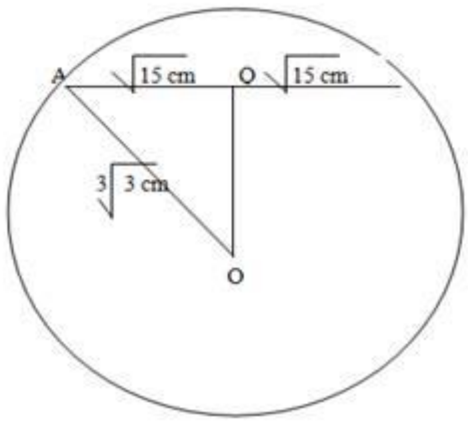
Therefore PS = SQ

∴PQ = 12cm + 12cm

= 24cm.

4. The chord AB of a circle with centre O radius 3cm long.

Find the distance of AB from O. give your answer in cm form



Soln:

By using Pythagoras theorem

$a^2 + b^2 = c^2$

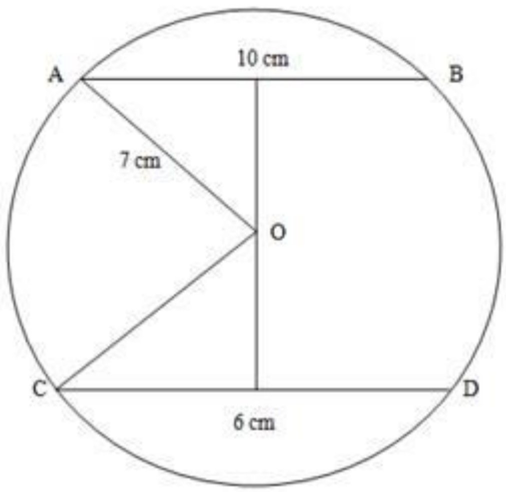
$15 + b^2 = 9 \times 3$

$b^2 = 27 - 15$

Apply square root both sides

$\therefore\ b^2 = \sqrt{12}cm$

5.Two chords AB and CD of a circle with centre O. if AB = 10cm, CD = 6cm, AO = 7cm. Find the distance between two chords



Triangle (i)

By Pythagoras theorem

a^2 x 6 = C^2

$5^2 + b^2 = 7^2$

$25 + b^2 = 49$

$b^2 = 49 - 25$

$b^2 = 24$

$b = 2\sqrt{6}cm$

triangle (ii) by Pythagoras theorem

$a^2 + b^2 = c^2$

$3^2 + a^2 = 7^2$

$9 + a^2 = 49$

$a^2 = 49 - 9$

$a^2 = 40$

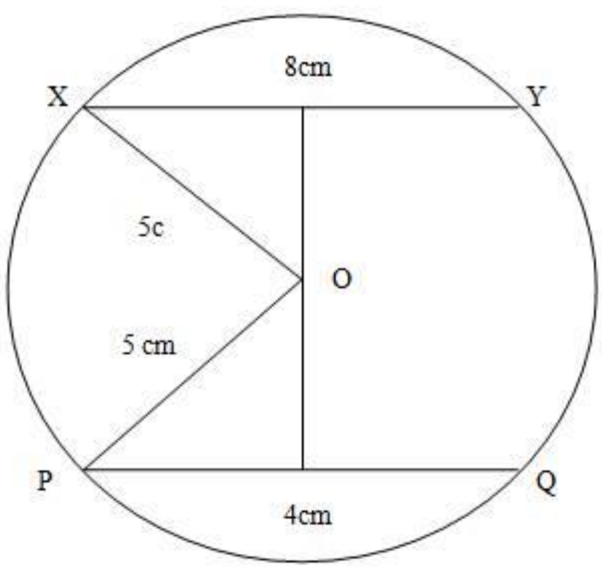
$$a = 2\sqrt{10}cm$$

∴ The distance between two chords is $2\sqrt{6}$ cm+$2\sqrt{10cm}$

Class Activity:

XY and PQ are parallel chords in a circle of centre O and radius 5cm.

If XY = 8cm and PQ = 4cm, find the distance between two chords.



Soln:

1st triangle

By using Pythagoras theorem.

$C^2 = a^2 + b^2$

$5^2 = 4^2 + b^2$

$25 = 16 + b^2$

$b^2 = 25 – 16$

$b^2 = 9cm$

square root both sides

b= 3cm

2nd triangle

By using Pythagoras theorem.

$C^2 = a^2 + b^2$

$5^2 = 2^2 + b^2$

$25 = 4 + b^2$

$b^2 = 25 – 4$

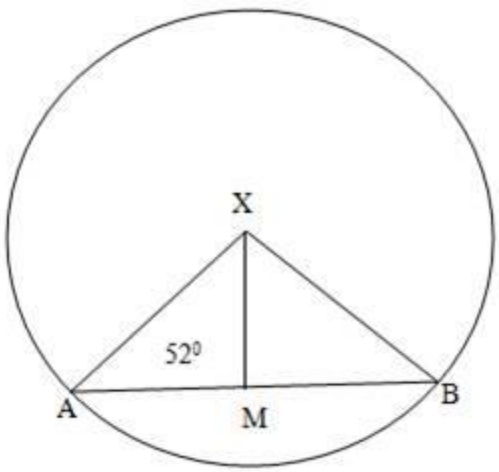
$$b = \sqrt{21}cm$$

∴ The distance between two chords is $3cm+\sqrt{21}cm$

Example:

1. AB is a chord of circle with centre X. the midpoint of AB is m.

If XAB find MXA



XAM + MXA = 180^{O}

$52^{o} + 90^{o}$ + MXA = 180^{o}

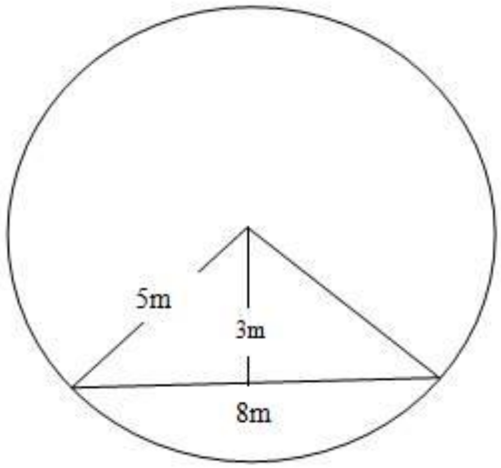
MXA = $180^{o} - 142^{0}$

= 38^{o}

2. AB is a chord in a circle with centre C. the length of AB is 8cm and the

Radius of the circle is 5cm. find,

The shortest distance of AB from C
ACB



By applying Pythagoras theorem.

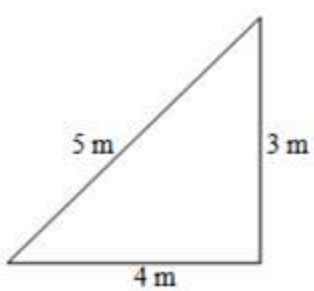
$4^2 + mc^2 = 5^2$

$Mc^2 = 25 - 16$

$Mc^2 = 16$

Square root both sides

$Mc = 4m$



Using,

$\frac{SOTOCA}{HAH}$

$\text{Sin } ACM = \frac{OPP}{HYP}$

Sin ACM = 0.8000

ACM = Sin^{-1} ((0.8)

= 53^{0} X 2

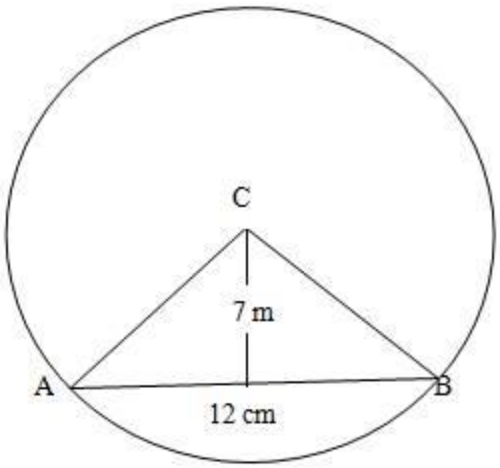
= 106^{0}

A chord AB has length 12m. it is 7m from the centre of the circle.

Find the (a) length of AC

(b)ACB

Soln:



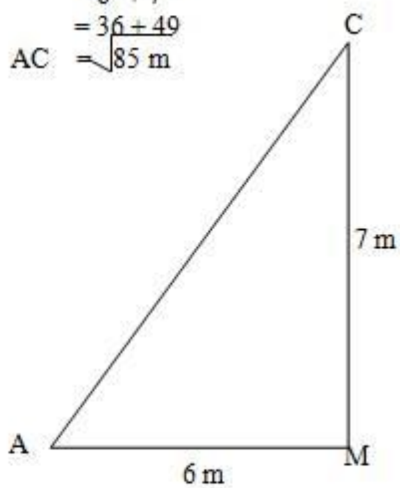
By using Pythagoras theorem.

$AC^2 = AM^2 + CM^2$

$= 6^2 + 7^2$

$= 36 + 49$

$AC = \sqrt{85}$ m



Using:

$\text{Tan ACM} = \frac{opp}{Adj}$

$\text{Tan ACM} = \frac{7}{6}$

Tan ACM = 0.8571

ACM = Tan^{-1}

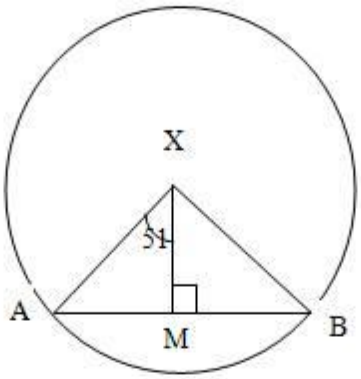
= 40^o

∴ACB =40^o x 2

= 80^o

EXERCISE

1. M is the Centre at the chord AB at a Circle with centre X if $< AXM = 51° \; Find < XAM$



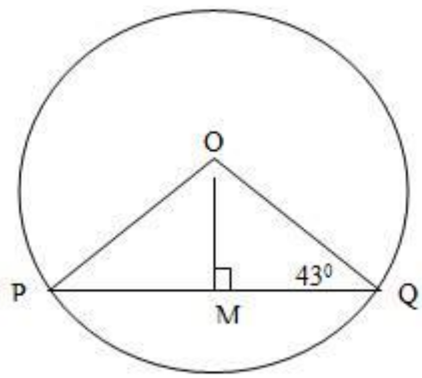
$< AXM + < XAM + AMX = 180°$

$81° + < XAM + 90° = 180°$

$< XAM = 180° - 141°$

$= 39°$

2. M is the centre at chord PQ at a circle with Centre O. if < PQO = 43°. Find



$< MOQ + < OQM + OMQ = 180°$

$< MOQ + 43° + 90° = 180°$

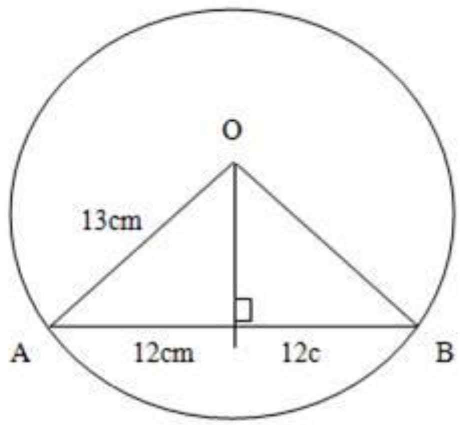
$< MOQ = 180° - 133°$

$= 47°$

3. A circle has radius 13cm and centre X. a chord AB has length 24cm. find:-

(a) T distance of the chord from the Centre

(b) < AXB



By Pythagoras theorem

$(AM)^2 + (MX)^2 = (AX)^2$

$12^2 + (MX)^2 = 132$

$144 + (MX)^2 = 169$

$(MX) = 169 - 144$

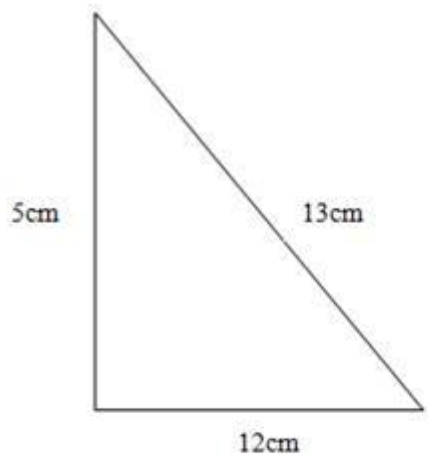
$(MX)^2 = 25$

Square root both sides

$(MX)^2 = 25$

MX = 5cm

The distance from the centre to chord is 5cm.



Using $\frac{SOTOCA}{HAH}$

$\sin\theta = \frac{Oposite}{Hypotenus}$

$= \frac{12}{13}$

$\sin\theta = 0.9230$

$\theta = \sin^{-1}(0.9230)$

$= 67°$

$< AXB = (< MXB)2$

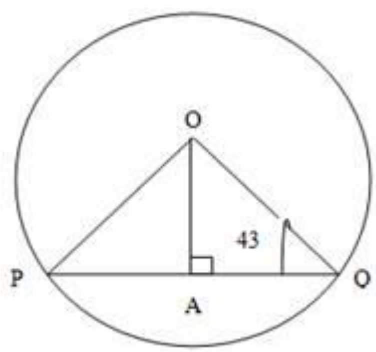
$= (67°)2$

$< AXB = 134°$

QUESTIONS:

1. Let A be the Centre at a chord PQ at a circle with Centre O. If < PQO = 43°, find < POQ

Soln



$< AOQ + < OAQ + < AQO = 180°$

$< AOQ + 90° + 43° = 180°$

$< AOQ + 133° = 180°$

$< AOQ = 47°$

$But \ < pOQ = (< AOQ)2$

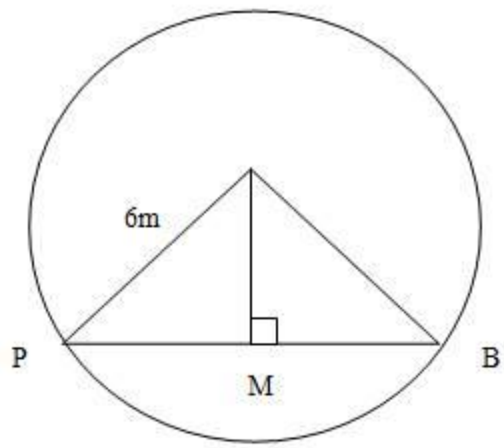
$47° x2$

$= 94°$

1. Q is the centre at a Circle and AB is a Chord

(a) The length at AB

(b) The distance at A from C



Soln:

Using $\frac{SOTOCA}{HAH}$

$\sin\theta = \frac{Adjacent}{Hypotenus}$

$\cos\theta = \frac{x}{6m}$

AM (Adjacent) $= 6\cos 50°$

$= 3.8568$

$AB = (AM)2$

$= 3.8568x2$

The length of AB = 7.77136m

Distance of AB from C

$\sin\theta = \frac{Opposite}{6m}$

$Opposite = 6\sin 50°$

$= 4.596$

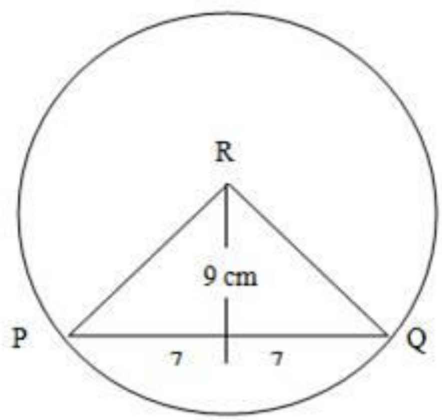
---

3. PQ is a Chord in a Circle with centre R. PQ = 14cm and the distance at R from PQ is 9cm, find:-

(a) The radius at a circle

(b) <PRQ

Soln:



Let RQ = Radius

By using Pythagoras theorem

$\overline{RQ^2} + \overline{RM^2} + \overline{QM^2}$

$\overline{RQ^2} = 9^2 + 7^2$

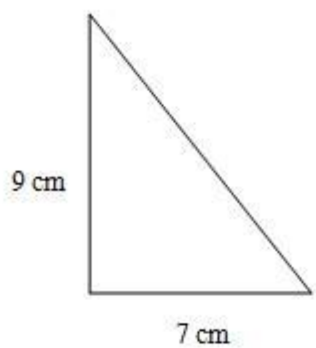
$\overline{RQ^2} = 130cm$

$RQ = \sqrt{130cm}$

$Radius = \sqrt{130cm}$

Using, SO TO CA

H A H



$\tan\theta = \frac{9}{7}$

$\theta = \tan^{-1}(0.777)$

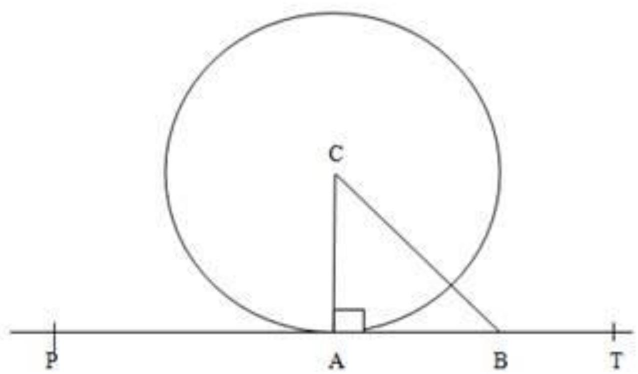
$< PBM = 37°$

TANGENT PROPERTIES

A tangent to a circle touches it at exactly one point

THEOREM:

A tangent to a circle the line perpendicular to the radius at the point of contact.



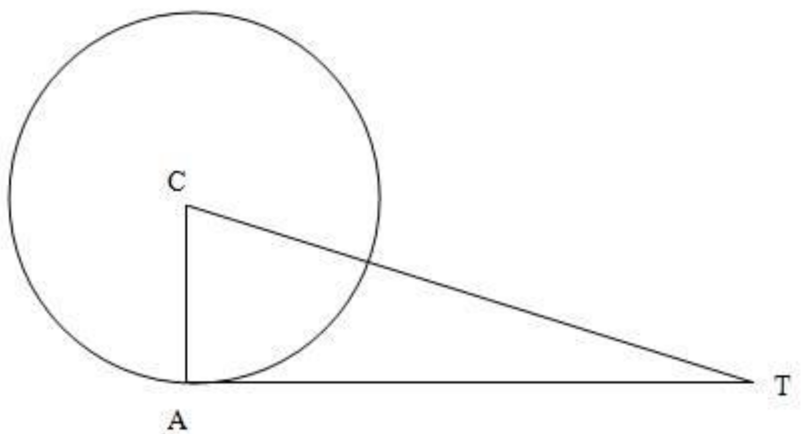
TAP is a line perpendicular to the radius CA show that TAP is a tangent as follows:-

If b is another point on TAP then CB is the hypotenuse at A CAB and hence CB is longer than CA it follows that B lies outside the Circle. Hence TAP needs the Circle only at A. TAP is a tangent to the Circle.

Hence a tangent is perpendicular to the radius.

Examples:

1. TA is a tangent to the Circle with centre C. If $< TCA = 49° \; Find \; < ATC$



Line AT⊥AC = 90°

<CAT+<ACT +<ATC=180°

90° + 49° + <ATC =180°

139° + <ATC =180°

<ATC=180° - 139°

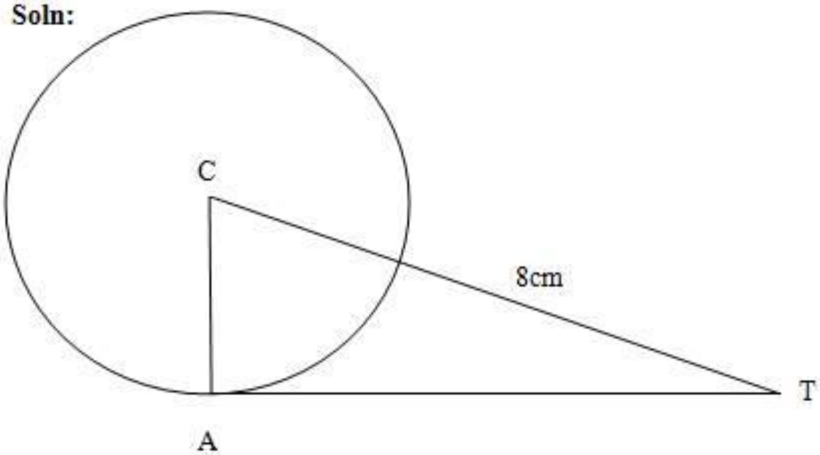
∴ <ATC = 41°

2. A point T is 8cm from the centre C of a circle of radius 5cm. Find

(a) The length of the tangent from T to the circle

(b) The angle between the tangent and TC

Soln:



By using Pythagoras theorem

$(AC)^2 + (AT)^2 = (TC)^2$

$5^2 + (AT)^2 = 8^2$

$25 + (AT)^2 = 64$

$AT^2 = 64 - 25$

$(AT)^2 = 39$

AT = 6.24

$$Sin < ATC = \frac{5}{8}$$

$$< ATC = \sin^{-1}\frac{5}{8}$$

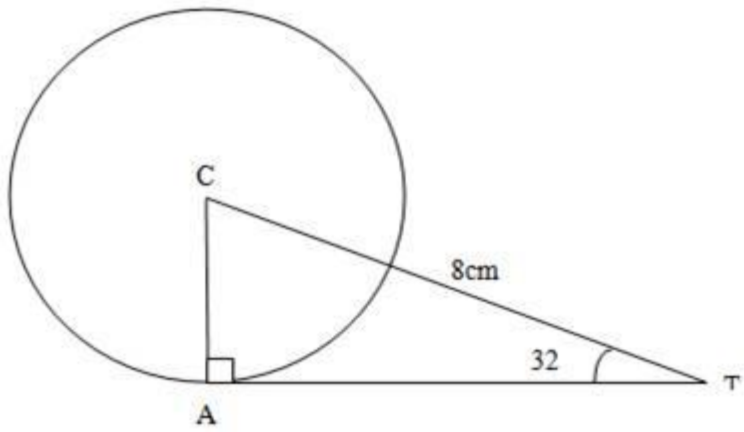
$=39^0$

QUESTIONS:

1. TA is a tangent to the circle at A. the centre is C. if <CTA=32^0

(a) Find <ACT

(b) If TC = 8cm, Find AT and radius of the circle



Solution: (a)

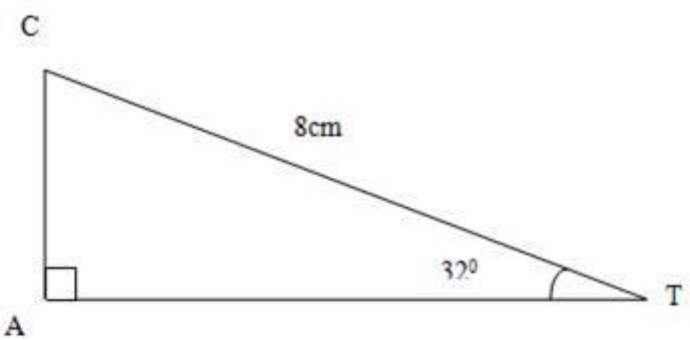
Line AT ⊥ AC = 90^o

<ACT +=180^o

<ACT +90^o + 32^0 =180^o

<ACT= 180^o– 122^0

<ACT= 58^o



Solution: (b)

Using, $\frac{SO}{H}\ \frac{TO}{A}\ \frac{CA}{H}$

$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\sin 32° = \frac{Opposite}{8}$$

= 8 (0.5299)

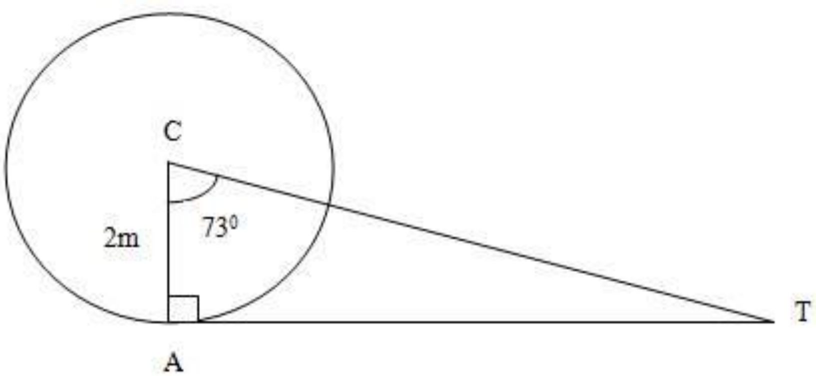
AT = 4.2392cm

2.TA is a tangent to a circle at A. The centre at the circle C if angle ACT = 73°and radius of the circle is 2m. find:-

(a) <ATC

(b) TA and TC

Soln: (a)



Line TA ⊥AC = 90°

<ATC+<ACT +<CAT=180°

90° + 73° + <ATC =180°

$<ATC + 163^o = 180^o$

$<ATC = 180^o - 163^o$

$<ATC = 17^o$

Solution: **(b)**

(i)

$$\text{Using}, \frac{\text{SO TO CA}}{\text{H A H}}$$

$$\text{Using} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 73° = \frac{\text{opposite}}{2\text{m}}$$

Opp = 2 Tan 73^o

=2(3.2709)

TA = 6.5418m

(ii) By using Pythagoras theorem

TA = 6.5418m

≈7m

$C^2 = a^2 + b^2$

$= 2^2 + 7^2$

$= 4+49$

$c^2= 53$

Square root both sides

$C^2 = 53$

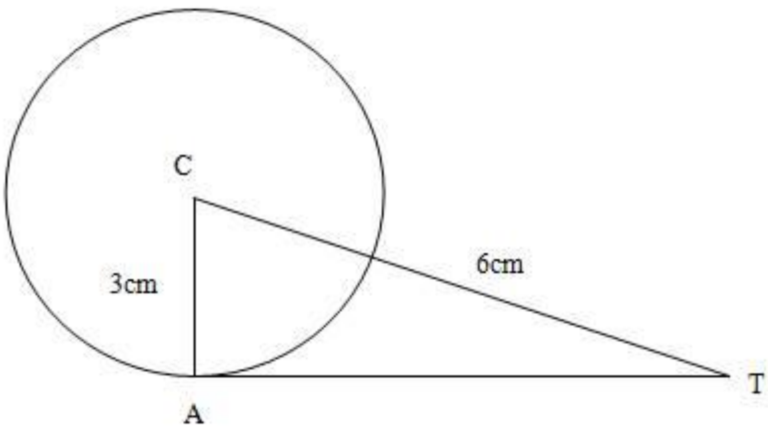
$$TC = \sqrt{53m}$$

3. A point T is 6m form the center C of a circle radius 3cm. Find:-

(a)The length of Tangent from T to the circle

(b)The angle between the tangent and TC

Solution: (a)



Using Pythagoras theorem

$A^2 + b^2 =C^2$

$3^2 + b^2 = 6^2$

$9 + b^2 = 36$

$b^2 = 36 - 9$

$b^2 = 27$

Square root both sides

$b^2 = 27$

$$b = 3\sqrt{3}\text{m}$$

$\therefore$ the length of tangent from T is $3\sqrt{3}\text{m}$.

Solution: **(b)**

Using , $\frac{\text{SO TO CA}}{\text{H A H}}$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{3}{6}$$

Cos = 0.5

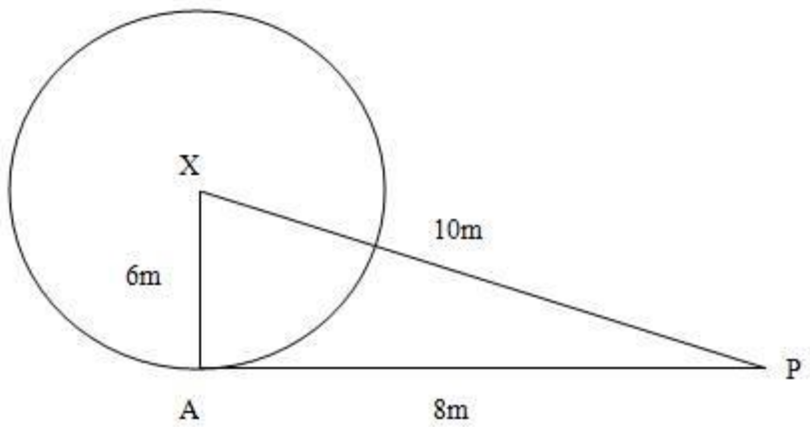
$= \cos^{-1}(0.5)$

$= 30^0$

Angel between tangent and TC is 30^0

Class Activity

1.A point, 10m from the center X of circle at radius 6m. A tangent is drawn from P to the circle touching at A. Find the length of the tangent from P to the circle.



By using Pythagoras theorem

$C^2 = a^2 + b^2$

$10^2 = 6^2 + b^2$

$100 = 36 + b^2$

$b^2 = 100 - 36$

$b^2 = 64$

Square root both sides

$b^2 = 64$

b = 8m

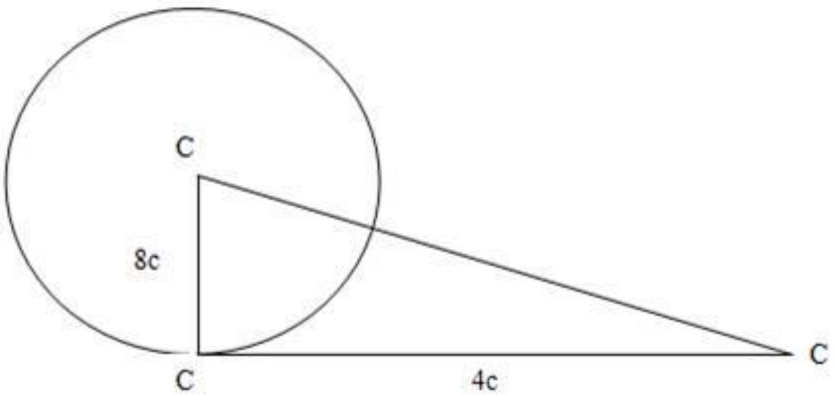
∴The length of the tangent from P to the circle is 8m

2.A tangent is drawn from T to a circle of radius 8cm. The length of the tangent is 4cm. Find,

(a) The distance of T from the Centre C of the circle

(b) The angle between TC and the tangent

Soln:



(a) By using Pythagoras theorem

$C^2 = a^2 + b^2$

$= 8^2 + 4^2$

$= 64 + 16$

$= 80$

Square root both sides

$C^2 = 80$

$\therefore C = 4\sqrt{5}m$

$$\text{Using}, \frac{\text{SO TO CA}}{\text{H A H}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{8}{4}$$

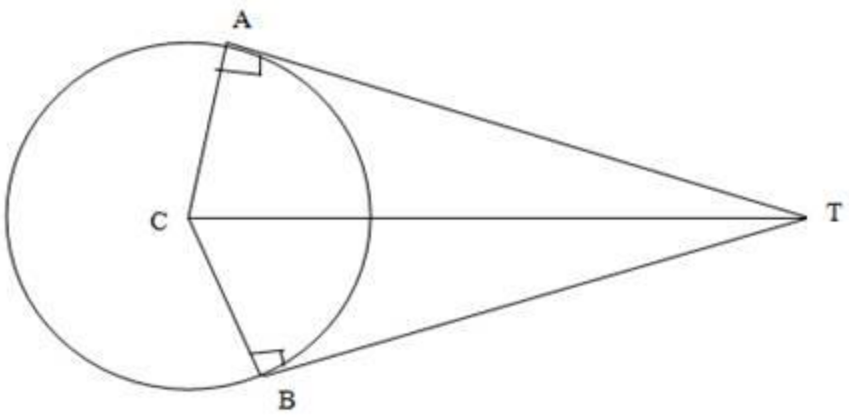
$\theta = \tan^{-1}(2)$

$= 6^{o}$

the angle between TC and tangent is 63^{o}

TANGENT FROM A POINT

Suppose T is outside a circle there are two tangent from T to the circle and they are equal in length



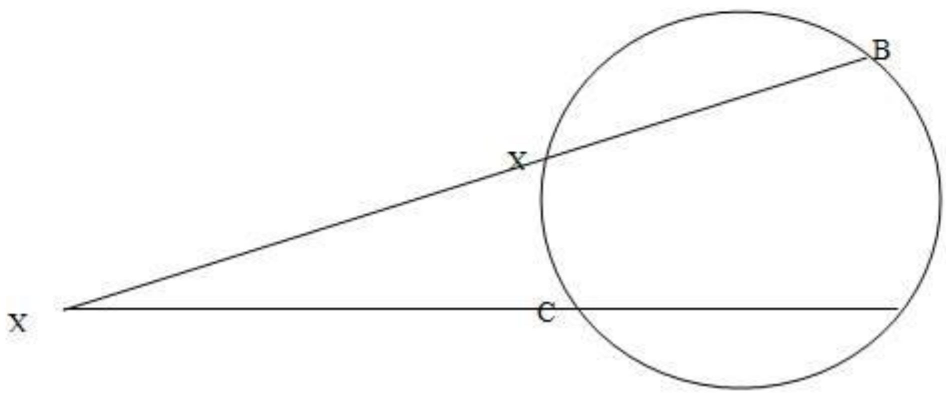
Proof:

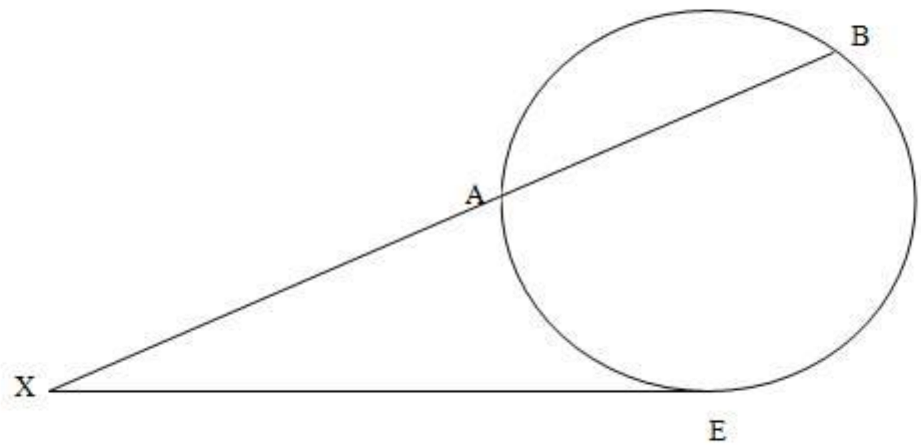
Consider the angle TCA and TCB

CA = CB (Both are radii)
TC = TC (Common)
<TAC=<TBC

CHORD AND TANGENT

Suppose the chord CD gets shorter and shorter is that C and D approach a common point E then the chord CD becomes the tangent at E, by interesting chord theorem.





XA x XB = AC x XD

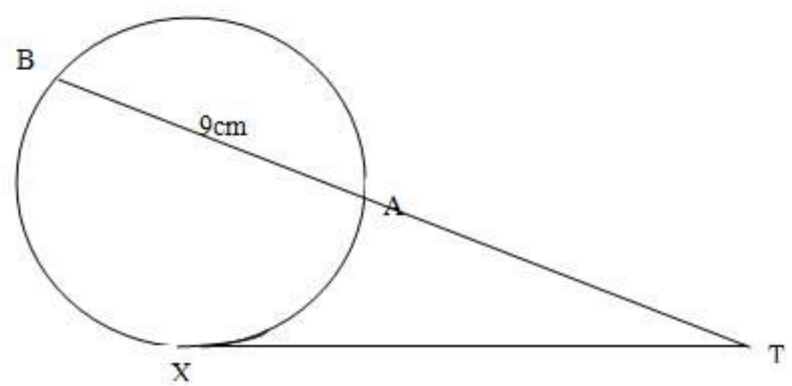
XA x XB = XE x XE

XA x XB = $(XE)^2$

Example

1.TX is a tangent to a circle. The line TAB cuts the circle at A and B with TA = 3cm and AB = 9cm. Find TX

Solution:



(TA) (TB) = TX x TX

(TA) (TB) = $(TX)^2$

Since, TB = (TA) + (AB)

= 3cm + 9cm

= 12cm

From the theorem

(TA) (TB) = $(TX)^2$

3cm x 12cm = $(TX)^2$

Square root both sides

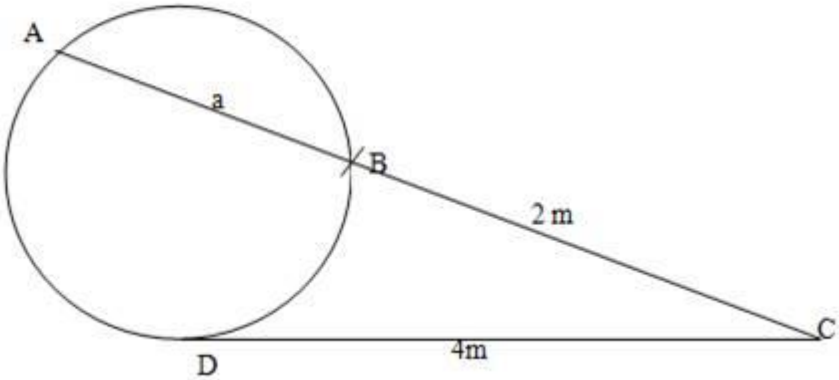
$36cm^2 = (TX)^2$

TX = ± 6cm

Since there is no -ve dimension therefore TX is 6cm

More Examples:

Find the length of unknown in the diagram.



Solution:

(CB) (CA) = (DC) (DC)

(CB) (CA) = $(DC)^2$

Since CA = (CB) + (AB)

= 2m + a

From the theorem.

(CB) (CA) = (DC)2

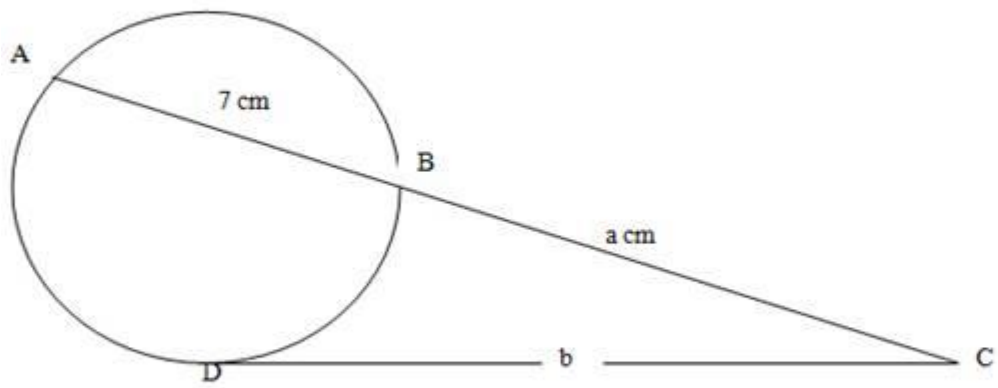
(2m) (2m +a) = $(4m)^2$

$4m^2 + 2ma = 16m^2$

$2ma = 16m^2 - 4m^2$

2ma = $12m^2$

a = 6m



Solution:

(CB) (CA) = $(CD)^2$

Since CA = (CB) + (AB)

= 9m + 7m

= 16

From the theorem

(CB) (CA) = $(CD)^2$

(9m) (16m) = $(b)^2$

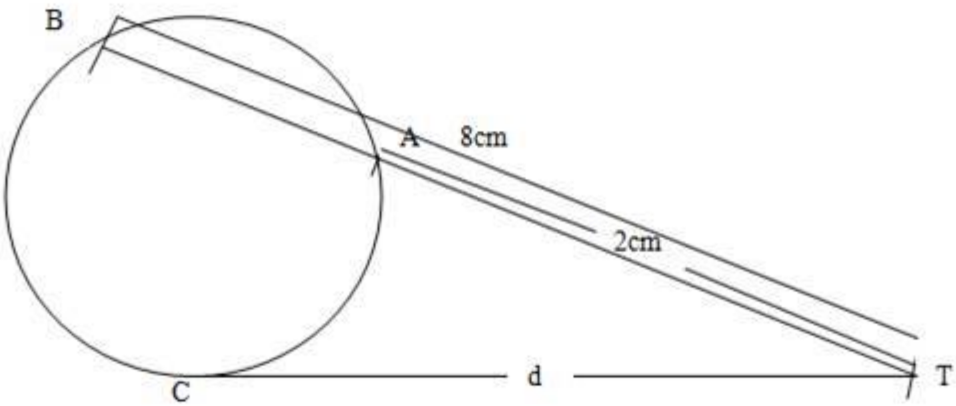
Square root both sides

$144m^2 = b^2$

b = ±12m

Since there is no -ve dimension **b= 12m**

2.TC is a tangent to a circle and Tab cuts at AB and B. if TA = 2cm and TB = 8cm, find TC



Soln:

From the theorem

(TA) (TB) = $(TC)^2$

2cm x 8cm = $(TC)^2$

$16cm^2 = (TC)^2$

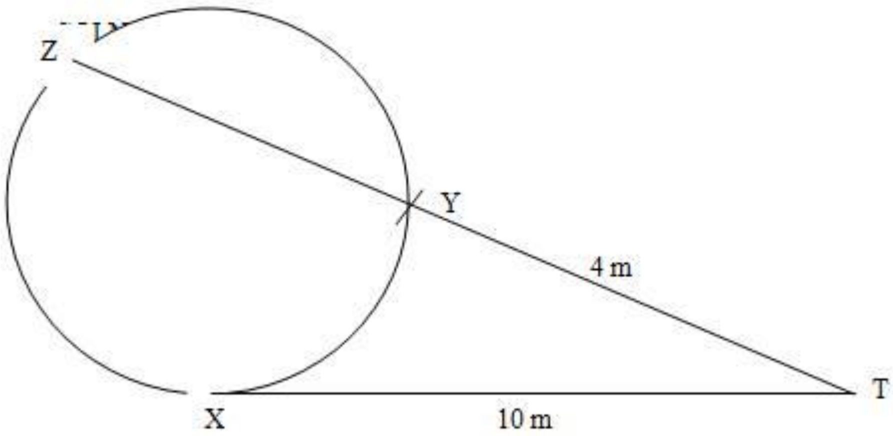
Square root both sides

$16cm^2 = (TC)^2$

TC = 4cm

3. TX is a tangent to a circle and TYZ cuts the circle and Y and Z. if TX= 10m and TY = 4m. Find TZ

Solution:



$(TY)(TZ) = (TX)^2$

Let $ZY = y$

Since $TZ = TY + ZY$

$= 4m + y$

From the theorem

$(TY)(TZ) = (TX)^2$

$(4m)(4m + y) = (10m)^2$

$16m^2 + 4my = 100m^2$

$4my = 100m^2 - 16m^2$

$4my = 84m^2$

$Y = 21m$

Since $ZY = y$

$TZ = TY + ZY$

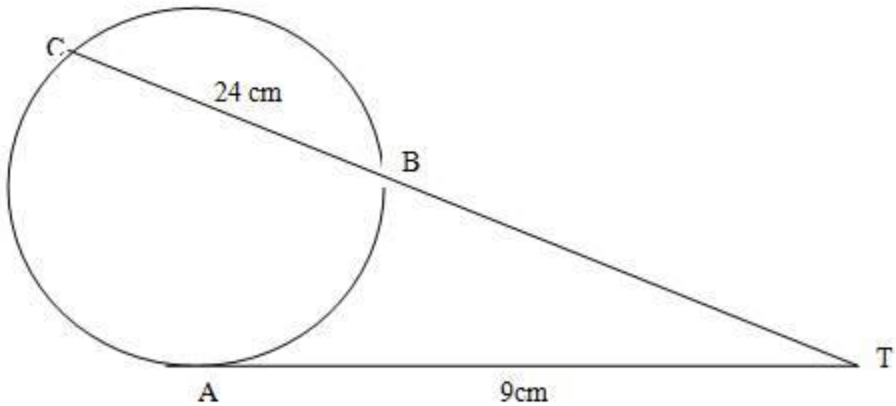
$= 4m + 21m$

$= 25m$

Class Activity

1.TA is a tangent to a circle and TBC meets the circle at B and C. TA = (9cm and BC = 24cm). Find TB

Solution:



$(TB)(TC) = (TA)^2$

Let $(TB) = y$

Since $TC = TB + CB$

$= y + 24cm$

From the theorem

$(TB)(TC) = (TA)^2$

$(y)(y + 24cm) = (9cm)^2$

$Y^2 + 24cmy = 81cm^2$

$Y^2 + 24cmy - 81cm^2 = 0$

By using General formula

Where a = 1, b = 24, c = 81

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-24 \pm \sqrt{24^2 - 4\,(1)(-81)}}{2(1)}$$

$$y = \frac{-24 \pm \sqrt{576 + 324}}{2}$$

$$y = \frac{-24 \pm \sqrt{900}}{2}$$

$$y = \frac{-24 \pm 30}{2}$$

$$y = \frac{-24 + 30}{2}$$

$$y = 3\text{cm}$$

Or

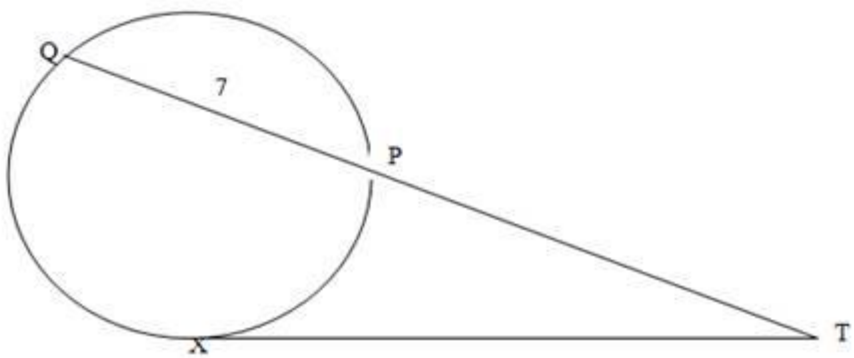
$$x = \frac{-24 - 30}{2}$$

$$y = -27\text{cm}$$

Since there is no negative dimension, the length at TB is 3cm

2.TX is a tangent to a circle and TPQ meets the circle at P and Q. TX = 12cm and PQ = 7cm, find TP

Solution:



$(TP)(TQ) = (TX)^2$

Let $(TP) = Z$

Since $TQ = TP + QP$

$= Z + 7cm$

From the theorem

$(TP)(TQ) = (TX)^2$

$(z)(z + 7cm) = (12cm)^2$

$z^2 + 7z = 144$

$z^2 + 7z - 144 = 0$

By using the general formula

Where $a = 1$, $b = 7$ and $c = -144$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-7 \pm \sqrt{7^2 - 4\,(1)(-144)}}{2(1)}$$

$$y = \frac{-7 \pm \sqrt{49 + 576}}{2}$$

$$y = \frac{-7 \pm \sqrt{625}}{2}$$

$$y = \frac{-7 \pm 25}{2}$$

$$y = \frac{-7 + 25}{2}$$

$$y = 9\text{cm}$$

Or

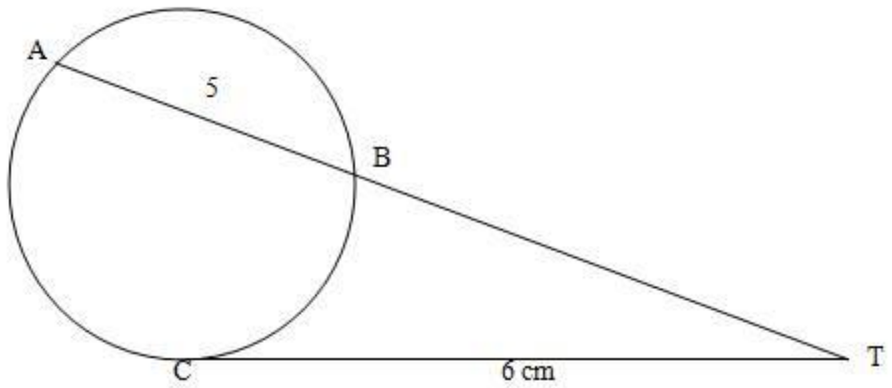
$$x = \frac{-7 - 25}{2}$$

$$y = -16\text{cm}$$

Since there is no negative dimension, the length at TP is 9cm

3.AB is a chord at a circle at length 5cm. C is another point on the circle. AB extended on the circle meets the tangents at C and T. if the TC = 6cm, find the possible value of TB.

Solution:



(TB) (TA) = $(TC)^2$

Let

(TB) = x

Since TA = TB + AB

= x + 5cm

From the theorem

(TB) (TA) = $(TC)^2$

(x) (x + 5) = $(6cm)^2$

$X^2 = 5x = 36$

$X^2 + 5x – 36 = 0$

By completing the square

$X^2 + 5x – 36 = 0$

$X^2 + 5x – 36 = 0$

$X^2 + 5x = 36$

Add $(\frac{1}{2} b)^2$ both sides

$X^2 + 5x + (½ x 5) = 36 + (½ x 5)^2$

$X^2 + (5/2)^2 = 36 + 25/4$

$(x + 5/2)^2 = 169/4$

Square root both sides

$(x + 5/2)^2 = ±169/4$

X + 5/2 = ± 13/2

X = -5/2 ± 13/2

= -5/2 + 13/2

= 4cm

Or

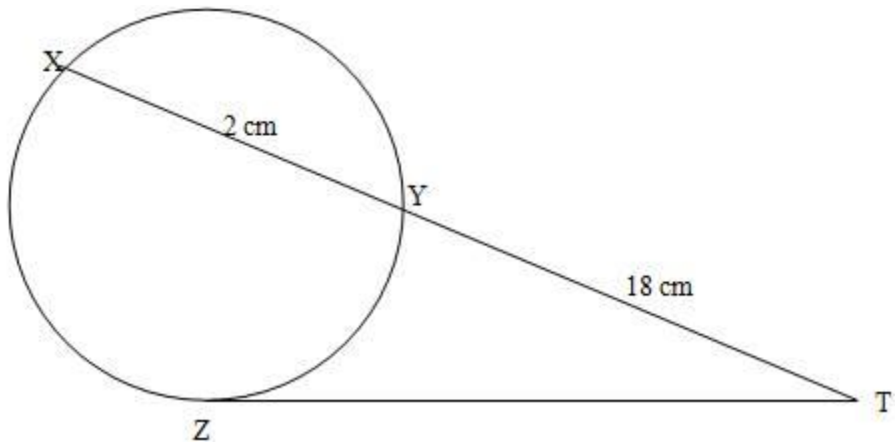
X = -5/2 – 13/2

= -9cm

Since there is no negative dimension, the length of TB is 4cm.

4. XY is a chord of a circle at length 2cm. z is another point on the circle. XY extended meets the length at z at T. if TX = 18cm, find the possible value of TZ

Solution:



$(TY)(TX) = (TZ)^2$

Since TX = TY + XY

18cm + 2cm= 20cm

From the theorem

$(TY)(TX) = (TZ)^2$

$(18cm)(20cm) = (TZ)^2$

$360cm^2 = (TZ)^2$ square root both sides

$(TZ)^2 = 360cm^2$

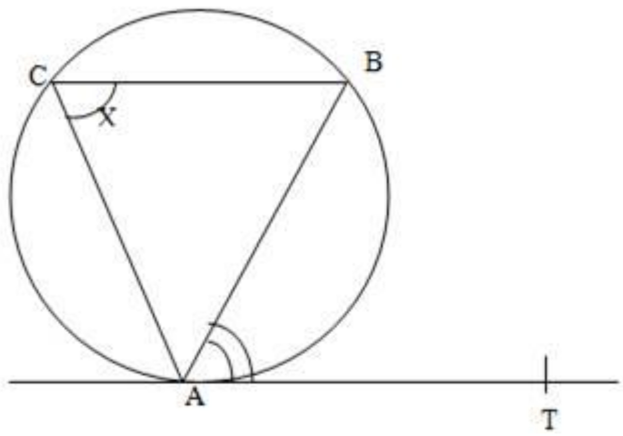
$$TZ = 2\sqrt{90cm}$$

ALTERNATE SEGMENT THEOREM

AT is a tangent to the circle and AB is a chord. The alternate segment theorem state that:-

THEOREM: The angle between the chord and tangent is equal to the angle in the alternate (others)

Segment i.e. < TAB = <ACB



Proof:

< TAB = <ACB

Aim: Is proving that <ACB= <BAT

Let ACB = X and Centre of a Circle to be

'O' AOB = 2x (< at the centre is twice the angle at the circumference).

AC,BC, and AB or chords in the Circle and AO = OB

∴Δ AOB is isosceles triangle since AO and OB are equal

<OAB=1/2(180° - 2x)

= 90° – x

<BAT+ <OAB =<OAT

<BAT=<OAT - <OAB

=90° - (90° - x)

$= 90^0 - 90^0 + x$

$= 0 + x$

$\therefore \angle BAT = x$

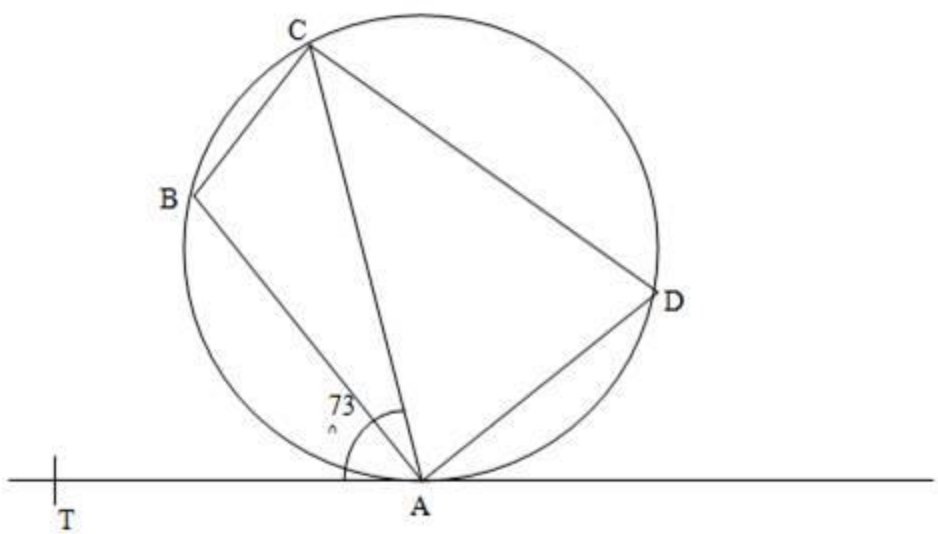
hence proved

$\therefore \angle ACB = \angle BAT$

$= x$

QUESTIONS:

1.ABCD is a cyclic quadrilateral TA is the tangent to the Circle at A. if <TAC = 73⁰, find <ABC



Solution:

TAC = 73^0

Angle at the same segment equal

TAC = ADC

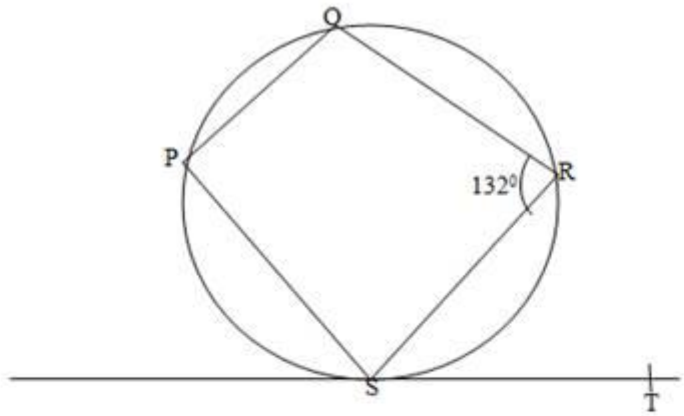
<ADC + 73^o = 180^o

Take out 73^o both sides

<ABC = $180^o – 73^o$

<ABC=107^o

2. PQRS is a cyclic quadrilateral. ST is the tangent to the Circle at S. if <QRS = 132^o.Find <RST



Soln:

<QRS +<RST =180^o

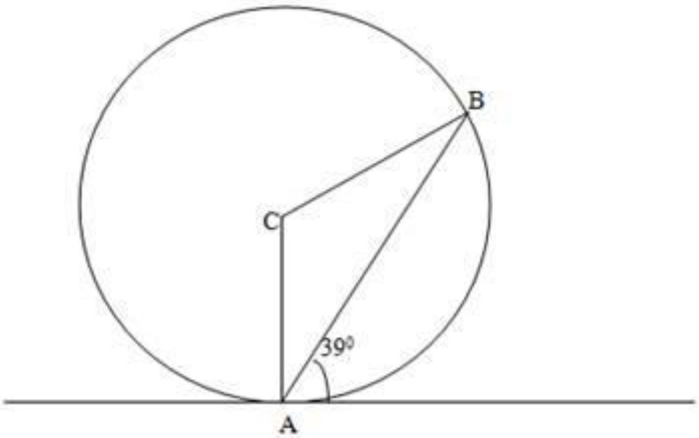
132^o +132^o=180^o

Take out 132^o both sides

<RST =180^o-132^o

=48^o

3. C is the centre of the Circle. If <BAT=39^o. find <ACB.



Soln:

Angle at the Centre is twice the angle at circumstances

<ACB = <BAT

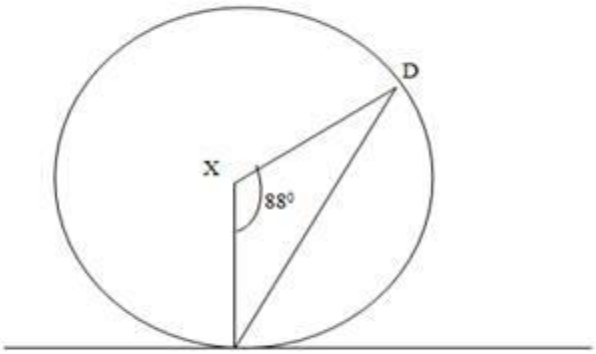
39^0=<BAT

<ACB x 2=< ABC

< ABC=39^0 x 2

=78^0

4.X is the Centre of the Circle if<CXD =88^0, find <TCD



Soln:

<DXC=<DCT/2

Angle at the Centre is twice the angle at the circumstances.

∴<TCD=88^0/2

$= 44^o$

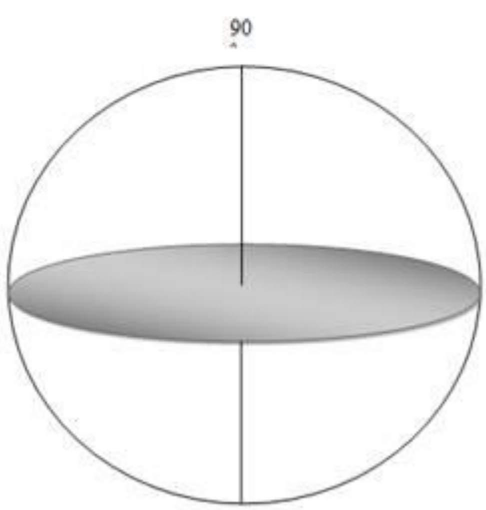
THE EARTH AS A SPHERE

SPHERE

Is a set of a point which equidistance (equal distance) from the fixed point called the centre of the Sphere.

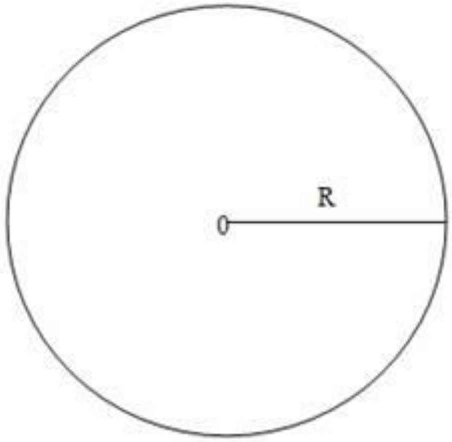
-The distance from the centre of the sphere to any point at the circumference of the sphere called Radius at the earth which is approximately as 6370km.

-The surface of the earth is not exactly spherical because it is flattened in its northern and southern pole or we say. The earth is not perfect sphere, as it is slightly flatter at the north and southern pole than at the equator. But for most purpose we assume that it is a sphere

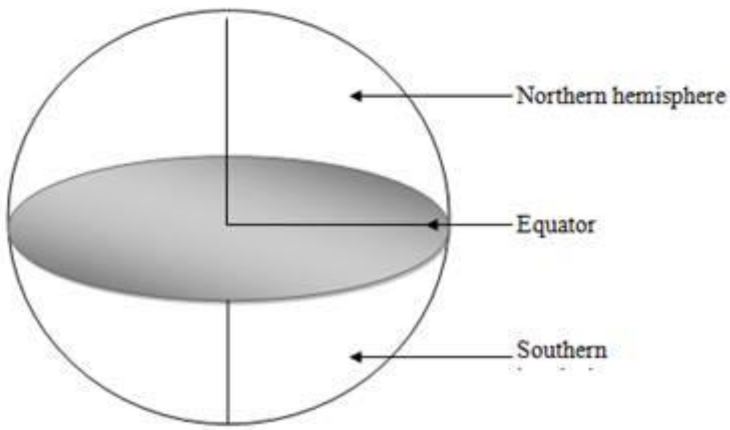


THE EARTH AS PERFECT SPHERE

We consider the earth to be a perfect sphere of radius 6370km at approximately 6400km.



-Whereby O is the center of the earth.
-R is the radius of the earth
-The earth rotates once a day about a line called polar axis (earth axis)
-This axis basses through the center and joins the northern and southern poles



The equator has 0°

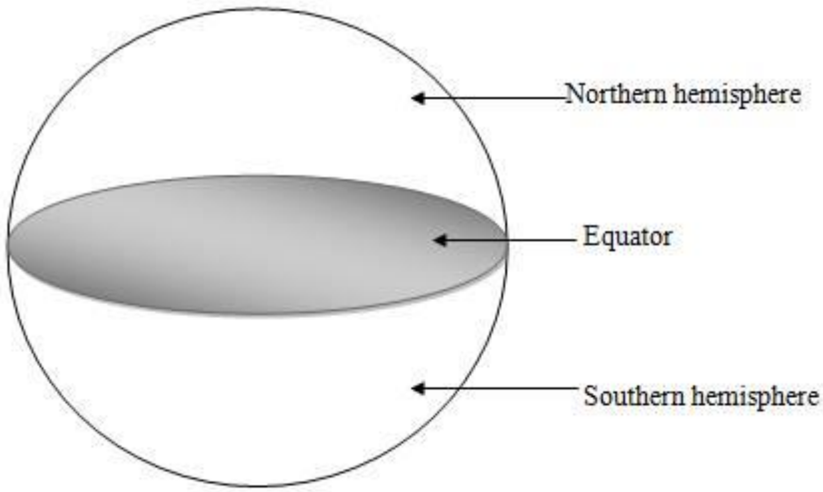
DEFINITIONS OF TERMS

I. **GREAT CIRCLE (EQUATOR)**

Is the line which drawn from west to east with 0°. Or

Is an imaginary line which divides the earth surface into two equal parts: Southern part and Northern part through the Centre of the earth called hemisphere.

The equator is the only Great Circle perpendicular to the earth axis



-Earth point on the earth's surface is said to be either in northern hemisphere or southern hemisphere.

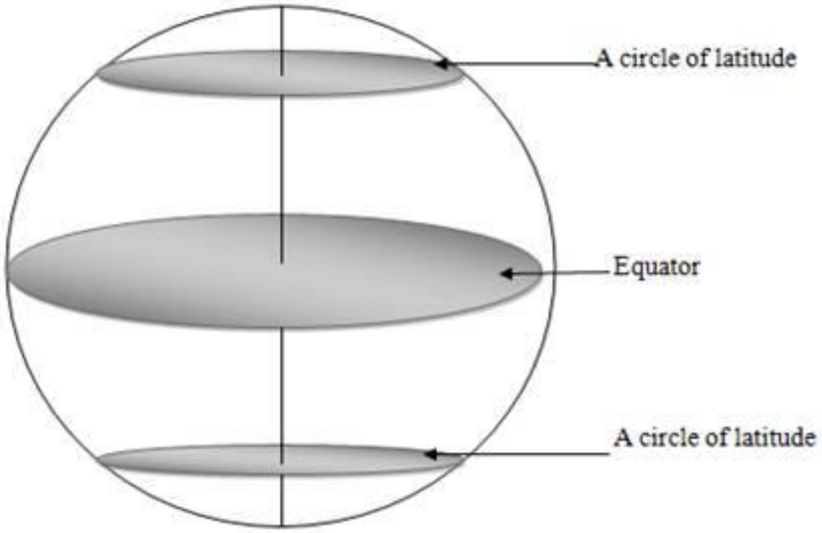
II. SMALL CIRCLE (LATITUDE)

Is the line drawn from West to east and measure in degree from the centre at the Earth (Equator) Northward or Southward.

- Latitude range from 0^oN or 90^oS

- The radius of parallel at latitudes becomes smaller as one moves towards the southern or northern pole

Example of the line of latitude that should be drawn from west to east .



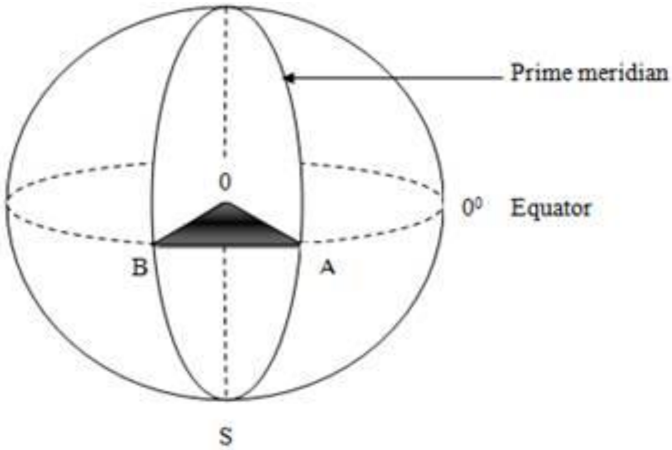
- The equator is the standard zero latitude from which other latitude are measured
- Any other line of latitude is named by the longitude basses through when rotating from the Equator to the line of latitude this angle is either north or south of the equator.

When naming a latitude it is essential to say whether it is north or south of the Equator.

III. **MERIDIAN (LONGTUDE)**

- These are lines drawn from north to south measured degree from the prime meridian westward or eastward.
- These circles are not parallel as they meet at the poles. These circles have radius equal to that of the earth and they are called great circle.
- Longitudes are also called meridian.

Diagram:



- In order to name lines at longitudes it has been necessary to choose a standard zero called the prime meridian.

This is a line at longitude which passes through Greenwich, London.

IV.**THE GREENWICH MERIDIAN**

-Greenwich meridian: Is a longitude whose degree measure is zero (0^0)

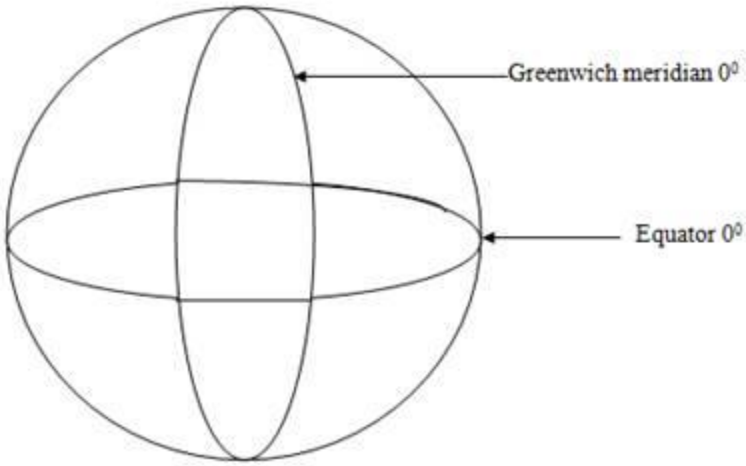
-Greenwich meridian is also considered as prime meridian

-Greenwich meridian is standard longitude in which other meridian are measured in degree from East for West.

- The Greenwich divides the earth's into two parts eastern part and western part

- Each point on the earth surface is said to be either on the eastern part or western part.

Diagram:

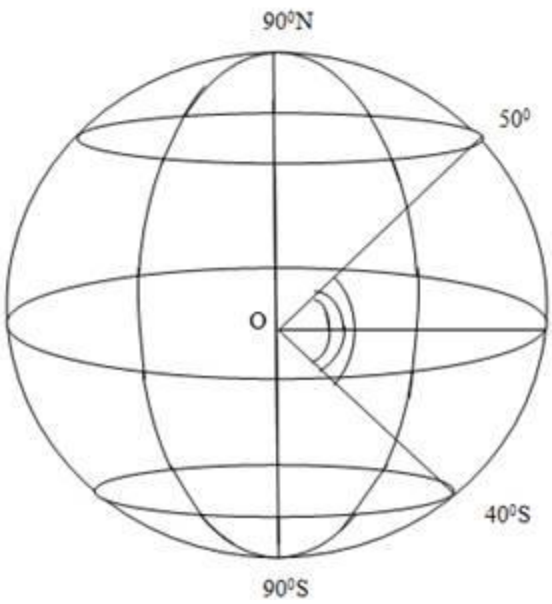


- The longitude of a place varies from 0° along the Greenwich meridian to 180°E or 180°W. The Radius of all longitude are equal to that of the earth.

LOCATION OF POINTS ON THE EARTH SURFACE

1.O is the centre of the earth. The latitude of P is 50°N and of Q is 40°S

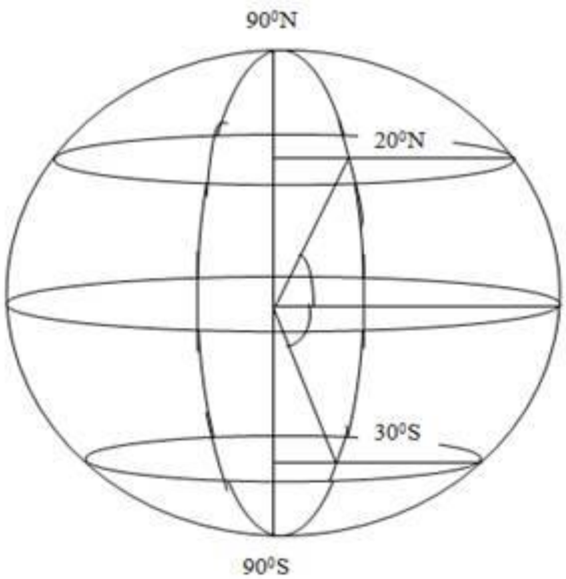
Solution:



Since the given point has been allocated into different hemisphere therefore the angle subtended by an arc PQ = $50^0 + 40^0 = 90^0$

2. Two towns are on the same circle of longitude. One town is 20^oN and other is 30^oS. What is the angle subtended by an arc of these two angles.

Solution:



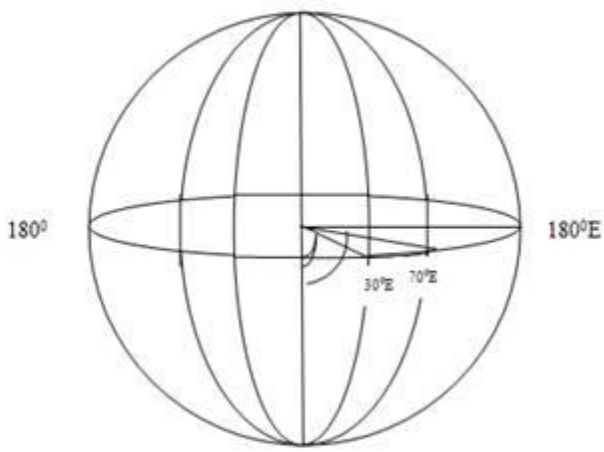
Since the two towns has been found into different hemisphere the angle subtended by an arc of the two angle are

$20^0 + 30^0 = 50^0$

3. Two towns C and D line on the equator. The longitude of C is 70^oE and for D is 30^0 E.

What is the angle subtended by an arc.

Solution:

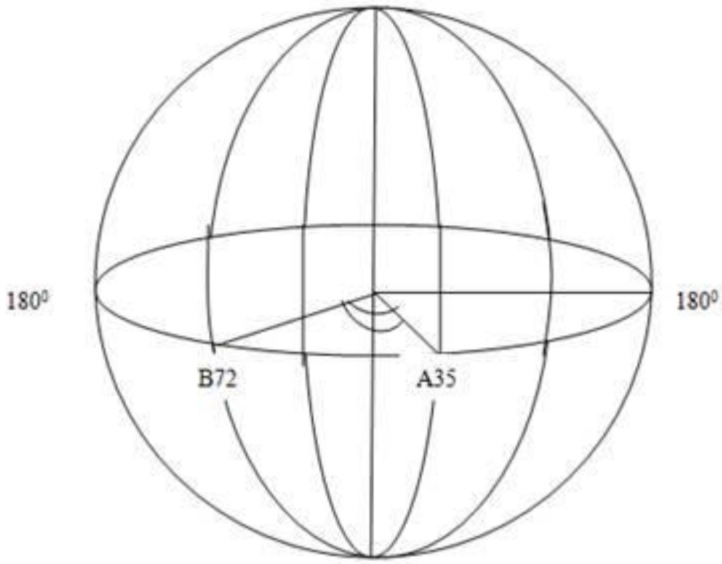


Since C and D are in the same hemisphere the angle subtended by an arc CD = 70⁰ – 30⁰

CD= 40⁰

4. Two towns A and B are on the equator.

The longitude of A is 35⁰E and the B = 72⁰W. Find the angle subtended by an arc AB



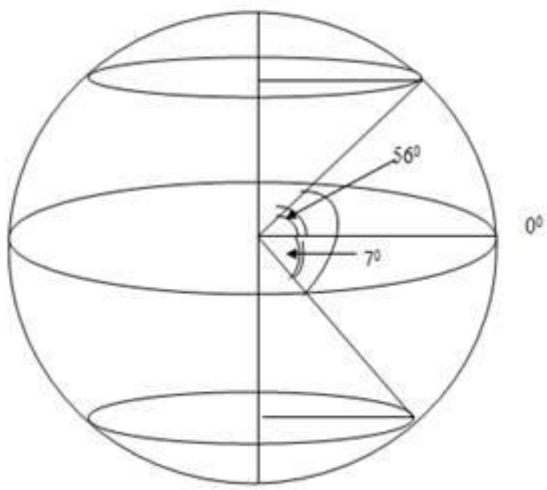
Since the point given has been found in the different hemisphere the angle subtended by an arc AB = $72^0 + 35^0$

$= 107^0$

HOME WORK

1.Given that Morogoro is (7^oS, 38^oE) and Moscow is (56^oN, 38^oE). Find the angle subtended by the area which connect the two places at the centre of earth.

Solution:

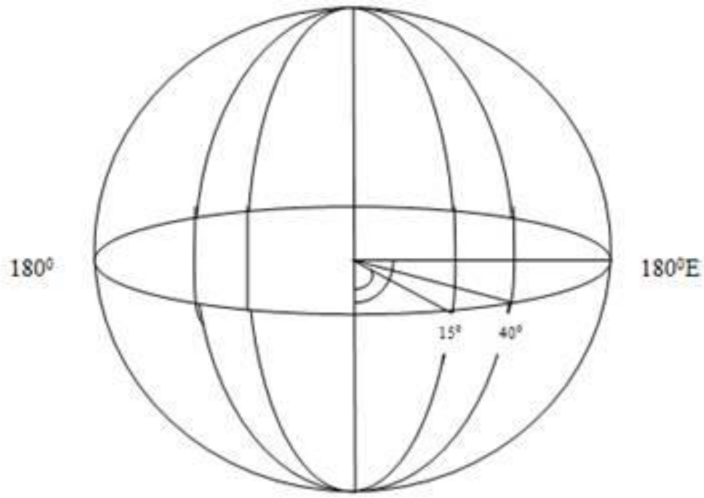


Since the given places have been alocated in the different hemisphere therefore subtended by an arc = $56^o + 7^o$

$= 63^o$

2. What is the difference in longitude between Brazivile Congo (4^oS, 15^oE) and Mombasa Kenya(4^oS, 40^oE)

Solution:



Since the given places have been allocated in the same hemisphere therefore the different between the two places = $40^o - 15^o$

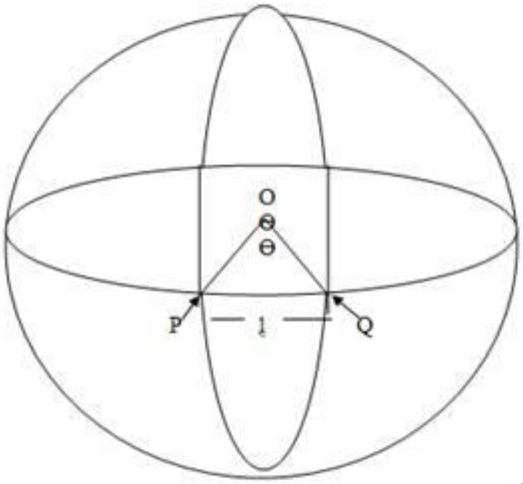
$= 25^o$

DISTANCE BETWEEN TWO PLACES MEASURED ALONG GREAT CIRCLE

Note: Great circles means either the equator or lines of longtudes.

Consider two points P and Q both found on the equator.

Diagram:



O is the centre of the earth OP = OQ = Radius of the earth angle POQ is the central angle line PO = 'l" is the length at arc PQ on the equator. θ is the difference in longitudes between points P and point Q.

Remember, If P and Q are on the same hemisphere. θ will be found by subtracting their respective longitudes and if P and Q are in different hemisphere, the value of θ will be obtained by taking their sum of the respective longtude.

θ = l

$360^o = 2\pi R$

$$l = \frac{\theta}{360°} \times 2\pi R$$

$$\text{But } \theta = \frac{\alpha \pm \beta}{360°} \times 2\pi R$$

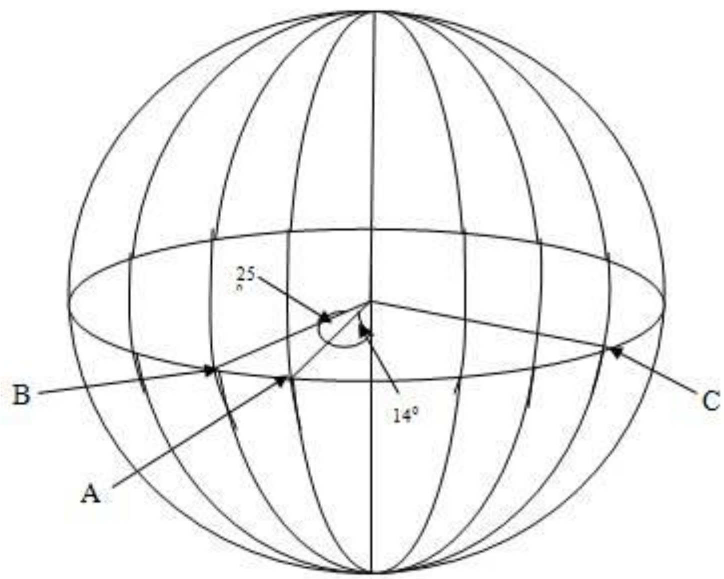
Whereby, π= 3.14

R = 6370km

Example:

Three points A(0°,14°W), B(0°,25°W) and C(0°, 46°E) are on the Earth's surface.

Calculate the length of the equator
(a)AB (b) AC (c) BC



$$\text{Length AB} = \frac{\alpha \pm \beta}{360°} \times 2\pi R$$

$$\text{Length AB} = \frac{25° - 14°}{360°} \times 2(3.14)6370\text{Km}$$

$$\text{Length AB} = \frac{11°}{360°} \times (6.28)6370\text{Km}$$

No	Log
1.1×10^1	1.0414
3.6×10^2	2.5563
	$\bar{2}.4851$
6.28×10	0.7980
6.37×10^3	3.8041 +
1.223×10^3	3.0872
1.223×1000	

∴The distance of points A and B is 1223km

(b)

$$\text{Length AC} = \frac{\alpha \pm \beta}{360°} \times 2\pi R$$

$$\text{Length AC} = \frac{14° + 46°}{360°} \times 2(3.14)6370\text{Km}$$

$$\text{Length AC} = \frac{60°}{360°} \times (6.28)6370\text{Km}$$

No	Log
6.0×10^1	1.7782
3.6×10^2	2.5563-
	$\bar{1}.2219$
6.28×10^0	0.7980
6.37×10^3	3.8041+
6.668×10^3	3 8240
6.668×1000	

∴The distance of point AC is 6668km

(c)

$$\text{Length BC} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$\text{Length BC} = \frac{25^\circ + 46^\circ}{360^\circ} \times 2(3.14)6370\text{Km}$$

$$\text{Length BC} = \frac{71^\circ}{360^\circ} \times (6.28)6370\text{Km}$$

No	Log
7.1×10^1	1.8513
$3.6. \times 10^2$	2.5563
	1.2950
6.28×10^0	0.7980
6.37×10^3	3.8041 +
7.891×10^3	3.8971
7.891 x 1000	
7891	

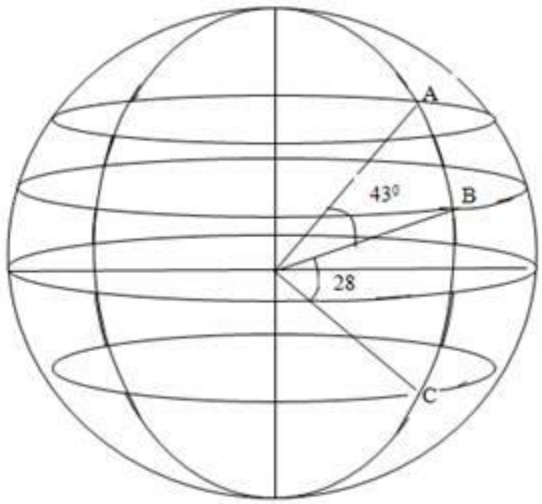
∴The distance of point BC is 7891

HOME WORK

1.Three points are such that A(43°N, 10°E), B(16°N, 10°E) and C(28°S, 10°E). calculates the lengths at the following arcs measured along the longitudes.

(a) AB (b) BC (c) AC

Solutions:



$$\text{Lenght AB} = \frac{\alpha \pm \beta}{360°} \times 2\pi R$$

$$= \frac{43° - 16°}{360°} \times 2(3.14)6370\text{Km}$$

$$= \frac{27°}{360°} \times 6.28 \times 6370\text{Km}$$

No	Log
2.7×10^1	1. 4314
3.6×10^2	2.5563 –
	2.8751
6.28×10^0	0.7980
6.37×10^3	3.8041+
3.000×10^3	3.4772
3.000×1000	= 3000

The distance at point AB is 3000km.

(b)

$$\text{Lenght BC} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$= \frac{28^\circ + 16^\circ}{360^\circ} \times 2(3.14)6370\text{Km}$$

$$= \frac{44^\circ}{360^\circ} \times 6.28 \times 6370\text{Km}$$

No	Log
4.4×10^1	1.6435
3.6×10^2	2.5563 –
	1.0872
6.28×10^0	0.7980
6.37×10^3	3.8041
6.156×10^3	3.7893
6.156×1000	
6156	

The length at point BC is 6156km.

(c)

$$\text{Lenght AC} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$= \frac{43^\circ + 28^\circ}{360^\circ} \times 2(3.14)6370\text{Km}$$

$$= \frac{71^\circ}{360^\circ} \times 6.28 \times 6370\text{Km}$$

No	Log
7.1×10^1	1.8513
3.6×10^2	2.5563
	$\bar{1}.2950$
6.28×10^0	0.7980
6.37×10^3	3.8041 +
7.891×10^3	3.8971
$7.891 \times 100^\circ$	
7891	

∴The distance of point AC is 7,891km.

Home Work

1.Two towns R and Q are 2813km apart R being direction of North of Q. If the latitude Q is 50S, find the Latitude of R.

Solution:-

Data

Distance RQ = 2813km

R = Required

Q = 5^oS

$$\text{Lenght RQ} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$2813\text{Km} = \frac{\alpha + 5^\circ}{360^\circ} \times 2(3.14)6370\text{Km}$$

$$\alpha + 5^\circ = \frac{2813 \times 360^\circ}{6.28 \times 6370}$$

	No	Log
Numerator:	2.813×10^3	3.4495
	3.6×10^2	2.5563 +
		6.0058
Denominator:	6.28×10^0	0.7980
	6.37×10^3	3.8041 +
		4.6021
Numerator:		6.0058
Denominator:		4.6021 –
	2533×10^1	1.4037

$$\alpha + 5^\circ = 25.33^\circ$$

$$\alpha + 5^\circ = 25.33^\circ$$

$$\alpha = 25.33^\circ - 5^\circ$$

$$\alpha = 20.33^\circ$$

Since R is due to North of Q,the latitude of R is 20.33°

Class Work

1.Calculate the distance between Tanga (5°, 39°E) and Ruvuma (12°S, 39°E) in;

(a) Nautical mile

(b) Kilometre

(a)Solution:

$(\alpha \pm \beta)\ 60^o = Nm$

$(12^o - 5^o)\ 60^o = Nm$

$7^o \times 60^o = Nm$

No	Log
7×10^0	0.8451
6×10^1	1.7782+
4201×10^2	2.6233

4.201 x 100

420.1 Nm

(b) 1Nm = 1.852km

420.1Nm =?

$$= \frac{420.1Nm \times 1.852km}{1Nm}$$

No	Log
4.201×10^2	2.6233
1.852×10^0	0.2677 +
7.780×10^2	2.8910

∴The distance between Tanga and Ruvuma is

(a) 420. 1Nm

(b) 778km

1.Find the distance between point x and y given that X(34^{o}N, 124^{o}E) and Y(41^{o}N,124^{o}E)

(a)In nearest Nautical mile

(b)In nearest Kilometers

Solutions:

$(\propto \pm \beta)\, 60^o = Nm$

$(41^o - 34^o)\, 60^o = Nm$

$7^o \times 60^o = Nm$

No	Log
7×10^0	0.8451
6×10^1	1.7782+
4.201×10^2	2.6233

= 420.1Nm

$$= \frac{1.852km \times 420.1Nm}{1Nm}$$

No	Log
1.852×10^0	0.2677
4.2×10^2	2.6232
7.925×10^2	2.8909

The distance between point X and Y is

(a)420 Nm

(b)793Km

Home Work

1.An air Craft took a height from town A(4^oN, 12^oE) moving southwards along a great circle for a distance of 2437Km. write the position of the town it landed.

Solution:

Distance = 2437Km

A = 4^oN, 12^oE

B = Required (5^o 12^oE)

$$2437\text{Km} = \frac{4^\circ + \beta}{360^\circ} \times 2(3.14)6370\text{Km}$$

$$4^\circ + \beta = \frac{2437 \times 360^\circ}{2(3.14)6370}$$

No	Log
2.437×10^3	3.3868
3.6×10^2	2.5563 +
	5.9431
6.28×10^0	0.7980
6.37×10^3	3.8041 +
	4.6021
	5.9431
	4.6021 –
	1.3410
2.193×10^1	
2.193×10	
21.93^0	

$4^o + \beta = 21.93^o$

$\beta = 21.93^\circ - 4^\circ$

=17.930

The position of town of Landed is 17.93^o

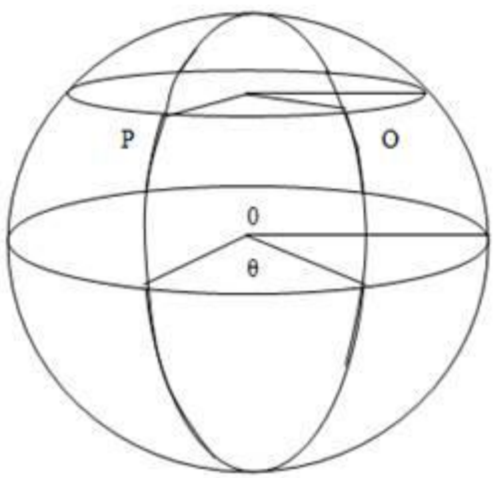
DISTANCE BETWEEN TWO POINTS ALONG THE PARALLEL OF LATITUDE.

Except the equator, other parallels of latitudes are small circles. Simply because their radii are less than the radius of the earth

Consider the point P and Q both found on the same parallel of latitude, let say α^oN and Q is the different in longitudes between P and Q.

R is the radius parallel latitude of α^oN

Diagram



Length at arc PQ = θ

Circumference of small circle 360°

$$\frac{PQ}{2\pi r} = \frac{\theta}{360^\circ}$$

But; r is the radius of small circle.

$$PQ = \frac{\theta}{360^\circ} \times 2\pi r \cos\alpha$$

But PQ is the distance between two points but θ = α ± β

Example

1.Calculate the distance between P(50°N, 12°W) and Q(50°N, 26°E)

Solution:

$$\text{Distance PQ} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R \cos\alpha$$

$$= \frac{12^\circ + 26^\circ}{360^\circ} \times 2(3.14)6370 \cos 50^\circ$$

$$= \frac{38^\circ}{360^\circ} \times 6.28 \times 6370 \times 0.6428$$

No	Log
$3.8\ 10^1$	1.5798
3.6×10^2	2.5563
	$\bar{1}.0235$
6.28×10^0	0.7980
6.37×10^3	3.8041
6.428×10^{-1}	$\bar{1}.8080$ +
2.714×10^3	3.4336
2.714 x 1000	
2714 x 1	
2714Km	

Home Work

1. Find the distance between point A(58°N, 23°E) and (58°N, 40°W)

   Solution

$$\text{Distance PQ} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R \cos\alpha$$

$$= \frac{23^\circ + 40^\circ}{360^\circ} \times 2(3.14)6370 \cos 50^\circ$$

$$= \frac{63^\circ}{360^\circ} \times 6.28 \times 6370 \times 0.6428$$

No	Log
$3.8\ 10^1$	1.5798
3.6×10^2	2.5563
	$\bar{1}.0235$
6.28×10^0	0.7980
6.37×10^3	3.8041
6.428×10^{-1}	$\bar{1}.8080$ +
2.714×10^3	3.4336
2.714 x 1000	
2714 x 1	
2714Km	

∴The distance between point A and B is 370Km

CLASS ACTIVITY

1. Calculate the distance from town P and Q along the parallel of latitude.

   If P (23°N,10°E) and Q (23°N, 54°E)

   Solution:

$$\text{Distance PQ} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R \cos\alpha$$

$$= \frac{54^\circ - 10^\circ}{360^\circ} \times 2(3.14)6370 \cos 23^\circ$$

$$= \frac{44^\circ}{360^\circ} \times 6.28 \times 6370 \times 0.9205$$

No	Log
4.4×10^1	1.6435
3.6×10^2	2.5563 –
	$\bar{1}.0872$
6.28×10^0	0.7980
6.37×10^3	3.8041
9.2015×10^{-1}	$\bar{1}.9640$ +
4.501×10^3	3.6533

4.501 x 1000 =4501 x 1

= 4501km

The distance from P to Q is 4501 Km

2. Two Towns both on latitude 45°S differ in longitude by 50°. Calculate the distance between two towns measured along the parallel of latitude.

Solution:

$$\text{Distance } = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R \cos\alpha$$

$$= \frac{50^\circ}{360^\circ} \times 2(3.14)6370 \cos 45^\circ$$

$$= \frac{50^\circ}{360^\circ} \times 6.28 \times 6370 \times 0.7071$$

No	Log
5.0×10^1	1. 6990
3.6×10^2	2.5563 –
	$\bar{1}.1427$
6.28×10^0	0.7980
6.37×10^3	3.8041
7.071×10^{-1}	$\bar{1}.8495$ +
3.929×10^3	3.5943

3.929 x 1000 = 3929 x 1

=3929Km

∴The distance between two towns is 3929km.

Example;

A plane flying at 595km/hour leaves Dar es Salaam (7°S, 39°E) at 8:00 am. When will it arrive at Addis Ababa at (9°N, 39°E)

Solution:

$$\text{Distance } = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$= \frac{7^\circ + 9^\circ}{360^\circ} \times 2(3.14)6370\text{km}$$

$$= \frac{16^\circ}{360^\circ} \times 6.28 \times 6370\text{km}$$

No	Log
1.6×10^1	1.2041
3.6×10^2	2.5563 –
	2.6478
6.28×10^0	0.7980
6.37×10^2	3.8041 +
1.778×10^3	3.2499

1.778 x 1000

1778km

The distance from Dar to Addis Ababa (Ethiopia) is 1778km

$$x = \frac{1778 \times 1}{595}$$

$$x = \frac{2.9}{10} \times 60$$

= 2:54

Since it spent 2:54 and left at Dar around 8:00 am now will reach Addis Ababa at 10:54am

CLASS ACTIVITY

1.An aero plane flies from Tabora (50S, 330E) to Tanga (5⁰S, 39⁰E) at 332 km/hour along parallel of latitude. If it leaves at Tabora at 3:00 pm. Find the arrival time at Tanga airport.

Solution.

$$\text{Distance} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R \cos \alpha$$

$$= \frac{39^\circ - 33^\circ}{360^\circ}\ 2 \times 3.14 \times 6370 \times \cos 5$$

$$= \frac{6^\circ}{360^\circ} \times 6.28 \times 6370 \times 0.9962$$

No	Log
6×10^0	0.7782
3.6×10^2	2.5563 –
	$\bar{2}.2219$
6.28×10^0	0.7980
6.37×10^3	3.8091
9.962×10^{-1}	$\bar{1}.9989$+
6.643×10^2	2.8224

664.3km

$\therefore$The distance between Tabora and Tanga is 664km.

$$x = \frac{664 \times 1}{332}$$

= 2 hours

Time arrival time at Tanga air port is 3:00pm + 2:00pm = 5:00pm

Home Work

1.A ship is Steaming in a eastern direction from town A to town B. if the position A is (32^oN, 136^oW) and B is (32^oN, 138^oW). What is the speed of sheep if it takes 3hours from town A to town B.

Solution:

$$\text{Distance } = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R \cos\alpha$$

$$= \frac{158^\circ - 136^\circ}{360^\circ}\, 2 \times 3.14 \times 6370 \times \cos 32^\circ$$

$$= \frac{22^\circ}{360^\circ} \times 6.28 \times 6370 \times 0.8480$$

No	Log
2.2×10^1	1.3424
3.6×10^2	2.5563-
	2.7861
6.28×10^0	0.7980
6.37×10^3	3.8041
8.480×10^{-1}	1.9284 +
2.019×10^3	3.3166

2.73 X 1000=2073Km

∴The distance from town A to B is 2073km.

$$x = \frac{2073 \times 1}{3}$$

∴The speed of sheep is 691km/hour.

In solving problems involving speed at ships, a term known as knot is usually used. By definition a speed of one nautical mile per hour is called knot
Therefore
1knote = 1Nm/hour = 1.852km/hour

Example: 1
When a ship is given 20knots is actually sailing at 20 nautical miles per hour or approximately 37 kilometer per hour.

Example: 2
A ship sails northwards to Tanga ($5^oS,39^oE$) at an average speed of 12 knots. If the ship starting points is Dar es Salaam (7^oS, 39^oE) at 12:00noon, when will it reach Tanga.

Solution: 1

How did 37km obtained.

1knote = 1.852km/hour

= 2 x 18.52km/hr

= 37.04km

= Approximately 37km

Solution: 2

$$\text{Distance} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$= \frac{7^\circ - 5^\circ}{360^\circ} \; 2 \times 3.14 \times 6370$$

$$= \frac{2^\circ}{360^\circ} \times 6.28 \times 6370$$

No	Log
2.0×10^0	0.3010
3.6×10^2	2.5563-
	3.7447
6.28×10^0	0.7980
6.37×10^3	3.8041+
2.222×10^2	2.3463
2.222 x 100	
222.2 km	

1knot = 1.852km/hr
12knot= x
1knot x = 12knots x 1.852km/hr

$$x = \frac{222 \times 1hr}{22}$$

X = 10.09
= 10:00
∴From 12:00 noon adding 10hours will sail at Tanga at 10:00pm.

Home Work

1. A ship is teaming at 15knots in western direction from Q to R. if the position of P is 40°S, 178°e and that of Q is 40°S, 172°E, how long will the journey take?

Solution:

$$\text{Distance} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R \cos\alpha$$

$$= \frac{178^\circ - 172^\circ}{360^\circ} 2 \times 3.14 \times 6370 \cos 40^\circ$$

$$= \frac{6^\circ}{360^\circ} \times 6.28 \times 6370 \times 0.7660$$

No	Log
6.0×10^0	0.7782
3.6×10^2	2.5563 -
	2. 2219
6.28×10^0	0.7980
6.37×10^3	3.8041
7.66×10^{-1}	$\bar{1}$.8842 +
5.107×10^2	2.7082
5.107 x 100	

∴510.7 km is the distance from point Q to R.

$$x = \frac{511 \times 1hr}{28}$$

= 18:12

∴The journey took = 18 hours and 12 minutes

Class Activity

1. A speed boat traveling from Zanzibar (6^oS, 45^oE) to Mwanza (9^oS, 45^oE) using 30knots left Zanzibar at 11:30am at what time did it reach at Mwanza?

Solution:

$$\text{Distance} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$= \frac{9^\circ - 6^\circ}{360^\circ}\ 2 \times 3.14 \times 6370$$

$$= \frac{3^\circ}{360^\circ} \times 6.28 \times 6370$$

No	Log
3.0×10^0	0.4771
3.6×10^2	2.5563
	3.9208
6.28×10^0	0.7980
6.37×10^3	3.8041 +
3.334×10^2	2.5229

3.334 x 100

333.4km

$$x = \frac{30 \times 1.852}{1}$$

$$x = \frac{333 \times 1hr}{56}$$

$$x = \frac{5.9}{10} \times 60$$

$\therefore$The distance from Zanzibar to Mtwara is 333.4km

= 5:54pm

Since it spent 5:54pm and left at Zanzibar at 11:30am. Now it will reach at Mtwara 5:24pm.

2. Find the time taken for a ship to sail from town P(80°N, 60°W) to town Q(60°S, 60°W) in 70knots

Solution:

$$\text{Distance} = \frac{\alpha \pm \beta}{360°} \times 2\pi R$$

$$= \frac{80° + 60°}{360°}\ 2 \times 3.14 \times 6370$$

$$= \frac{140°}{360°} \times 6.28 \times 6370$$

No	Log
1.4×10^2	2.1461
3.6×10^2	2.5563
	$\bar{1}.5898$
6.28×10^0	0.7980
6.37×10^3	3.8041 +
1.553×10^4	4.1919

1.553×10000

15530km

The distance from town P to town Q is 15530km

$$x = \frac{70 \times 1.852}{1}$$

=129.64km/hr

≈130km/hr

$$x = \frac{1553 \times 1hr}{130}$$

$$x = \frac{119.6}{13} \times 60$$

$= 119:36$

The ship will take 119hours and 36 minutes from town P to town Q

Class Activity.

1. A ship sails from A (0^o,20^oW) to B (10^oN, 20^oW) at 16 knots. If it leaves A at 8:00am on Tuesday when will it reach B?

Solution:

$$\text{Distance } = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$= \frac{0^\circ + 10^\circ}{360^\circ} 2 \times 3.14 \times 6370$$

$$= \frac{10^\circ}{360^\circ} \times 6.28 \times 6370$$

No	Log
1.0×10^1	1.0000
3.6×10^2	2.5563 –
	$\bar{2}.4437$
6.28×10^0	0.7980
6.37×10^3	3.8041+
1.111×10^3	3.0458

1.111 x 1000

1111km

∴The distance from town A to B is 1111km.

1knot = 1.852km/hour

16knots =?

$$x = \frac{16 \times 1.852}{1}$$

= 16 x 1.852km/hour

= 29.632km/hour

30km = 1hour

$$x = \frac{1111 \times 1\text{hr}}{30}$$

= 37hours

37 - 24hour = 13hours

8:00 + 24hr = 8:00am

8:00am + 13hours = 9:00am

The ship will reach town B at 9:00 am on Wednesday

Home Work

A ship sails from point A (10^oS, 30^oW) to B(10^oN, 30^oW) at 20 knots. If it leaves point A at 12:00 midnight on Monday when will arrive at B?

Solution:

$$\text{Distance} = \frac{\alpha \pm \beta}{360^\circ} \times 2\pi R$$

$$= \frac{10^\circ + 10^\circ}{360^\circ}\, 2 \times 3.14 \times 6370$$

$$= \frac{20^\circ}{360^\circ} \times 6.28 \times 6370$$

No	Log
2.0×10^1	1. 3010
3.6×10^2	2.5563 –
	$\bar{2}.7447$
6.28×10^0	0.7980
6.37×10^3	33.8041 +
2.222×10^3	3.3468

2.222 x 1000

2222km

The distance from point A to B is 2222km.

1kont = 1.852km/hour

20knot = x?

knot x = 20knot x 1.852km/hour

$$x = \frac{20 \times 1.852\text{km/hr}}{1}$$

≈37km/1hour

2222km = x?

37km x = 2222km x 1hour

$$x = \frac{2222 \times 1\text{hr}}{37}$$

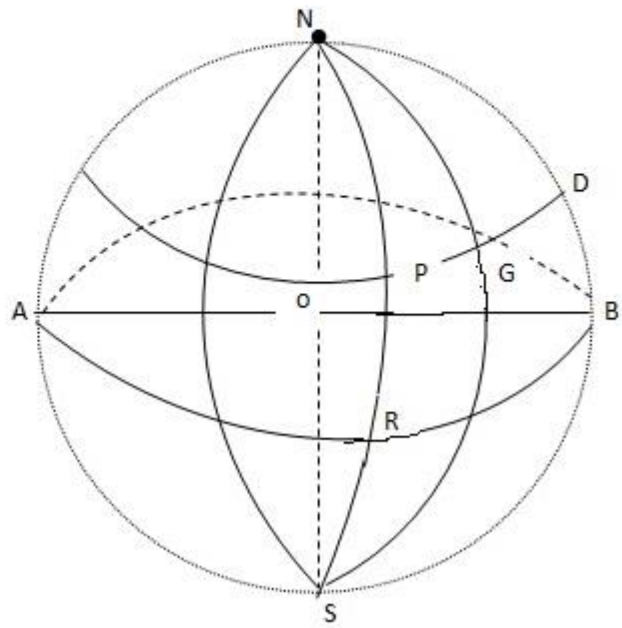
$$x = \frac{2222\text{hr}}{37}$$

=60hours

Since 12:00pm on Monday the ship has spend 2 and ½ days where by it will arrive at B at 12:00 afternoon on Thursday.

THE EARTH AS A SPHERE

The earth surface is very close to being a sphere. Consider a sphere representing the shape of the earth as below.



NS is the Axis of the earth in which the earth rotates once a day.

O is the center of the earth.

The radius of the Earth is 6370 Km.

GREAT CIRCLES.

Is that which is formed on the surface of the earth by a plane passing through the center of the Earth. Its radius is equal to the radius of the Earth.

Examples;

ARB and ANB.

The equator is also a great circle.

NDBS, NGS and NPRS all are meridians. (Longitude).

EQUATOR: Is the line that circles the Earth midway between the North and the South Pole.

PRIME MERIDIAN (**:** is a meridian (longitude) which passes through Greenwich, England.

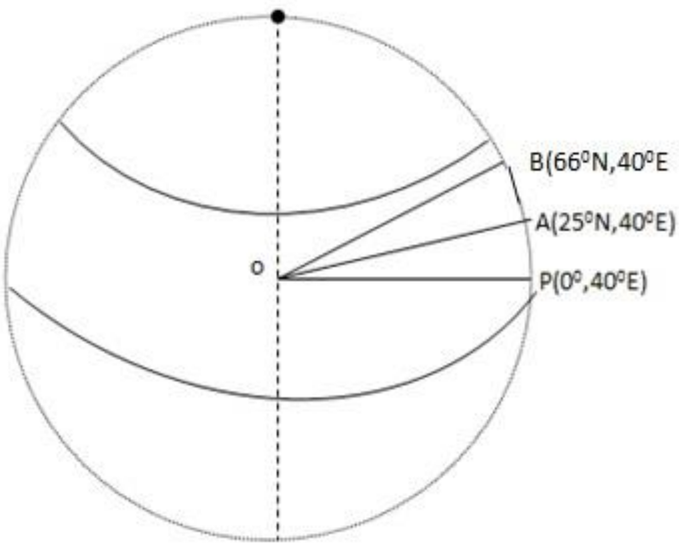
SMALL CIRCLES : is one formed on the surface of the Earth by a plane that cuts through the Earth but does not pass through the centre of the Earth. Eg. ARB.

LATITUDE: The angular distance of a place north to south of the earth's equator or of a celestial object north or south of the celestial equator. It is measured on the meridian of the point.

Example 1.

CASE 1

1. 1. Points A and B have the same longitudes but different latitudes. Point A (66° N, 40^{0}E) B, Point B (25°N ,40^{0}E)A. Find the angle subtended at the center of the Earth by arc AB if A is (25^{0}N,40^{0}E) and B is (66°s ,40^{0}E)



2

AOB =BOP- AOP

AOB =66^0- 25°
=41^0

Exercise 1

In questions 1 to 4 consider the town and cities indicated an d the answer the questions that follow.

1. Which of the following towns and cities lie on the same meridian?

•Tabora (5^0s, 33^0E) Dar-es salaam (7^0s, 39^0E) Mbeya (9^0 s,33^0E) Chahe Chahe (5^0s, 40^0 E), Tanga (5^0S, 39^0E) Moshi (3^0 s, 37^0 E) , Zanzibar (6^0 S, 40^0 E),Mwanza (3^0 S, 33^0 E) Morogoro (7^0 S, 38^0 E), Nakuru (0^0, 36^0 E) , Kampala (0^0,33^0E) , Gulu (3^0N,32^0E)

2. Which town and cities have latitudes like that of;

a. a) Moshi?

Mwanza

b. b) Chahe Chahe?

Tanga and Tabora

3. Which towns and cities have longitudes like that of

a. Mbeya? Tabora , Mwanza and Kampala.

b. Dar-Es-Salaam? Tanga

4. Find the angle subtended at the center of the Earth by arc AB if A is Mwanza and B is Mbeya.
5. Find the angle subtended at the centre of the Earth by arc XY if X is Nakuru and Y is Kampala

Solution

1. A Mwanza (3^0 S, 33^0 E)

B Mbeya (9^0 S, 33^0 E)

SOB = BOA- AOS

SOB= $9^0 – 3^0$

SOB = 6^0

5. c. X Nakuru (0^0 , 36^0 E)

d. Y Kampala (0^0 , 33^0 E)

e. TOX = XOY –TOY

f. TOX= $36^0 – 33^0$

g. TOX = 3^0

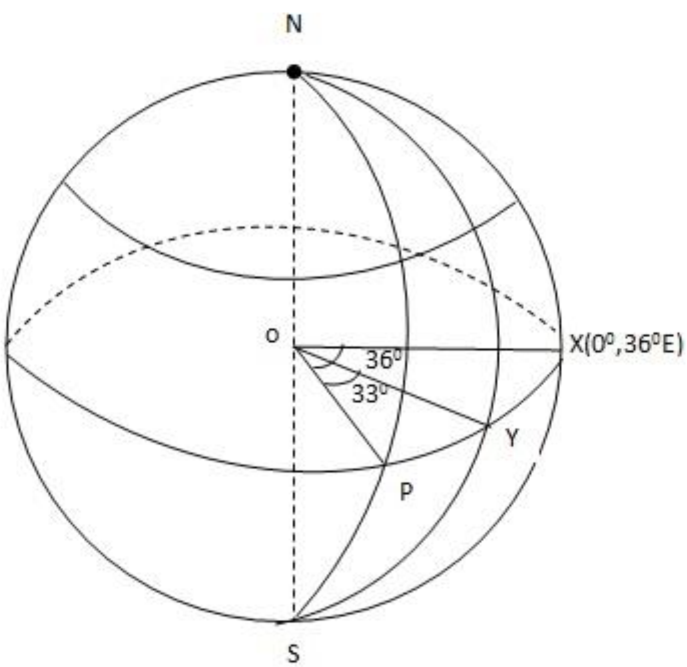
CASE 2

Point A and B are on the same latitude but different longitudes.

Examples

1. Find the angle subtended at the center of the Earth by arc XY if X is Nakuru (0^0, 36^0 E) and Y is Kampala (0^0 , 33^0 E)

XOY = XOP- YOP



$= 36^0 – 33^0$

$= 3^0$

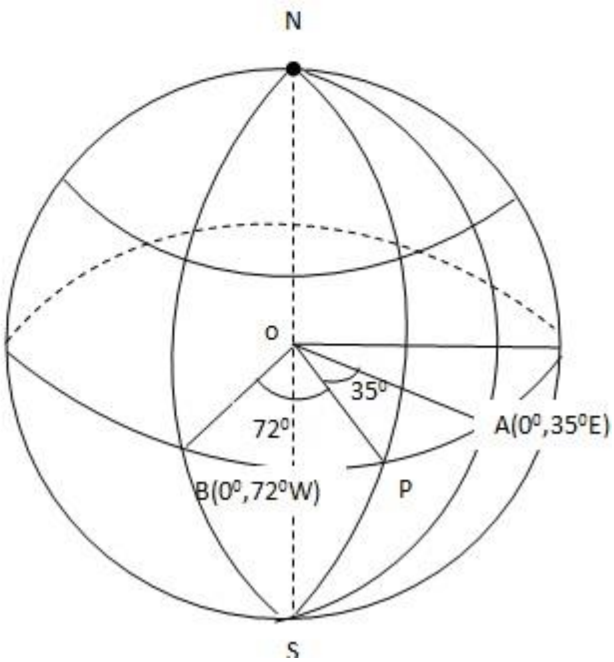
2. Two towns A and B are on the equator.The longitude of A is 35^0 E and that of B is 72^0 W. Find the angle subtended by the arc AB at the center of the Earth.

Solution

AOB = AOP + POB

$= 35^0 + 72^0$

$= 107^0$



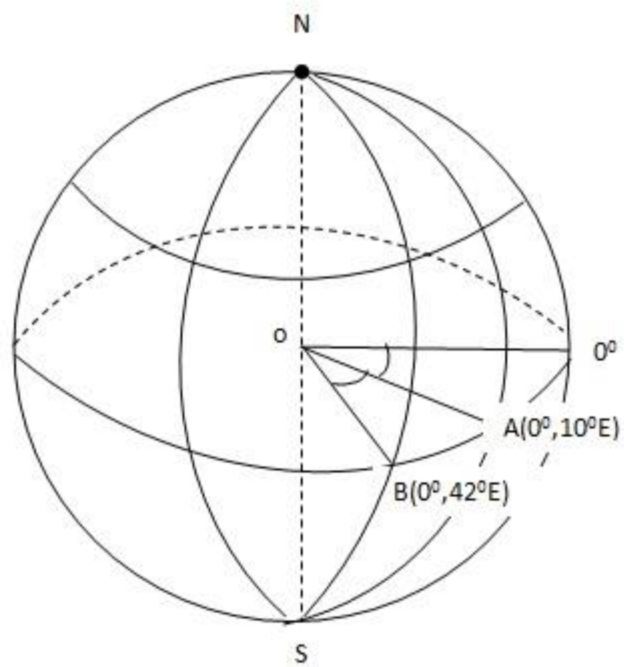
3. Two towns A and B in Africa are located on the Equator. The longitude of A is 10^0 E and that of B is 42^0 E . Find the angle subtended by the arc AB at the center of the Earth.

   Solution

AOB = SOB – SOA

$= 42^0 – 10^0$

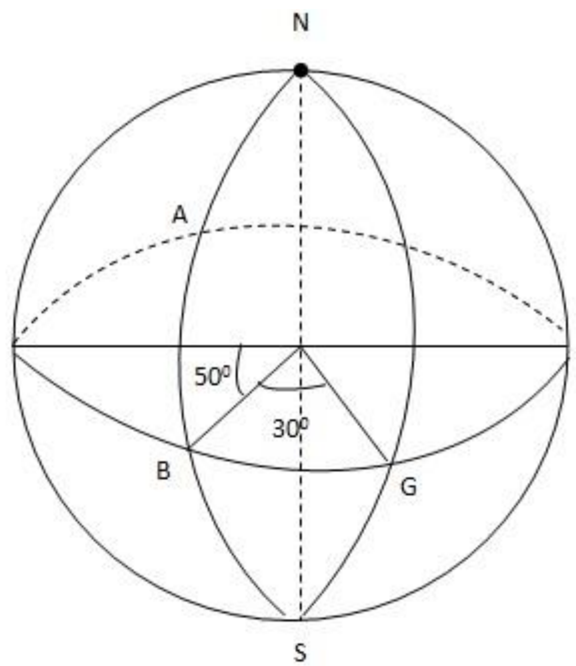
$= 32^0$



The angle formed by arc AB is 32^0

Exercise 2

1. In the figure below, if the center through N,G,S is the prime meridian , the center of the Earth and the Equator passes through B and G , the longitude and latitude of A.



Longitude of A = (0^0 , 30^0 W)

Latitude of A = (50^0 N, 0^0)

There after draw a figure similar to that of question 1 to illustrate the position of point H (60^0S,45^0E).

2. P and Q are towns on latitude 0^0 . if the longitude of P is 116^0 E and that of Q is 105^0 W, find the angle subtended by the arc connecting the two places at the center of the Earth . Draw a figure to illustrate their positions.

LENGTH OF A GREAT CIRCLE.

Distance on the surface of the Earth are usually expressed in nautical miles or in kilometers. A nautical mile is the length of an arc of a great circle that subtends an angle of 1 minute at the center of the Earth.

The length of arc AB is 1 nautical mile;

1^0 = 60 minutes
1^0 = 60 nautical miles

For a great circle, angle at the center if the Earth is 360^0 .

Length of a great circle.
1^0 = 60 nautical miles
360^0 = ?
$360^{0 \times} 60^0$ = 21600 nautical miles.

Therefore, equator and all meridians are great circles , distance (length) is equal to 21600 nautical miles.

In kilometers.

1 nautical mile = 1.852 Km
21600 nautical miles = ?
21600x 1.852 = 40003.2 km

OR

We can use the following formula to find the distance.(length)

C= 2π r

Where “r” is the radius of the Earth.

r= 6370
π=3.14
C= 2x3.14x6370

C= 40003.6 km

Example

1. If the latitude of Nakuru is O^0 , find the distance (length) in nautical miles from this town to the North Pole.

Solution.

From O^0 to North Pole , the angle is 90^0

$1^0 = 60^0$ nautical miles.

$90^0 = ?$

$90^0 \times 60^0 = 5400$ nautical miles

1 nautical mile = 1.852 km
5400 nautical mile = ? km

Distance = 1. 852 x 5400
= 10000.8km

2. calculate the distance of the prime meridian from south to North pole in

a. nautical mile

b. kilometers.

Solution

a. $1^0 = 60$ nautical miles.

180^0 = ? nautical miles

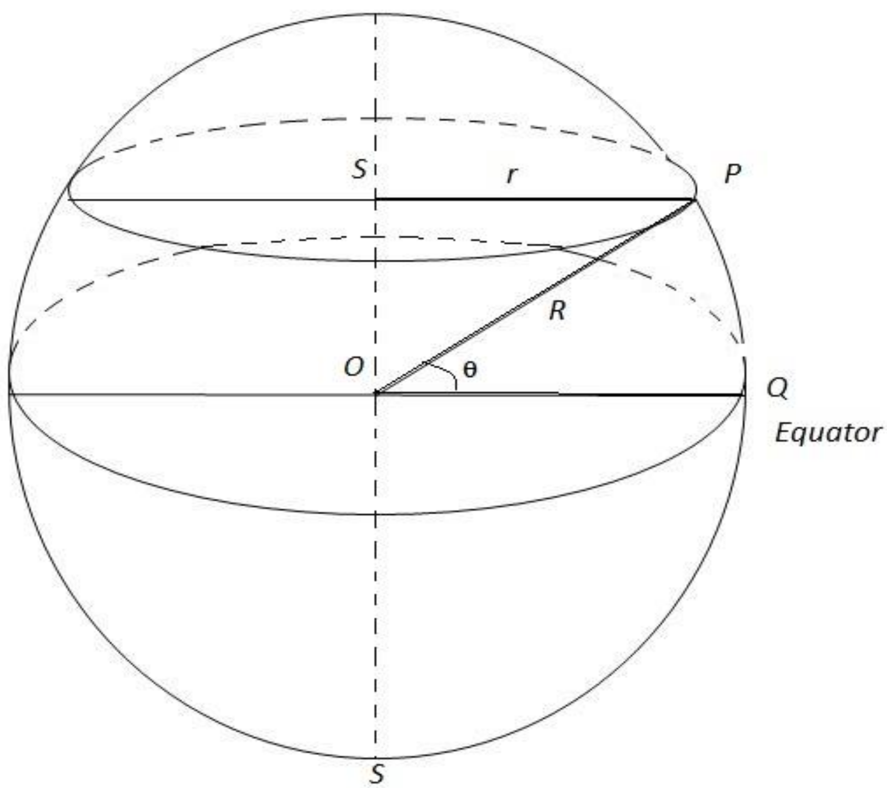
$180^0 \times 60^0$ = 10800 nautical miles.
∴ The distance of prime meridian from south to north pole is 10800Nm

b. 1 nautical mile = 1.852 km
10800 Nautical miles =?km

10800x 1.852= 20001.6 km
∴ The distance of prime meridian from south to north pole is 20001.6 km

3. Calculate the distance of the equator from east to West in Nautical Miles.
1^0 = 60 nautical miles.
180^o ?
$180^0 \times 60^0$= 10800 nautical miles.
∴ The distance of the equator from East to West in Nm is given by 10800Nm

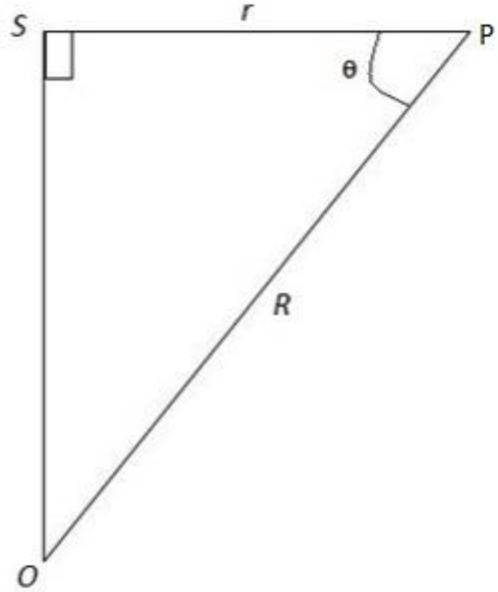
Length of small circle.



Let P be any point on the surface of the earth through this point a small circle is drawn with parallel of latitude θº as shown above. The radius of the earth as R and the radius of the parallel latitude (r) are both perpendicular to the polar axis.
Note: SP is parallel to OQ (Both are perpendicular to NS)

$\therefore \theta^{\circ}$ = OPS

Then we have



From trigonometrical ratios

r= R cos θ where R is the radius of the earth and r is the radius of the small circle of latitude θ.

∴ Distance of parallel of latitude θ = 2πr

= 2πRcos θ

Example 1.

1. Calculate the circumference of a small circle in kilometers along the parallel of latitude 10^0 S.

   Soln

   C = 2πR cos θ

   = 2x 3.14x 6370x cos 10^0

   =39395.54528km

2. Calculate the length of the parallel of the latitude through Bombay If Bombay is located 19^0 N, 73^0 E

   C= 2πR cos θ

   = 2x3.14x6370xcos19^0

   =37823.40km

In nautical miles.

3782km/1.852km/miles = 20423 nautical miles.

Exercise

In the questions below , take the radius of the Earth , R= 6370km and π = 3.14

1. The city of Kampala lies along the equator. Calculate the distance in kilometers from the city of Kampala to the South Pole

2. How far is B from A if A is 0^0 ,0^0 and B is 0^0, 180^0 E.

3. What is the latitude of a point P north of the Equator if the length of the parallel of the latitude through p is 28287 kilometers.(give your answer to the nearest degree.

4. What is the radius of a small circle parallel to the equator along latitude 70^0 N

Solution

1. 1^0= 60 nautical miles.

90^0 = ?

60^0 x90^0 = 5400 nautical miles.

In kilometers .

5400nm x 1.852km/nm= 10000.8km

2.

1^0 = 60nm

180^0 = ?

= 180^0 x 60^0 = 10800 nm

1nm = 1.852 km

10800nm =?

10800x 1.852= 20001.6 km

3.

C= 2πR cos θ

28287= 2 x 3.14x 6370 x Cos θ

Cos θ = 0.7071

θ= 45º

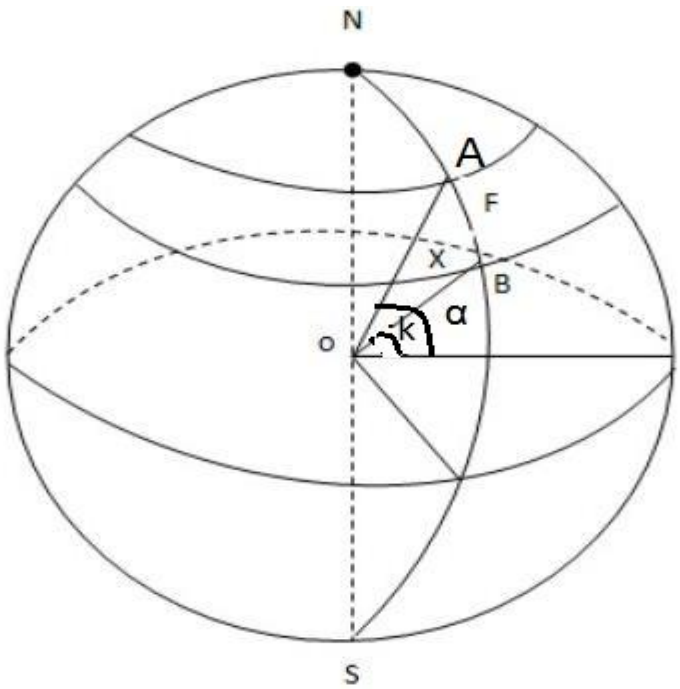
4.

r = Rcos θ

= 6370 x 0.3420

=2178.4 km

Distance between points along the same meridian.



A (60^0 N , 30^0 E)

B (20^0 N , 30^0 E)

K=20^0

∝= 60^0

= 600 - 200

1^0 = 60 nautical miles

40^0 = ? Nm

40^0 x 60^0 = 2400 nautical miles

Examples

1. 1. Find the distance between A (30^0 N, 139^0 E) and B (45^0 N , 139^0 E) in

a. Nautical miles.
b. Kilometers.

Solution

Since A and B have the same longitudes , they are on the same meridian. The difference between their latitudes is

= $(45-30)^0$

=15^0

=15^0 x 60 nm/ ^{0} = 900 nautical Mile

OR

Distance AB = πRθ/ 180^0

= 1666.8 km

1nautical mile = 1.852 km

= 900 nautical miles

2. 2. Find the distance in kilometers between A (9^0 S, 33^0 E) and B (8^0 S, 33^0 E)

Solution

The points have the same longitudes but their latitudes differ.

The difference in latitude is

=9^0 - 8^0

= 1^0

$1^{0=}$ 60 nm

The distance is 60nm

3. 3. Find the distance in nautical miles on the same meridian with latitude

a. 10^0 N, 35^0 N

b. 20^0 N, 42^0 S

Solution

a. $35^0 - 10^0 = 25^0$

$1^0 = 60$ nm

$25^0 = ?$

60x25= 1500 nm

b. $20^0 + 42^0 = 62^0$

$1^0 = 60$ nm

$62^0 = ?$

60x 62 = 3720 nm

4. 4. Find the distance AB In nautical miles between each of the following pairs of places.

a. A (18^0 N, 12^0 E)
B (65^0 N, 12^0 E)

$(65-18)^0 = 47^0$

1^0 = 60 nautical miles

$47^0 = ?$

60 x 47^0 = 2820 nautical miles

b. A(31^0 S, 76^0 W) and B (22^0 N , 76^0 W)
31^0 S + 22^0 N = 53^0

$1^0 = 60$ nm

$53^0 = ?$

=3180nm

5. Find the distance in kilometers between;

a. Tanga (5^0 S, 39^0 E) and Addis Ababa (9^0 N, $39\ ^0$E)
Solution:

$(9+5) = 14^0$

$1^0 = 60$ nm

$14^0 = ?$

=840 nautical miles

1nm = 1.852 km

840nm =?

Distance = 1555.68 km

b. Mbeya (9^0 S, 33^0E) and Tabora (5^0S, 33^0E)
Soln.

$(9-5)\ ^0 = 4^0$

$1^0 = 60$ nm

$4^0 =?$

= 240nm

=240 nmx 1.852km/nm

=444.48 km

6. A ship sails northwards to Tanga (5^0 S, 39^0 E)at an average speed of 12 nm/ hr. If the ships starting point is Dar Es salaam (7^0S, 39^0E) at 12:00 noon , when will it reach Tanga?

Solution

$(7-5)\ ^0 = 2^0$

$1^0 = 60$ nm

$2^0 = ?$

Distance = 120 nm

Velocity = 12nm/hr

$$Velocity = \frac{distance}{time}$$

$$Time = \frac{120}{12}$$

Time = 10 hours

It will reach tanga at 10:00 pm

7. A plane flying at 595 km/hr leaves dar-es-salaam (7^0 s , 39^0 E) at 8:00 am. When will it arrive at Addis Ababa (9^0 N, 39^0 E)?

Solution

9^0 N + 7^0 S = 16^0

1^0 = 60 nm

16^0 =?

Distance = 960 nm=1777.92km

$$Time = \frac{distance}{velocity}$$

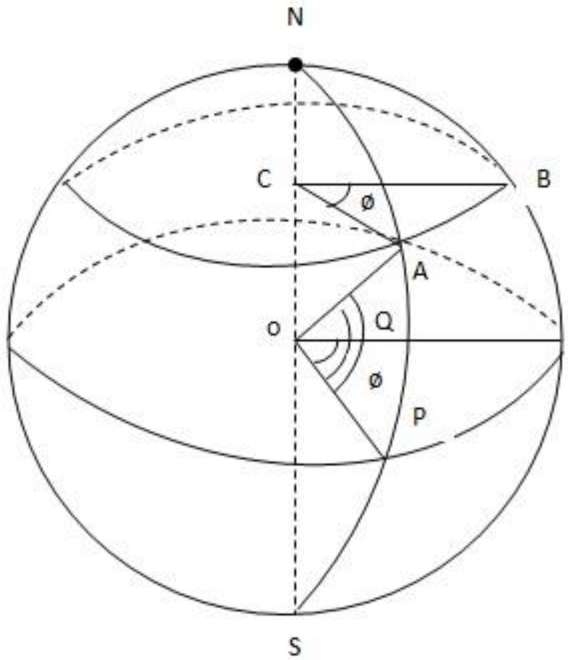
$$= \frac{1777.92}{595}$$

Time = 2.988 hours

It will arrive at 11: 00 am

DISTANCE BETWEEN POINTS ALONG THE PARALLEL OF LATITUDES.

Consider the figure below, points A and B are two points having the same latitude 0^0, since both lie on the parallel of latitude but they are different in their longitudes i.e. That point A is on a different longitude from that of point B. the difference between their longitudes is θ.



$360^0 = 2\pi r$

$\theta = ?$

$$\therefore length\ of\ arc\ AB = \frac{2\pi r\theta}{360} = \frac{\pi r\theta}{180}$$

Example.

1. A ship is streaming in a western direction from Q and P. if the position of P is (40^0 S, 178^0E) and that of Q (40^0 S, 172^0E). how far does the ship move from Q to P?

Solution

Difference in longitude = 178-172= 6^0

$$Arc\ length = \frac{2\pi \times 6370 \times 6 \times \cos 40}{360} = 510.7Km$$

Exercise

1. Two points on latitude 50^0 N lie on longitudes 35^0E and 40 ^{0}W. what is the distance between them in nautical miles.

2. An airplane flies westwards along the parallel of latitude 20^0 N from town A on longitude 40^0 E to town B on a longitude 10^0W. find the distance between the two towns in kilometers.

3. An aeroplane flies from Tabora (5^0S, 33^0E) to Tanga (5^0S, 39^0E) at 332 kilometers per hour along a parallel of latitude. If it leaves Tabora at 3 pm, find the arrival time at Tanga airport?

4. The location of Morogoro is (7^0 S , 38^0E) and that of Dar-Es-Salaam is (7^0S, 39^0E). find the distance between them In kilometers.
5. A ship after sailing for 864 nautical miles eastwards find that her longitude was altered by 30^0. What parallel of the latitude is the ship sailing?
6. An aeroplane takes off from B (55^0 S, 33^0E) to C (55^0 S, 39^0E) at a speed of 332 km/ hr . if it leaves B at 3:00 pm , at what time will it arrive at C airport?
7. A ship sails due North from latitude 20^0 S for a distance of 1440km. find the latitude of the point it reaches.

Solution

1.

$35^0E + 40^0W = 75^0$

$1^0 = 60 \cos \theta$

$75^0 = ?$

$60^0 x\ 75^0 x \text{ Cos } 45º = 2892.6$ nm

2.

$10^0W + 40\ ^0E = 50^0$

$1^0 = 60 \cos 20^0$

$50\ ^0 = ?$

= 2819.1 nm

1nm =1.852 km

2819.1nm= ?

Distance = 5221km

3.

$(39\text{-}33)^0 = 6^0$

$1^0 = 60^0 \cos\theta$

$6^0 = ?$

$= 6^0 \text{x } 60 \text{ xcos } 5^0$

=359nm

Velocity = 332 km /hr

Distance = 664.86 km

$$Time = \frac{distance}{velocity}$$

$$= \frac{665}{332}$$

Time = 2 hrs

Arrival time = 5:00 pm

4.

Longitude difference = $39^0 - 38^0 = 1^0$

$$Distance\ between\ the = \frac{1 \times 2\pi \times 6370 \times 6 \times \cos 7}{360}$$

= 110.29Km

5.

L = 864 nm =1600.128km

$$L = \frac{30 \times 2\pi \times 6370 \times 6 \times \cos\theta}{360}$$

$$\cos\theta = \frac{1600.128 \times 360}{30 \times \pi \times 2 \times 6370}$$

Cos θ = 0.4799952

θ = 61°

6.

$(39-33)^0 = 6^0$

$1^0 = 60 \cos\theta$ nm

$6^0 = ?$

Where $\theta = 55^0$

= 358.632nm

But 1 nm = 1.852 km

359nm =?

Distance =665 km

$$Time = \frac{distance}{velocity}$$

$$= \frac{665}{332}$$

Time = 2 hours

3:00 pm + 2 hours = 5:00 pm

7.

1 nm = 1.852 km

? = 1440 km

= 777.54 nm

$1^0 = 60$ nm

? = 777.54 nm

=12.95 ^{0}

The latitude it reaches will be 12.95 ^{0}

ACCOUNTS

Accounts is a place in a ledger where all the transactions are relating to a particular asset, liabilities and capital, expenses,or revenue items were recorded.
Accounts is a wider concept with identifying measuring and communicating economic information to permit informed judgments and decisions by users of the information. The part of accounting that is concerned with recording data is known as **book keeping**.

Book keeping - is the art of recording financial business transactions on a set of books in terms of money or money's worth.

<u>Why do we need to keep business records?</u>

- To determine whether the business is making profit or loss.
- To determine financial strength of the business.
- To enable the government to access reliable resources.
- To enable different stake holders to make reliable decisions about the business i.e. investors, bankers, customers etc.

THE DOUBLE ENTRY SYSTEM

Before looking at the system of double entry let us look at the meaning of business transactions.
Transactions means movements of money,goods or services from one part/person to another. **For example**: Selling goods in cash Tshs 10,000/= or selling goods worth Tshs 10,000 on credit to Aisha.

The double entry system - Is the process of recording these business transactions twice.

LEDGERS

A ledger is a main principle book of accounts in which business transactions are recorded in double entry system.

The ledger contains section called "**Accounts**" which contain detail of transactions for specific items each account bears a title and a number called folio i.e. page of the ledger.

Each account should be shown on a separate page. The double entry system divides each page into two values;

a. The left hand side is called the **debit side**
b. The right hand side is called the **credit side**.

The title of each account is written across the top of the account at the center so double entry system needs every debit entry should have corresponding credit entry. An example of a ledger;

NAME OF THE ACCOUNTS

Date	**Particulars**	**folio**	**Amount**	**Date**	**particular**	**folio**	**Amount**

Each side of account should have four column i.e. date, particulars, folio and amount.

Use of columns;

i) **Date column**
For writing year, month and date.

ii) **Particulars**
For short descriptions of the transactions

iii) **Folio column**
For pages of reference

iv) **Amount column**
For writing amount of money

Worked examples on how to record transactions in double entry system;

1. Juma starts business with Tshs 20,000/= in cash on 1st January 2006. The transaction needs to debit cash account and credit capital account as follows**.**

Dr **CASH ACCOUNT** **Cr**

Date	**Particular**	**folio**	**Amount**	**Date**	**particular**	**folio**	**Amount**

1/1/2006	capital		20,000				

Dr **Capital account** **Cr**

Date	Particular	folio	Amount	Date	particular	folio	Amount
				1/1/2006	Cash		20,000

2. John started a business on 1st Jan 2000 with capital at Tshs 5,000,000 in cash

January

2) 2 purchased goods and paid in cash T.shs. ………… 1,000,000/=

3) 3 Brought goods for cash. ………………… 500,000/=

5 paid wages in cash …………… 50,000/=

7 sold goods in cash …………… 300,000/=

8 brought goods in cash ………. 800,000/=

9 received loan from C.R.D.B………….. 70,000/=

12 bought parking materials in cash…….. 20,000/=

28 paid transport charges ……… 30,000/=

28 drew cash for burial Tshs……….. 10,000/=

Enter the above transaction in the ledger and complete the double entry

Solution

Dr CASH ACCOUNT Cr

Date	Particulars	Folio	Amount	Date	Particular	Folio	Amount
1/1/2000	Capital	P1	5,000,000	2/1/2000	Purchase	P2	1,000,000
7/1/2000	Sales	P3	300,000	3/1/2000	Purchases	P2	500,000
9/2/2000	Loan	P5	70,000	5/1/2000	Wages	P4	50,000
28/1/2000	Drawing	P8	10,000	8/1/2000	Purchases	P2	800,000
				12/1/200	P material	P6	20,000
				28/1/200	Transport	P7	30,000
				31/1/200	Balance	c/d	2,980,000
			5,380,000				5,380,000
1/2/2000	Balance	b/d	2,980,000				

Dr CAPITAL ACCOUNT. P1 Cr

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
31/1/2000	Balance	c/d	5,000,000	1/1/2000	Cash	P1	5,000,000
				1/2/2000	Balance	b/d	5,000,000

Dr PURCHASE ACCOUNT. P2 Cr

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
2/1/200	Cash	P1	1,000,000	31/1/2000	Balance	c/d	2,300,000
8/1/2000	Cash	P1	500,000				
8/1/2000	Cash	P1	800,000				
			2,300,000				2,300,000
1/2/2000	Balance	b/d	2,300,000				

DR SALES ACCOUNT. P3 Cr

Date	Particular	Folio	Amount	Date	Particular	Folio	Amount
31/1/2000	Balance	c/d	300,000	7/1/200	Cash	P1	300,000
				1/2/2000	Balance	b/d	300,000

Dr WAGES ACCOUNT. P4 Cr

Date	particulars	Folio	Amount	Date	Particulars	Folio	Amount
5/1/2000	Cash	P1	50,000	31/1/2000	Balance	c/d	50,000
1/2/2000		b/d	50,000				

Dr LOAN ACCOUNT. P5 Cr

Date	Particular	Folio	Amount	Date	Particular	Folio	Amount
31/1/2000	Balance	c/d	70,000	9/1/200	Cash	P1	70,000
				1/2/2000	Balance	b/d	70,000

Dr P. MATERIAL ACCOUNT. P6 Cr

Date	Particular	Folio	Amount	Date	Particular	Folio	Amount
12/1/2000	Cash	P1	20,000	31/1/2000	Balance	c/d	20,000
1/2/2000	Balance	b/d	20,000				

Dr TRANSPORT ACCOUNT. P7 Cr

Date	Particular	Folio	Amount	Date	Particular	Folio	Amount
28/1/2000	cash	P1	30,000	31/1/2000	Balance	c/d	30,000
1/2/2000	Balance	b/d	30,000				

Dr DRAWING ACCOUNT. P8 Cr

Date	Particular	Folio	Amount	Date	Particular	Folio	Amount
31/1/2000	Balance	c/d	10,000	28/1/2000	Cash	P1	10,000
				1/2/2000	Balance	b/d	10,000

EXERCISE

1. Mark commenced business on 1st June 2001 with capital Tshs. 1,000,000.

June 2 bought for cash………	500,000/=
2 paid transport charges………….	50,000/=
3 bought parking charges………….	10,000/=
4 sold goods for cash …………	300,000/=
5 sold goods for cash…………..	100,000/=
6 purchased goods and paid cash…	180,000/=
8 paid wages …….	38,000/=
10 cash sales……	200,000/=
12 cash purchases…………	180,000/=
15 cash sales to date ……….	250,000/=
20 paid rent …………	50,000/=

Enter the above transactions in the cash account complete the double entry balance the accounts at the end of the month and extract a trial balance

Dr CASH ACCOUNT. P1 Cr

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
1/6/2001	Capital	P2	1,000,000	2/6/2001	Purchases	P4	500,000
4/6/2001	Sales	P5	300,000	2/6/2001	Transport	P6	50,000
5/6/2001	Sales	P5	100,000	3/6/2001	Parking	P3	10,000
10/6/001	Sales	P5	200,000	6/6/2001	Purchases	P4	180,000
15/6/001	Sales	P5	***250,000***	8/6/2001	Wages	P7	38,000
				12/6/001	Purchases	P4	***180,000***
				20/6/001	Rent	P8	50,000
				30/6/001	Balance	c/d	***842,000***
			1,850,000				1,850,000
1/7/2001	Balance	b/d	***842,200***				

Dr CAPITAL ACCOUNT. P2 Cr

Date	Particular	Folio	Amount	Date	Particulars	Folio	Amount
30/6/2001	Balance	c/d	1,000,000	1/6/2001	Cash	P1	1,000,000
				1/7/2001	Balance	b/d	1,000,000

Dr PACKING ACCOUNT. P3 Cr

DATE	PARTICULAR	FOLIO	AMOUNT	DATE	PARTICULAR	FOLIO	AMOUNT
3/6/001	Cash	P1	10,000	30/6/2	Balance	c/d	10,000
3/7/001	Balance	b/d	10,000				

Dr **PURCHASES ACCOUNT. P4**

Date	Particulars	Folio	Amount	Date	Particulars	Folio
2/6/2001	Cash	P1	500,000	30/8/2001	Balance	c/d
6/6/2001	Cash	P1	180,000			
12/6/001	Cash	P1	180,000			
			860,000			
1/7/2001	Balance	b/d	860,000			

Dr **SALES ACCOUNT. P5** Cr

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
30/6/001	Balance	c/d	850,000	4/6/2001	Cash	P1	300,000
				5/6/2001	Cash	P1	100,000
				10/6/2001	Cash	P1	200,000
				15/6/2001	Cash	P1	250,000
			850,000				850,000
				1/7/2001	Balance	b/d	850,000

Dr **TRANSPORT ACCOUNT. P6** Cr

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
2/6/2001	Cash	P1	50,000	30/6/2001	Balance	c/d	50,000
1/7/2001	Balance	b/d	50,000				

Dr WAGES ACCOUNT. P7 Cr

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
8/6/2001	Cash	P1	38,000	30/6/2001	Balance	c/d	38,000
1/6/2001	Balance	b/d	38,000				

Dr **RENT ACCOUNT. P8** **Cr**

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
20/6/2001	Cash	P1	50,000	30/6/2001	Balance	c/d	50,000
1/7/2001	Balance	b/d	50,000				

TRIAL BALANCE AT THE END OF THE MONTH 30TH JUNE 2001

NO.	Details	Debit	Credit
1	Cash	872,000	
2	Capital		1,000,000
3	Purchases	830,000	
4	Transport	50,000	
5	Packing	10,000	
6	Sales		850,000
7	Wages	38,000	
8	Rent	50,000	
	Total	1,850,000	1,850,000

FIRMS NAME

DR Trading account for the year ended CR

Opening stock xxx	Sales xxx
Add purchases xxx	Less. RIN xxx
Less outwards xxx	Net sales xxx
Net purchases xxx	
Cost of goods available	
For sale xxx	
Less : closing stock xxx	
Cost of goods sold xxx Gross profit c/d xxx Xxx	Xxx

PROFIT AND LOSS ACCOUNT

This is an account prepared in order to ascertain the net profit and loss by the business. All expenses are debited to this account while gains or profits are credited to this account and debited with all incurred expenses.

Note

If the business has made profit it increases the capital and if it has suffered loss it reduces the capital.

It's lay out;

FIRM'S NAME

DR PROFIT AND LOSS ACCOUNT FOR THE YEAR ENDED CR

Expense	xxx	Gross profit b/d from trading account	xxx
Net profit	xxx	Other gains [income]	xxx
			xxx
	xxx		

Note

The excess of expenses over incomes is formed as **net profit.**

Example

1. From the following trail balance you are required to prepare a trading profit and loss account. For the year ended 30th June 2006 B Samanga.

Trial balance as at 30th June 2006

s/n	NAME OF THE ACCOUNT	DEBIT	CREDIT
1	Sales		3,850
2	purchases	2,900	
3	Rent	240	
4	Lighting expenses and general expenses	150	
5	Fixtures and fittings	60	
6	Debtors	300	
7	Salaries	680	
8	creditors		910
9	Cash in hand	1,710	
10	Bank	20	
11	Drawings	700	
12	Capital		2000

Note
Closing stock was valued at Tshs 300. B Samanga and profit account.

DR TRADING ACCOUNT B SAMANGA AS AT 30TH JUNE 2006 CR

Opening stock 0000	Sales 3,850
Add purchases 2,900	R.Inward 0000
Less R outwards - 0000	Net sales 3,850
Net purchases 2,900	Add all receivables
Cost of goods available for sale 2900	
Less closing stock -300	

Cost of goods sold 2,600	
Gross profit c/d 1,250	
3,850	3,850
	Balance b/d 1,250
Expenses	
Rent 240	Gross profit 1,250
L and G 150	
Salaries 680	
Total expenses 1,070	
Net profit/balance c/d 180	
1,250	1,250
	Balance b/d net profit 180

BALANCE SHEET FOR THE YEAR ENDED 30TH JUNE 2006

ASSETS	**LIABILITIES**
Fixed assets	**Long term liabilities**
Fixture and fittings 60	Capital 2,000
Current assets	Creditor 910
Stock 300	Net profit 180

Debtors 300	3,090
	Less drawing - 700
Bank 20	
Cash hand 1,710	
Total current assets 2,330	
Total fixed asset 2,390	2,390